AGARD-AG-313

AGARD

ADVISORY GROUP FOR AEROSPACE RESEARCH & DEVELOPMENT

7 RUE ANCELLE 92200 NEUILLY SUR SEINE FRANCE

AGARDograph No.313

The Anatomy of the Gyroscope
Part III

NORTH ATLANTIC TREATY ORGANIZATION

DISTRIBUTION AND AVAILABILITY
ON BACK COVER

AGARD-AG-313

NORTH ATLANTIC TREATY ORGANIZATION

ADVISORY GROUP FOR AEROSPACE RESEARCH AND DEVELOPMENT

(ORGANISATION DU TRAITE DE L'ATLANTIQUE NORD)

AGARDograph No.313

THE ANATOMY OF THE GYROSCOPE – PART III

by

Frank W.Cousins
43 Emmanuel House
18 Rochester Row
London, SW1
United Kingdom

Edited by

John L.Hollington
Smiths Industries/Aerospace
and Defence Systems Company
Bishop's Cleeve
Cheltenham, Glos GL52 4SF
United Kingdom

This AGARDograph has been prepared and edited at the request
of the Guidance and Control Panel of AGARD.

THE MISSION OF AGARD

According to its Charter, the mission of AGARD is to bring together the leading personalities of the NATO nations in the fields of science and technology relating to aerospace for the following purposes:

— Recommending effective ways for the member nations to use their research and development capabilities for the common benefit of the NATO community;

— Providing scientific and technical advice and assistance to the Military Committee in the field of aerospace research and development (with particular regard to its military application);

— Continuously stimulating advances in the aerospace sciences relevant to strengthening the common defence posture;

— Improving the co-operation among member nations in aerospace research and development;

— Exchange of scientific and technical information;

— Providing assistance to member nations for the purpose of increasing their scientific and technical potential;

— Rendering scientific and technical assistance, as requested, to other NATO bodies and to member nations in connection with research and development problems in the aerospace field.

The highest authority within AGARD is the National Delegates Board consisting of officially appointed senior representatives from each member nation. The mission of AGARD is carried out through the Panels which are composed of experts appointed by the National Delegates, the Consultant and Exchange Programme and the Aerospace Applications Studies Programme. The results of AGARD work are reported to the member nations and the NATO Authorities through the AGARD series of publications of which this is one.

Participation in AGARD activities is by invitation only and is normally limited to citizens of the NATO nations.

The content of this publication has been reproduced
directly from material supplied by AGARD or the authors.

Published March 1990

ISBN 92-835-0549-2

Printed by Specialised Printing Services Limited
40 Chigwell Lane, Loughton, Essex IG10 3TZ

ACKNOWLEDGEMENTS AND INTRODUCTION

Part III of the report seeks to give a little relief by way of picture and comment to the largely numerical data of Part I and the bibliographical data of Part II. It is highly selective, being but a personal abstraction of material gathered over many years of research. The selection has been made to cover certain features that are neglected in the general literature that is directed to the gyroscope and gyroscopic phenomena, yet abundantly illustrated from *inter alia* the patent literature of the United Kingdom and the United States of America.

The original material abstracted for Part III has had to be reduced and the skillful editing of the present part of the report is due to my friends Mr J.L.Hollington and Mr D.S.Markham of Smiths Industries Aerospace and Defence Systems. Their interest in the presentation of the whole report to a wider public has brought it to a successful conclusion.

Frank W.Cousins
Westminster 1988

EDITORIAL COMMENT

It has been a pleasure and a privilege to edit this fine work by Mr Frank W.Cousins. Naturally, it is a personal document that reflects his deep understanding of the topic and the great attention to detail evidenced throughout the whole work. The painstaking manner in which the material has been collected and collated over the years has made the task of editing straightforward and rewarding.

These volumes are commended not only to a newcomer to this field of enquiry, but to those established practitioners in the art who will find between these pages much that is of interest, and that which will enhance still further their appreciation of it.

J.L.Hollington
Cheltenham 1988

PREFACE to Parts I and II (published in February 1988)

The purpose of this report is to direct the student of the gyroscope and gyroscopic phenomena to that *terra incognita** of technical literature that resides primarily in the patent literature of the United Kingdom and the United States of America**, augmented by that which resides in the technical journals of each of those nations and those of the U.S.S.R. I have tried to review all of the British patent specifications, but I am well aware that there may be some lacunae.

I have not been able to extend my researches into a complete examination of the patent specifications of the United States of America, but where I do record them, and provided they have a number higher than No.2415067 of c.1947, then each U.S. specification will itself provide a review of the related prior art. Hence each U.S. specification is itself a valuable reference to a much deeper field of enquiry. It is the same with the learned journals of the World, each paper will carry a useful bibliography and again the field of enquiry is remarkably extended thereby.

I have seen all of the entries I have made and I have given the names of the Journals in full to try to save the confusion that surrounds the present poor state of bibliography.

To produce this report has taken fourteen years of research, and I think I may be allowed to draw attention to that part that deals *inter alia* with gyroscopic gears. The subject has not previously appeared in any text on gear design, and is to be found, as far as I am aware, solely in the patent literature. I offer it here, for the first time to a wider audience.

* * *

PREFACE aux chapitres I et II (edités en Février 1988)

L'objet du présent rapport est d'orienter l'étudiant du gyroscope et des phénomènes gyroscopiques vers cette *"terra incognita"*† de la littérature technique. Celle-ci se trouve principalement dans la documentation concernant les brevets au Royaume Uni et aux Etats Unis†† et dans les revues et journaux techniques de chacune de ces nations ainsi que de l'URSS. Je me suis imposé comme tâche de passer en revue toutes les spécifications de brevets britanniques, mais je suis conscient du fait qu'il pourrait y avoir des lacunes.

Je n'ai pas été en mesure d'élargir le domaine de mes recherches afin de présenter une revue exhaustive des spécifications de brevets des Etats-Unis, mais là ou j'y fais référence — et pourvu qu'il lui soit attribué un numéro supérieur au No.2415067 du c.1947 — chaque spécification US fournira d'elle-même un aperçu de l'état de l'art pré-existant. Chaque spécification US sert donc de référence précieuse à une activité de recherche plus approfondie. Il en est de même pour la littérature savante du reste du monde, où chaque communication comporte des références bibliographiques de valeur, qui serviront aussi à élargir le champ des recherches de façon considérable.

J'ai examiné personnellement tous les documents inclus dans mon rapport et j'ai cité les noms des différentes revues en toutes lettres, en espérant ainsi éviter la confusion qui caractérise la situation actuelle médiocre de la bibliographie dans ce domaine.

Le présent rapport représente un travail de recherche de quatorze ans, et je pense qu'il me serait permis de signaler la partie qui traite *inter alia* des engrenages gyroscopiques.

Ce sujet ne parait nulle part ailleurs dans les textes concernant la conception des engrenages, et à ma connaisance, il n'est traité que dans la littérature des brevets. Je le propose, pour la première fois, à un public plus large.

Frank W.Cousins
Westminster 1987

* The term is not too extravagant since The British Library in London holds in excess of twenty two million patent specifications.

** The research has been primarily in British and American patent specifications and in consequence corresponding foreign patents may exist. The reader is directed to study the problem from Derwent Patent No. Family Index which began in c.1974.

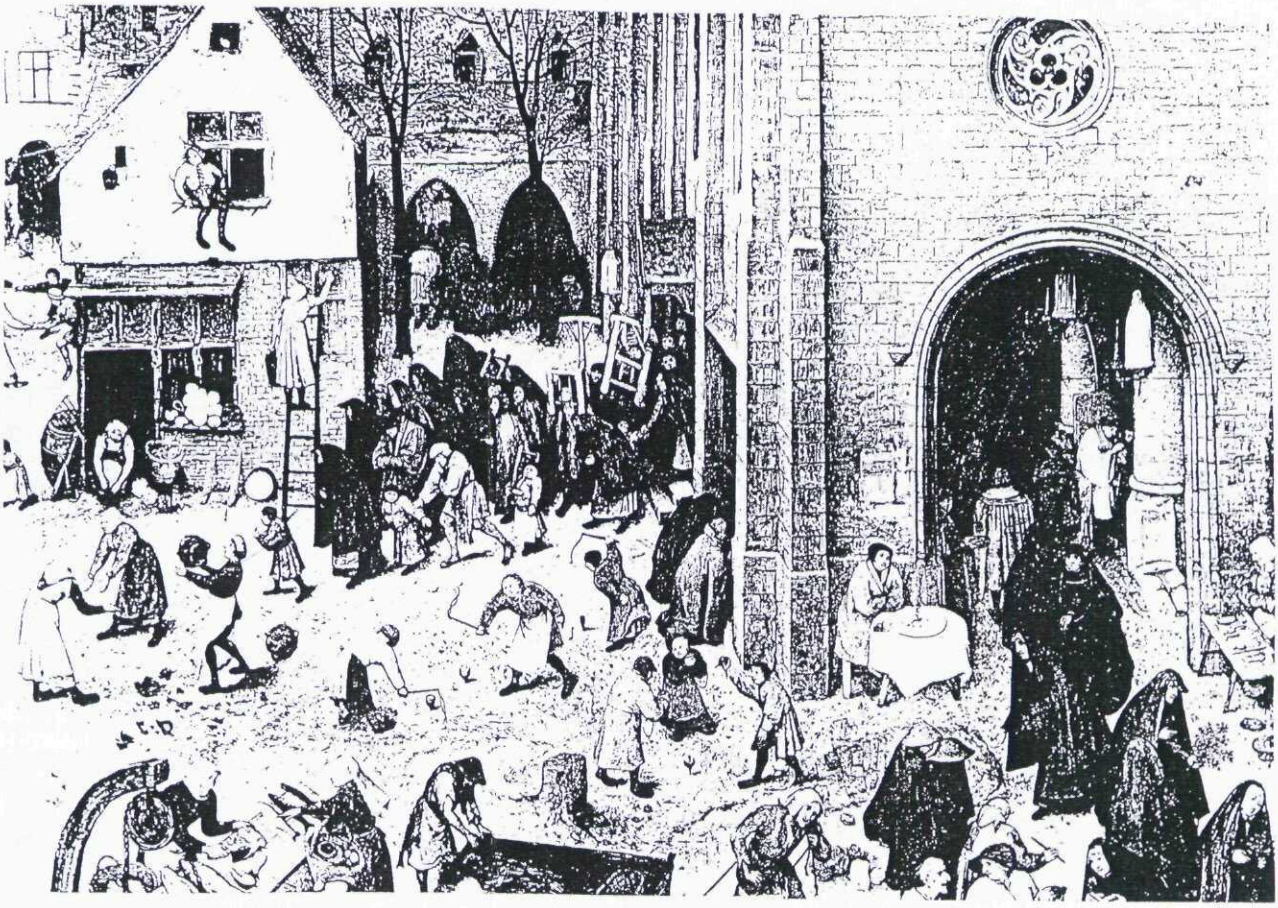
† Le terme n'est pas trop fort puisque la British Library de Londres contient plus de 22.000.000 spécifications de brevets.

†† Nos recherches ont porté principalement sur les spécifications de brevets britanniques et americains, et par conséquent, il se peut qu'il existe des spécifications étrangères équivalentes. Nous attirons l'attention du lecteur sur le Derwent Patent No. Family Index, qui date de c.1974.

PART III

CONTENTS

Page



Frontispiece. The Fight between Carnival and Lent
1559 Pieter Brueghel the Elder (by courtesy of the Vienna Kunsthistorisches Museum)

S'engage puis on voit

Napoléon I

1. DEFINITIONS AND ANTECEDENTS OF THE GYROSCOPE

1.1 DEFINITIONS

Gyr-o-scope (yr as ir),s. [Gr. γῦρος (guros) = a ring, or circle, and σλοπεω (skopeo) = to look at, or behold.]

Astron. Mach.: An instrument constructed by a Frenchman, called M. Foucault, to make the rotation of the earth visible. The principle on which it proceeds is this - that, unless gravity intervene, a rotating body will not alter the direction in which its permanent axis points. In the gyroscope there is a rotating metallic disc, the middle point of whose axis is also the centre of gravity of the machine. By this device the action of gravity is eliminated. The instrument, moreover, is so constructed that the axis of rotation can be made to point to some star in the sky. Then, as the heavy disc whirls round, it is found that the axis continues to point to the moving star, though, in consequence of this, apparently altering its direction relatively to bodies on the earth. If, again, the axis be pointed to the celestial pole, which is fixed, no alteration in its position relative to bodies on the earth takes place. The only feasible explanation of these appearances is that the earth is revolving on its axis. (Sir George Biddell Airy, 7th Astronomer Royal: Popular Astronomy, 8th ed., pp. 78, 89, 282-285.)

Lloyds's Encyclopaedic Dictionary*. (1895)

Gyroscope (Foucault, 1852): An instrument designed to illustrate the dynamics of rotating bodies, and consisting essentially of a solid rotating wheel mounted in a ring, and having its axis free to turn in any direction.

A New English Dictionary on Historical Principles. Henry Bradley. (1900).

(The lexicographic genesis of the term).

Gyroscope. A machine embodying one or more masses, each of which can rotate simultaneously about two axes which are not parallel to each other.

C.H. Van Asperen.
U.K. Patent Specification 422577 Page 2.
(1933).

Gyroscope. Gyroscopic action is manifested by any body which is free to turn about three axes, each of which is perpendicular to the other two. Its chief characteristic is the fact that a force exerted on the body does not turn it about the axis about which the force is applied, but about another axis at right angles to it. A secondary characteristic is the fact that a constant force does not cause the body to develop an increasing angular velocity, but to "precess" with constant angular velocity. In these respects, the gyroscope appears to contradict the laws of motion, but the contradiction is, however, only apparent.

Baker C.C.T.
Dictionary of Mathematics. (1961).

Gyroscope - essentially a spinning wheel, suspended generally upon bearings and having at least a partial freedom to carry out a change of orientation of its spin axis.

Savet. Celerina. 45. (1962).

* This is one of the earliest entries known. There is no entry for, example in <u>The Penny Cyclopaedia.</u> 27 vols (1833-43).

Any rotating body having freedom in one or more planes at right angles to the plane of rotation is called a gyro or gyroscope. A gyro having complete freedom in three planes at right angles to each other is called a free gyro. Mechanically complete freedom of a wheel in three planes can be realized by mounting it in a system of gimbals. However, a rotating ball held in the air would also be a gyroscope; in fact, the Earth itself is a gyroscope.

Leimanis E. (1965).

Gyroscope. Also used, in various modifications or in conjunction with other equipment, to provide a horizontal or vertical reference direction (as in the artifical horizon and the gyro-compass), to stabilize ships, mono-rail vehicles, etc., and to measure angular velocity and angular acceleration (as in the turn-and-bank indicator and other navigational devices).

A supplement to the Oxford English Dictionary.
R.W. Burchfield. (1972).

A conventional gyroscope is a mechanism comprising a rotor journaled to spin about one axis, the journals of the rotor being mounted in an inner gimbal or ring, the inner gimbal being journaled for oscillation in an outer gimbal which in turn is journaled for oscillation relative to a support. The outer gimbal or ring is mounted so as to pivot about an axis in its own plane determined by the support. Hence the outer gimbal possesses one degree of rotational freedom and its axis possesses none. The inner gimbal is mounted in the outer gimbal so as to pivot about an axis in its own plane which axis is always normal to the pivotal axis of the outer gimbal. Hence the inner gimbal possesses two degrees of rotational freedom and its axis possesses one. The rotor is journaled to spin about an axis which is always normal to the axis of the inner gimbal. Hence the rotor possesses three degrees of rotational freedom and its axis possesses two. The centre of gravity of the rotor is thus in a fixed position. The rotor simultaneously spins about one axis and is capable of oscillating about the two other axes, and thus except for its inherent resistance due to rotor spin, it is free to turn in any direction about the fixed point.

Some gyroscopes have mechanical equivalents substituted for one or more of the elements, e.g., the spinning rotor may be suspended in a fluid, instead of being pivotally mounted in gimbals. In some special cases, the outer gimbal (or its equivalent) may be omitted so that the rotor has only two degrees of freedom. In other cases, the centre of gravity of the rotor may be offset from the axis of oscillation, and thus the centre of gravity of the rotor and the centre of suspension of the rotor may not coincide.

U.S. Patent Office. (c1977).

My definition of a gyroscope is any spinning body whose axis of spin is capable of being rotated about a non-parallel axis. Precession is thus the opting for such motion and I would term this "rotation of a live body" as opposed to that of a "dead mass", c.f. an excited electric motor as opposed to an "unexcited" one.

Professor E. R. Laithwaite.
Electrical Review Vol. 207 No. 3. (18 July 1980).

1.2 PRECURSORS

Precursors lie in three widely separated parts of the world structure, in the macrocosmos, in the microcosmos and in one solitary class of the animal kingdom, Insecta.

The first part is discussed mathematically in the works inter alia of POISSON. S.D. (1827) BRAUNBEK. W. (1953) and WIEBELITZ. R. (1955). In astronomy the main interest lies in the perturbations of the Earth in its axial rotation and orbital revolution about the Sun under complex forces from both the Sun and the attendant planets of the Solar System that cause the Earth both to precess and nutate. For a short discussion of the subject see CONDON E.U. and CLEMENCE G.M.*

The second part lies in atomic, sub-atomic and nuclear physics where spin is the sine qua non of a complex behaviour involving atomic and nuclear 'gyroscopes', subject to periodic forces and manifesting itself inter alia in nuclear induction.** These subjects have today (1988) generated a large body of literature and I give below only the most important early authors such as BLOCH. F. (1946) BLOCH. F. et.al. (1940)(1946) KIRCHNER. F. (1955) and WANGSNESS. R.K. and BLOCH. F. (1953). For an accurate discussion of the subject see CONDON E.U.+

The third part lies in the class Insecta of the animal kingdom in the order Diptera, the true two-winged flies that exhibit more than eighty five thousand species. These animals form three sub-orders:

1. the Nematocera
2. the Brachycera-Orthorrhapha, and
3. the Brachycera-Cyclorrhapha.

The second sub-order escape from the pupa via a rectangular slot and the third sub-order escape via a circular cap. The very tiny flies of the third order of the family of the Pipunculidae are noted for their amazing flying techniques including precise hovering. All species possess halteres, said to be reduced hind wings that vibrate and provide balance and flight information by gyroscopic forces. Some entomologists would allow this gyroscopic refinement to exist in the Strepsiptera once considered a part of the beetle family Meloidae, but now given separate order status. The animals of the Diptera and the Strepsiptera are the only known animals to possess gyroscopic aids to assist guidance in free flight.

* CONDON E.U. ORBITAL MOTION
Handbook of Physics. McGraw Hill (1967)
2nd Edition pp. 2.28-2.32
CLEMENCE G.M. DYNAMICS OF THE SOLAR SYSTEM
IBID pp. 2.60-2.68

** See N.M.R. nuclear magnetic resonance, discovered 1945. ABRAGAM. A and GOLDMAN. M. (1982).

\+ CONDON E.U. QUANTUM MECHANICS AND ATOMIC STRUCTURES
Handbook of Physics, McGraw Hill (1967) pp. 7.3-7.37
CONDON E.U. ELECTRONIC STRUCTURE OF MOLECULES
IBID pp. 7.108-7.125

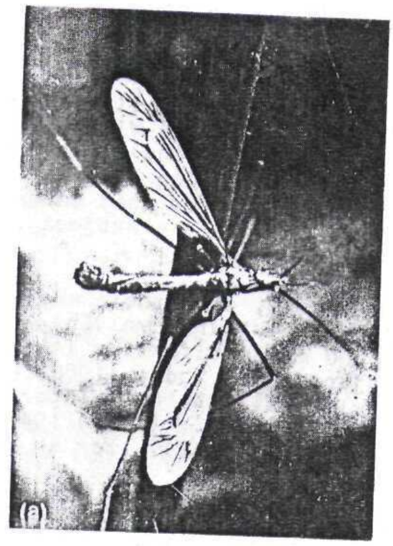


Figure 1.2.1
Crane Fly Tipula sp (daddy-long-legs showing halteres in motion)

Figure 1.2.2
Model haltere of Lucilia sericata dorsal side

1.3 THE TOP

The top is generally considered to be a child's toy, and indeed it is said to be known as such from Egyptian times and to be depicted on 5th Century Attic vases. It is clearly shown in its traditional use by the Flemish master Pieter Brueghel the Elder (1559)*. For some understanding of its inner arcane nature we have to await the mathematical analyses of Isaac Beeckman** (1588-1637) the Dutch philosopher and mathematician whom Gassendi called "the best philosopher I have met up to now" (1629). The top has generated a large literature from the pens of some of the most famous of philosophers, including Plato + who in The Republic, when dealing with the difficult question of the nature of contraries - states:-

> 'And suppose the objector to refine still further, and to draw the nice distinction that not only parts of tops, but whole tops, when they spin round with their pegs fixed on the spot, are at rest and in motion at the same time (and he may say the same of anything which revolves in the same spot), his objection would not be admitted by us, because in such cases things are not at rest and in motion in the same parts of themselves; we should rather say that they have both an axis and a circumference; and that the axis stands still, for there is no deviation from the perpendicular; and that the circumference goes round. But if, while revolving, the axis inclines either to the right or left, forwards or backwards, then in no point of view can they be at rest.'

Both Shakespeare++ and Milton were aware of the top's propensities and each refers to its condition of sleep, Milton in the beautiful lines from Paradise Lost. (8.lines 160-165).

> 'Whether the Sun predominate in Heav'n
> Rise on the Earth or Earth rise on Sun,
> He from the east his flaming mode begin
> Or Shee from west her silent course advance,
> With inoffensive pace that spinning sleeps,
> On her soft axle'

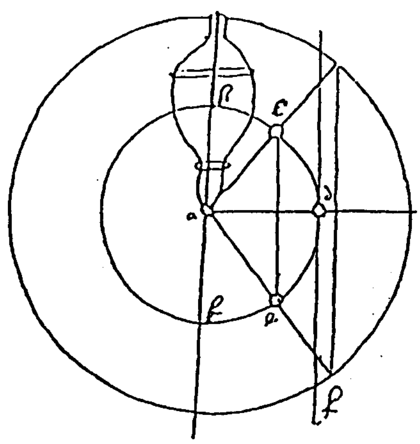


Figure 1.3.1 A diagram from the first mathematical investigation of the top from the works of Isaac Beeckman c.1604-1634.

* See Frontispiece

** See Beeckman, Isaac by Hooykaas. R.
Scribner's Dictionary of Scientific Biography 1 (1970) pp 566-568.

\+ The Republic Book IV c.436.

++ I shall sleep like a top else, The Two Noble Kinsmen. 3.04.26.

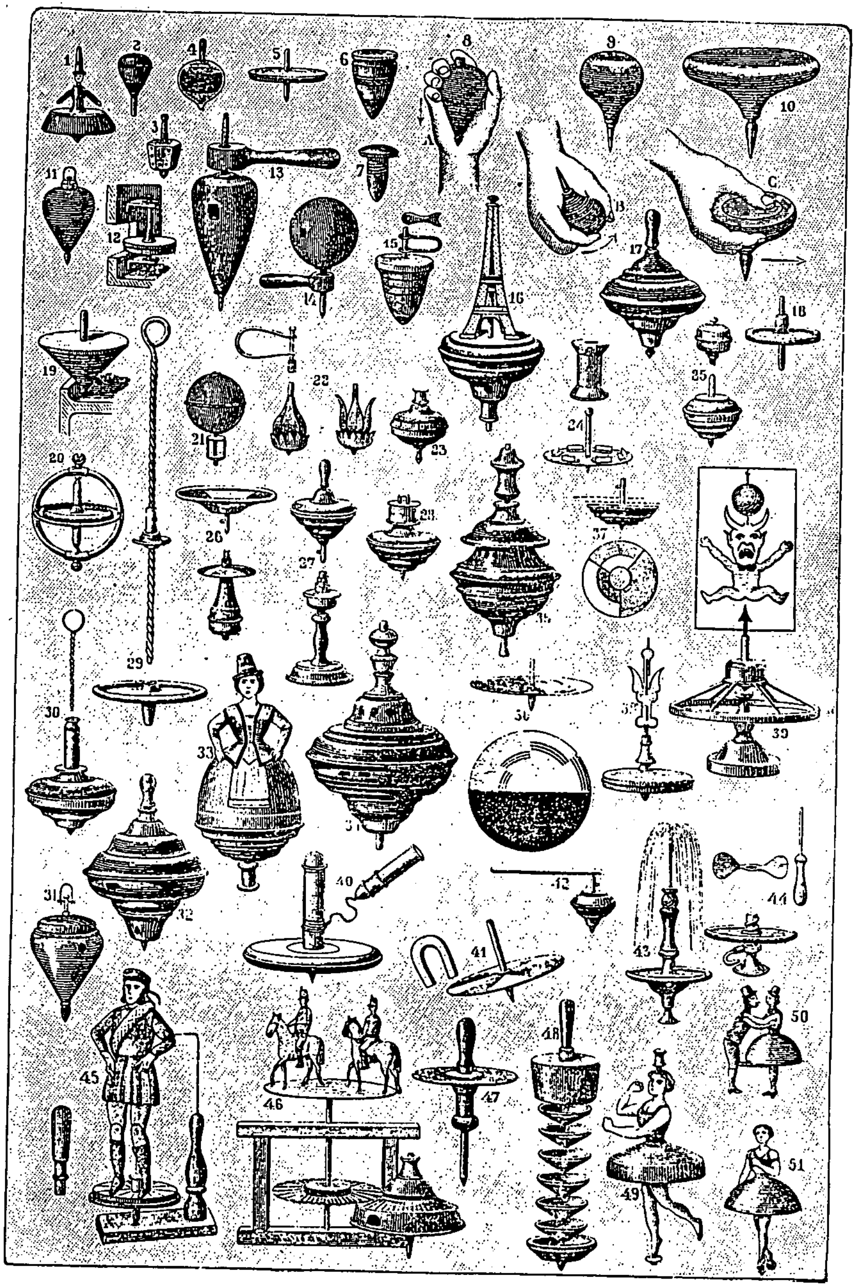


Figure 1.3.2 Curious Tops - Scientific American 18 January 1896.

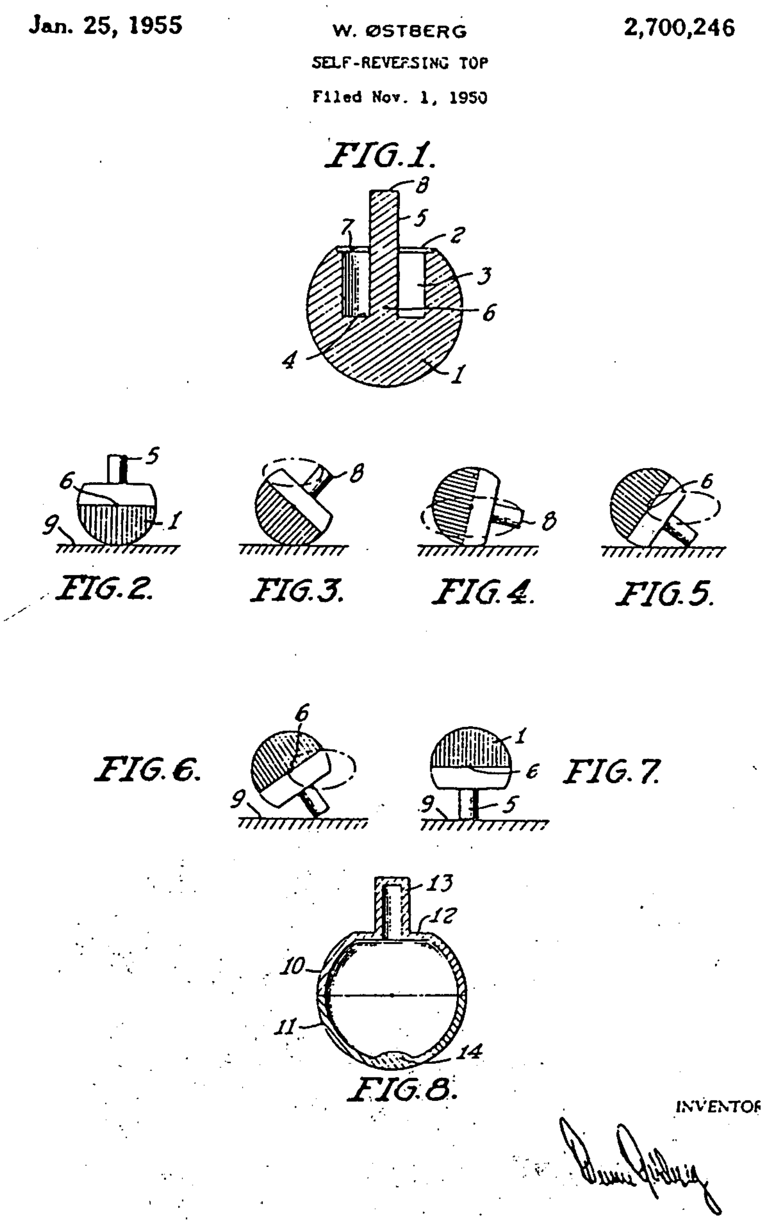


Figure 1.3.3 The Tippe Top

From an anonymous source (c.1754) we have:-

'When a top is set up truely erect the point of contact will be in its axis of rotation on a point in the horizontal plane, but its extremity being something spherical so soon as the centre of gravity begins to descend the point of contact will describe a circle round its axis on another in the horizontal plane, whose tangent being continually oblique to the plane of the inclination of its axis, let its force be resolved into two forces, whereof one acting to the said plane, will continually change the position thereof, whereby the centre of gravity will describe a circle parallel to that of contact; the other acting in the plane of inclination, will impell the centre of gravity forward in the direction of its axis, which being inclined to the horizon, let this force be again resolved into two others, whereof one acting horizontally, will not change the inclination of the axis, but the other acting to the horizon will cause it to ascend, while by the force of gravity it endeavours to descend and as long as this force exceeds that of gravity it will preserve it in a nearly erect position, or being put out of such position by any external force it shall again accede to it till the forces shall equiponderate, but with the velocity, this power also diminishing, the power of gravity shall by degrees lessen its inclination, and, in the end, bring it to intire rest.'

An indication of the ingenuity and diversity that has been applied to the design of the top is aptly illustrated in Fig 1.3.2.

The most recent innovation is the sublime Tippe Top of DAILEY. ØSTBERG and HUMMEL. See Fig 1.3.3 which has excited the interest of several mathematicians and arrested the interest of at least two Nobel Prizemen. See Fig 1.3.4.

The Tippe-top is the subject of many investigations but it needs to be made clear that the top is not only inverting but self-reversing in its rotation of spin.

See Hart. J.B. (1959)

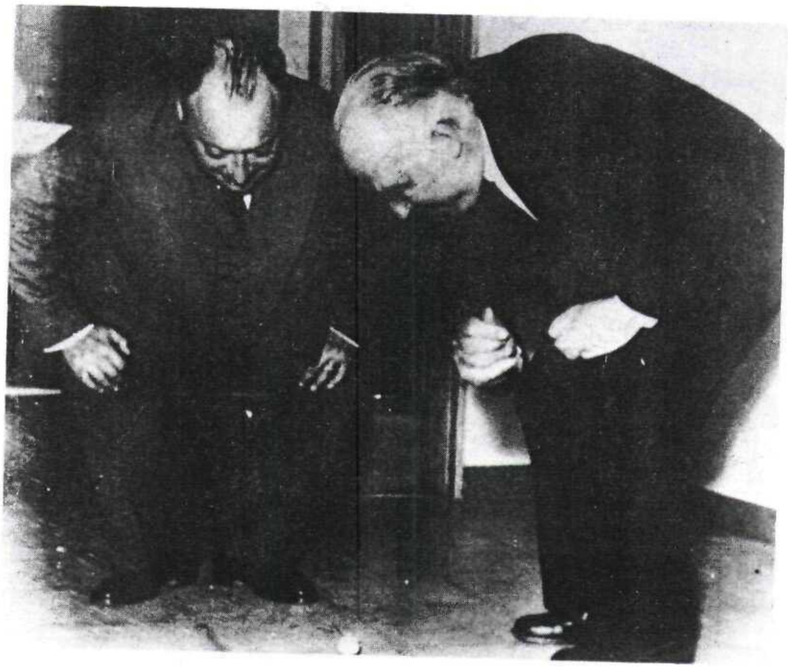


Figure 1.3.4 Nobel Prizemen admire the tippe-top (Wolfgang Pauli & Niels Bohr) by courtesy of AIP Niels Bohr Library.

1.4 SERSON'S GYROSCOPIC SEXTANT

Without doubt the top's most prestigious rôle, from the scientific standpoint, which at once takes it from the humble toy to a macrocosmic entity, is exhibited by SERSON's gyroscopic sextant, that was perfected for the French navy by PONTHUS and THERRODE. See Figs 1.4.1 and 1.4.2.

The following is an account of Serson's top, or gyroscopic sextant from The Gentleman's Magazine for October 1754 Vol. 24 p. 446-448 entitled Description of Mr. Serson's Whirling Speculum, with its use in navigation.

> It is generally allowed that no instrument has been invented for taking the Sun's altitude at sea with so much ease of certainty as that commonly called Hadley's quadrant, tho really Sir. Isaac Newton's. This valuable instrument is however quite useless in foggy weather, when the edge of the sea or visible horizon is hidden, or very indistinct. A spirit level properly affixt thereto might perhaps supply the want of a horizon on land but will not answer at sea, on account of the unsteadiness of the vessel, as has been found by experience.
>
> Eleven or twelve years ago Mr. Serson, an ingenious mechanick, took a hint from the property of a top set a spinning, that the axe of its rotation affects a vertical position, and got a kind of top made, whose upper surface perpendicular to the axe was a circular plane of polished metal, and found, as he had expected that when this top was briskly set in motion, its plane surface would soon become horizontal; that all objects at rest, and reflected by that that surface to an eye also at rest, did appear entirely without motion, and that if the whirling plane were disturbed from its horizontal position, it would soon recover it again, and preserve it unless disturbed anew, or that its velocity was so far diminished; Encouraged by his first success, he, assisted by the advice of the late Mr. Geo Graham FRS, got his apparatus improved in several respects, so as to be thought a fit appendage to Hadley's quadrant, for rendering it capable of taking altitudes at sea, without an horizon. For if the Sun's image, reflected from the speculums of the quadrant, could be made to coincide with his image reflected from the horizontal speculum, instead of coinciding with the visible horizon, it would then manifestly follow, from the constant law of reflection, that the index would show on the limb, the double angle of the Sun's apparent altitude. Not long after Mr. Serson procured this scheme to be laid before the honourable commissioners of the navy, who were pleased to appoint Capt. Russel and Capt. Christopher Middleton FRS to make tryal of it on board one of his majesty's yachts. Accordingly Sept. 12, 1743 these gentlemen went down to the Nore in company with Mr. Serson; Mr. Simpson now mathematical master of his majesty's academy at Woolwich, and some others. They had set a stop watch of Mr. Graham's make that morning, by his regulator, to the apparent time, allowing for difference of meridians between London and the Nore. All things being disposed in due order at a proper distance from noon, an observation was taken without the horizon, and the time of the day thence computed by Mr. Simpson, allowing for refraction, which was found to agree very well with the watch. Afterwards several other altitudes were taken, which, according to Mr. Simpson's calculations, generally answered to the intervals of time between them; the greatest deviations of errors never exceeding 3 or 4 minutes of a degree, notwithstanding the swell of the sea was considerable. The night proving clear they, in like manner, took the meridian altitudes of several of the brightest of the fixt starts, and found the differences of their altitudes agreeable to their known differences of declinations. The next day Mr. Serson himself undertook to draw up an account of these tryals, but being an illiterate man, did it so improperly that the company refused to sign it. However, the aforesaid captains reported to the commissioners that in their opinion, Mr. Serson's contrivance was highly deserving their encouragement, as likely to prove very useful in foggy weather. Mr. Serson having afterwards thought of a method for securing his speculum from tarnish and the force of the wind, was at length ordered on board his majesty's ship the Victory, a first rate,* to make observations with his instrument during the voyage, which were to be compared with those taken, in the usual way, by the

ship's officers. But the Victory was soon unfortunately lost* with all on board, and so perished poor Mr. Serson and in some sort his invention to; for I cannot learn that it has been at all prosecuted since, nor do I know that so much as a print of it has hitherto been published. The late ingenious Mr. Graham indeed procured one to be made which he kept till his death, and used frequently to express great indignation at the unaccountable disregard of so promising a discovery, having himself made many tryals of its properties, as appeared by a note found in the box that contain'd it, certifying the exact time of its whirling, as well in vacuo as in the open air. Mr. Shelton also finished one which was purchased by an agent of the duke of Chaulnes, president of the royal academy of science at Paris, and sent into France, and these are perhaps the only instruments of the kind in the world.

'Tis not improbable, therefore, that the French may make some insignificant alteration in it, and in time, as usual** venture to call it their own. However, to secure, if possible, the invention to the name and memory of its true author, and in the hope of exciting somebody, at least, to improve it, into a lasting benefit to mankind, I have hereto annexed a figure of the instrument, taken from Mr. Serson's curious model, with the manner of setting it in motion, and applying the quadrant to it in observation.

The Speculum described

It is made of the metal used for reflecting telescopes, something more than three inches diameter, but no thicker than is necessary for grinding and polishing it to an exquisitely true plane; that being essential to its just performance, as also in the perpendicularity of its axe, which much be of hardened steel, whose lower end, which extends but a small matter beneath the speculum's lower surface, terminates in a cone whose point is a little rounded off. Its other end, which rises half an inch above the polished surface, is filed square.

The speculum is let into the upper edge of a brass hoop, half an inch deep, and thick enough to bear being turned away thinner and thinner, in a lathe, till the conical point of the axe, or the point of rotation, be found to be precisely in the common centre of gravity of the speculum, hoop, and axe, taken all together; this being the main intention of the hoop; for if the centre of gravity be higher than the point of rotation, the top will not spin so long and will be more easily put out of its position, tho it will recover it again; and if the centre of gravity be below the point of rotation the speculum will never recover it again; and if the centre of gravity be below the point of rotation the speculum will never recover its once lost horizontality, but keep in a kind of vibratory rotation, till it ceases to move.

The speculum is spun up to speed by a string means as shown in Figure 1*** the handle and spinning contrivance then being removed. Figure 2 shows the agate support that supports the point of the speculum it is wrought and polished to a shallow concavity.

The modus operandi is as follows:-

The observer places himself so as to see the image of the Sun in the speculum then looking through the sight of his quadrant, he moves the index till the image of the Sun, reflected by the speculums of the quadrant, is perfectly united with his image reflected by the whirling speculum, and then the index Q Figure 3 shows on the limb, an angle equal to the angle Sas, which is the double of the Sun's apparent altitude above the true horizon.

* HMS Victory 1st rate 100 guns, a rebuild of The Royal James, wrecked 5 Oct. 1744 on the Casquets.

** M. Du Hamel has even showed an inclination by what he has just published in the last transactions of the R. Acad of Sciences at Paris to divide the honour of the invention of artificial magnets with our countryman Dr. Knight, tho' notoriously upon idle pretences.

*** See Fig 1.4.2

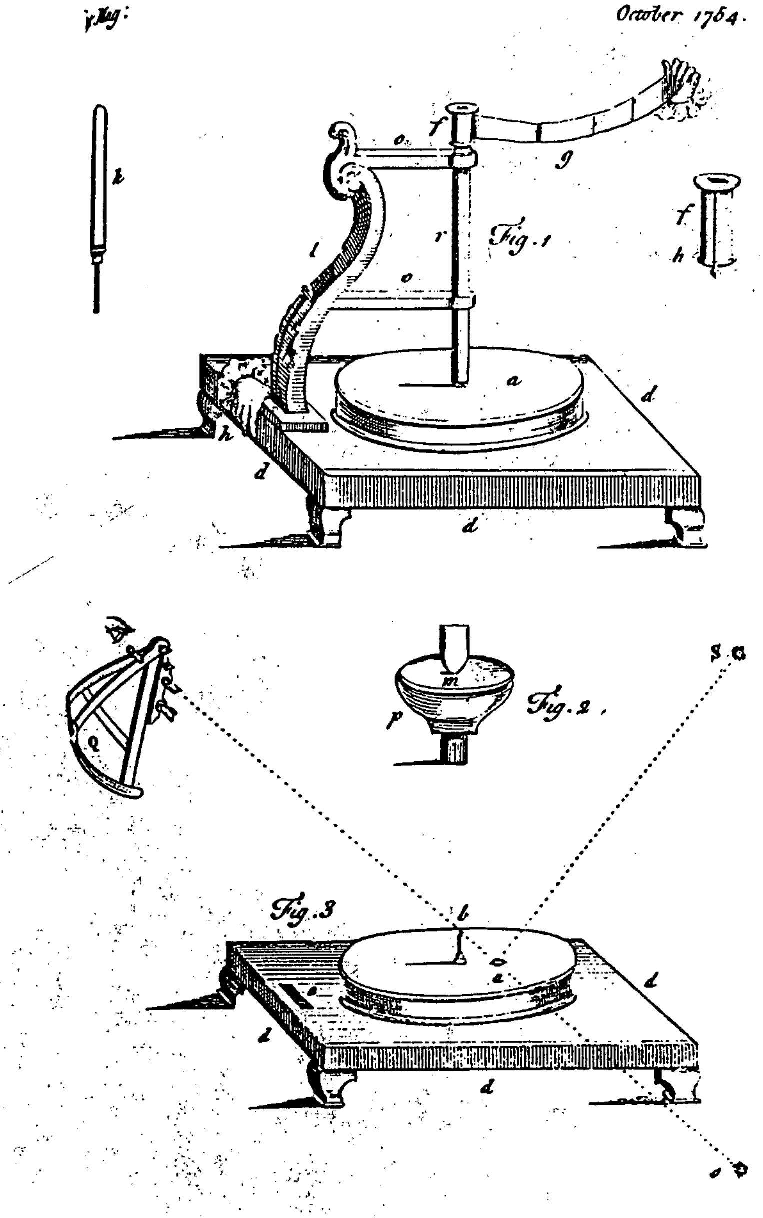


Figure 1.4.1 <u>SERSON'S SPECULUM</u>

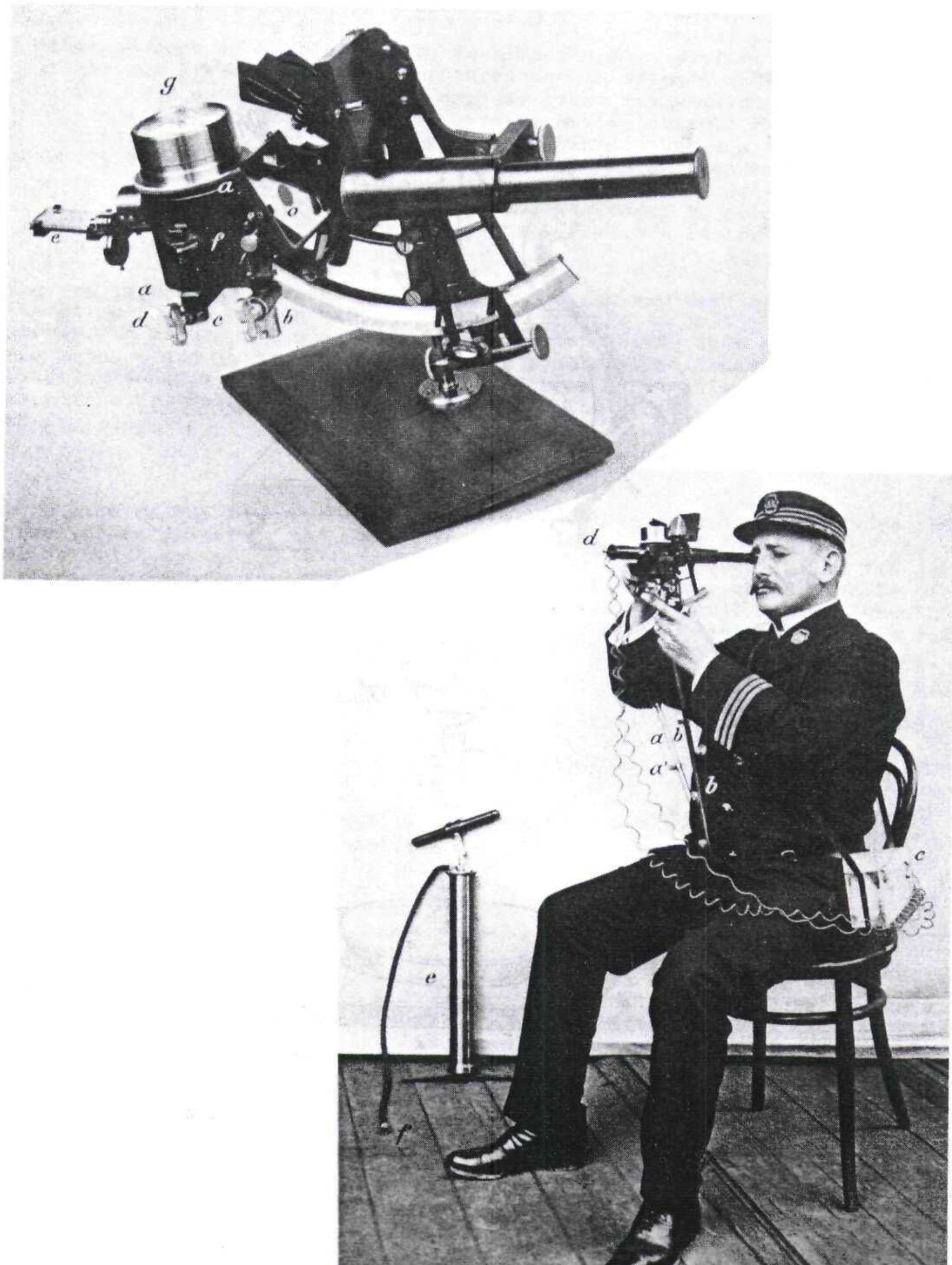


Figure 1.4.2 Gyroscopic Sextant as used in the French Navy.
made by Ponthus. P. and Therrode. L.
(See, for example, U.S. Patent Specification. 705702. (1902)).

1.5 GENERAL COMMENTS

Works on the gyroscopic per se, before the turn of the 19th Century, were of a very general nature. The more arcane investigations were those of pure mathematicians concerning the integrability of the equations of motion of a single rigid body about a fixed point, as epitomized by the mathematical works of EULER. L. (1707 - 1783) POINSOT. L. (1777 - 1859) KOVALEVSKAYA. S. (1850 - 1891) and ROUTH. E.J. (c1884). but, in 1897 the academic world was presented with the first part of the fruits of a memorable collaboration between two of the most experienced and dedicated mathematicians, Christian Felix Klein* (1849-1925) and Arnold Johannes Wilhelm Sommerfield* (1868-1951); the former, professor of mathematics at Gottingen and the latter, professor of mathematics at Clausthal** and at Aachen. This work in its complete form (Über die Theorie des Kreisels (1897-1910) grew out of Klein's lectures of 1895-96 and became a one thousand page treatise. Klein had in 1890 become especially interested in mathematical physics and engineering + and this was true also for Sommerfield as evinced by his address to the Kassel congress of 1908 where he pointed to the felicitous collaboration between engineers and mathematicians.

Surprisingly this large and thorough work of Klein and Sommerfield, remains to this day (1988) the locus classicus of its subject yet it refers to but three German patent specifications. ++

Since the turn of the century we have several texts of some erudition from the pens of both European and Slavonic authors - notably -

SIR GEORGE GREENHILL. (1914).
GRAY. A. (1918).
BOUASSE. H. (1923) in French.
FERRY. E. S. (1932).
BULGAKOV. B.V. (1939 in Russian (trans. 1960).
GRAMMEL. R. (1950) in German.
GOLUBEV. V.V. (1953) in Russian (trans 1960).
SCARBOROUGH. J.B. (1958).
ARNOLD. R.N. MAUNDER. L. (1961).
COCHIN. I. (1963).
WRIGLEY. W. HOLLISTER. W.M. DENHARD. W.G. (1969).
MAGNUS. K. (1971) in German.

With the exception of FERRY. E.S. (1932), who refers to a small number of United States patents, all significantly ignore the escalating wealth of technical information to be found in the patent literature, especially since the years immediately preceding the First World War and after. In 1889 patent literature gives accurate details of gyroscopic torpedoes, to be followed by disclosures of the gyroscopic guidance compass of c1909 and the gyro-horizon of c1917. This open disclosure of valuable technical information has continued up into the years of the Second World War and beyond, and it forms the subject of Part I of this report.

* For excellent biographies of KLEIN and SOMMERFIELD see respectively Burau. W. Schoenberg B. Scribner's Dictionary of Scientific Biography 7 (1973) p396-399. ibid Forman. P. Hermann. A. 12 (1975) p525-531.

** Now Clausthal-Zellerfeld.

\+ See KLEIN. F. Vorlesungen über die entwicklung der mathematik IM 19 Jahrhundert. Berlin (1928).

Translation into English by Ackerman M. Development of mathematics in the 19th Century. Lie Groups Vol. IX. (1979).

++ See Part IV footnotes on Pages 846-848 to Nos. 34513, 178814 and 182855 respectively of VandenBus and Anschutz Kaempfe.

PAOLO FRISI (1728-1784)

JOHANN GOTTLIEB FRIEDRICH von BONHNENBERGER

(1765-1831)

Figure 2.1.1

ANATOLE HENRI ERNEST LAMARLE

(1806-1875)

JEAN BERNARD LEON FOUCAULT

(1819-1868)

2. DISCOVERY

2.1 BOHNENBERGER'S MACHINE

It is clear from a large number of references in the literature that Foucault generally is credited with the discovery of the gyroscope in 1852 whereas in fact his lasting contribution in limine was lexicographic only. We see clearly from the second definition 1. in Section 1.1 taken from the forerunner of the O.E.D. - The New English Dictionary of 1900, that Foucault coined the word gyroscope, but in the authoritative Scribner's Dictionary of Scientific Biography (V (1972) p 84-87) Harold. L. Burslyn erroneously, in my view, claims too much for Foucault. This is shown from a closer inspection of the evidences. Prior to Foucault's birth in Paris on September 18th 1819 a 'machine' fully worthy of the undiscovered name 'gyroscope' was demonstrated in 1817 by Johann Bohnenberger, professor of mathematics and astronomy at the University of Tübingen. It was reserved to Bohnenberger to make the discovery of the gyroscope from his observations of the movements of the planet Earth.

Again, in 1831, some two decades before the date of Foucault's experiment in 1852 Walter R. Johnson had described his Rotascope before the Franklin Institute of America and drawn attention to the pioneering work in dynamics of Paolo Frisi.

Yet another claimant for the honour of discovery is Anatole Lamarle professor at the University of Gent. Neither Bohnenberger nor Lamarle receive attention from the encyclopaedists, both are ignored in toto by the McGraw Hill Encyclopedia of Science and Technology (1977) and by the even more influential Encyclopaedia Britannica (1986).

Bohnenberger receives a passing reference in the excellent volumes of Scribner's Dictionary of Scientific Biography but merely to disclose D.B. Herrmann's interest in his founding with B.A. von Lindenau of the famous Zeitschrift für Astronomie und verwandte Wissenschaften in 1816. Lamarle also rates but a minor reference where J. Pelsenser records that he assisted the eminent Belgian mathematician J.M. de Tilly to attack Euclid's 5th postulate. In fairness to the encyclopaedists it should not go unnoticed that under the entry Kreisel, Der Gross Brockhouse (6 (1955) p629) records Bohnenberger's name and gives him a couple of lines for his astronomical prowess. In what follows I seek to restore the balance:-

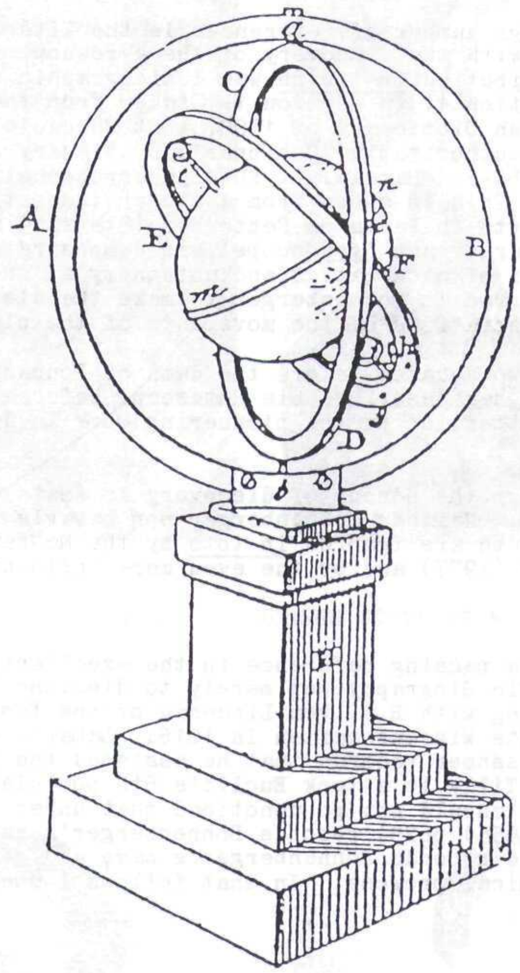


Figure 2.1.2 GILBERT'S ANNALEN der PHYSIK
reference "Figure 4 of Table 1"

Translation from GILBERT'S ANNALEN der PHYSIK 60 1818 p. 60 et seq.

"Description of a machine which is used to explain the laws of the rotation of the Earth around its axis and the change in the position of the Earth's axis"

by Professor von Bohnenberger of Tübingen.*

'Figure 4 of Table I illustrates this machine (one third of its actual size) as it is very accurately and neatly made by University Mechanic Mr. Buzengiger of Tübingen for the price of 18 Gulden. Quite apart from the fact that it explains and demonstrates a noteworthy movement in our solar system, it shows so many striking phenomena when suitably set in movement that it deserves the attention of physicists. I shall now give clear instructions for its use and, to the extent that this is possible without calculation, set forth the reasons for its movements, even although this can be done only very incompletely without the assistance of higher analysis.**

The machine consists of a flattened round body K which can move very easily around an axis ef and is so suspended by means of the three metal rings AB, CD, EF that only its centre, at which its whole weight may be imagined to be concentrated, is supported, and its axis ef can move very freely in all directions, since the ring AB is fixed to the base H of the machine. Inside the same the second ring CD rotates around the two steel tips ab, disposed at the end points of the diameter of the first ring, which stands vertical in the usual position of the machine. The third ring EF rotates in a similar manner inside the second ring around two steel tips, of which only one c is shown in the drawing, the tips being so disposed that the straight line between them makes a right-angle with the line drawn through the tips ab. The round body K which I shall refer to from now on as "the ball" for short although it departs from the spherical, is so suspended by the steel spindle ef in the third ring EF that the spindle makes a right-angle with the straight line through the tip and the opposite tip.

Since the ring EF rotates around the horizontal axis, the axis ef of the ball can make any required angle with the horizon, and since the ring CD rotates around the vertical axis, the ring EF can move into the position of any straight line which may be imagined drawn through the centre of the ball and can therefore move in all directions.

Connected to the spindle of the ball at one of its ends f is a brass roller which has a short pin and is only partly shown in the drawing. If a loop is then made in a strong silken thread, the thread is suspended by means of the loop from the pin, and the thread is rolled up onto the roller by rotating the ball, the ball can be given a very rapid rotary movement continuing for sometime, by holding the three rings together in one's left hand, without touching the ball or its spindle, and rapidly unrolling the thread from the roller by pulling strongly with one's right hand. To make this movement last

* Taken from the valuable Tübingen Journal for Science and Medicine of von Anthenrieth and von Bohnenberger Volume 3, Number I (1817).

** For anyone who has the necessary basic knowledge, a detailed theory of this machine has been set forth in French by M. Poisson in his Memoire sur un cas particulier du movement de rotation des corps pesans in Journal de l'Ecole Polytechnique, Vol 9, page 247 (1813).

This memoire by Poisson makes it clear that a device according to von Bohnenberger was in Paris in the year 1813. Poisson's words are as follows at page 259:-

"Il existe au cabinet de physique de l'Ecole Polytechnique une machine tres - ingenieuse imaginee par M. von Bohnenberger qui represent les divers circonstances du mouvement qui nous considerons - "

longer, the inside of the ball is provided with lead adjacent its equator which is shown by the metal strip mn on its surface, in such a way that the ball remains in equilibrium in every position and its whole mass is uniformly distributed in relation to its axis, so that the axis of rotation of the ball becomes an axis known as free in mechanics. Any engineer intending to make a machine of this kind must pay particular attention to the last mentioned circumstance. Even a small air bubble, which may easily form during the casting of the lead, is enough to cause irregularity in the movement of the machine and a loud noise and to shake the frame considerably.

The machine is used in the following way. The ball must be given rapid rotary movement around the axis ef by means of the thread in the manner indicated, the thread always being completely pulled off the roller, so that it leaves the roller pin. Then, without impeding the movement of the ball, the ring EF is pressed by one finger into any desired position. It will be found that no small force is required for this, although so long as the ball remained immobile, a very slight pressure was enough to move the ring and overcome the slight friction at its pins. While the ball rotates around its axis, the axis will always tend to retain that position which it was given by being set in rotation, even if the whole machine is grasped at its base H and set in motion.

The machine can be carried about in any direction and at any speed, but the axis of the ball will always remain in a parallel position; for instance, if it has originally been turned toward the north in all places it will always direct itself toward the north, like a magnetic needle. The ball, therefore, not only continues undistubidly the rotary movement conferred on it, but it also maintains that position of its axis which it was initially given, no matter how the machine is moved as long as no pressure is exerted on the spindle itself or its supporting rings CD and EF.

The weight having the reference G in the drawing should then be attached to the ring EF adjacent to end point f of the ball spindle; this can be very easily done by attaching the weight by means of pins g and h which fit into two holes (also having the references g and h) in the ring. As long as the ball has no axial rotation, the weight presses the ring EF down to the side F, and after a few oscilations, the ring EF comes to rest only in a vertical position and consequently also brings the axis of rotation ef of the ball into a vertical position. However, if the ball is given a rotary movement by means of the thread, and then the ring EF is so placed that it is inclined by any angle to the horizontal and its weighted side is the lower one, this angle of inclination to the horizon, of both the ring and the ball axis remains unchanged, but the axis itself no longer remains in a parallel position, but moves together with the ring CD at the same time very slowly round in a direction opposite to the direction of rotary movement of the ball. Due to the speed of the axial rotation of the ball, the later direction can be judged only by the direction in which the thread was wound up onto the spindle roller; the ball rotates in a direction opposite to that in which the thread was wound up. As the axial rotation of the ball becomes gradually slower, the retrograde movement of the axis progressively accelerates. It is also noticed that then the ring EF gradually approaches the vertical position, although this last mentioned change is merely due to the friction at the pins ab of the ring CD.

It will also be noticed that the angle that the ring EF makes with any vertical plane usually decreases quickly if only a slight resistance is offered to the movement of the ring CD, but on the other hand increases immediately if the ring CD is acted on in the direction of its movement - i.e., its movement is accelerated.

If lastly, as before, the whole machine is set in movement, even then, this causes no change in the movement of the ball, either in axial rotation or in the retrograde movement of its axis. If, for instance, a distant object is noted, to which one of the poles of the ball was turned before the whole machine was moved away, it will be observed that this pole will change its position in relation to that object in precisely the same way as is observed if the machine remains in one place, on condition that in the meantime the ball has substantially maintained the same speed of rotation.

The machine therefore clearly shows that if a ball is given a rotary movement accompanied by a repelling movement, the axis of rotation will always remain constant during such repelling movement, as long as there is no force tending to change the position of its axis. Even an impact is incapable of causing any considerable change in the position of the axis, unless the impact is a fairly strong one. If the ball is rotating rapidly, small weights can be dropped onto the ring EF without noticably changing the position of the axis, merely on condition that the weights leave the ring again immediately after impact. If, on the other hand a force acts continuously on the axis, as the weight G laid on this machine, it is true that the angle which the axis of rotation makes with a plane assumed to be immobile, for instance, a horizontal plane, but a different movement of the axis takes place from that which, at first glance, would have been expected from the force acting on the axis, since in that case the axis moves in such a way as to describe the surface of a cone whose axis extends parallel with the direction of that disturbing force, that is to say vertically, or perpendicularly to the horizontal plane in the case of the machine here described.

The reason for this strange modification of the movement is to be found in the so-called inertia of bodies - i.e., their tendency to remain in the state of rest or movement in which they find themselves. Thus, for instance, the particles of the water flowing out of a firebrigade hose can not readily be deflected from the rectilinear direction of their movement, and adjacent the mouth of the hose, the emerging water jet feels very hard, as if it had been turned into ice. Similarly, in this machine the ball continues the rotary movement conferred on it in accordance with the law of inertia; its particles describe large or smaller circles extending parallel with one another and a force is required to deflect them from the circles.

A mean movement is composed of the movement which each of the particles has and the movement which the disturbing force (the laid on weight G) tends to produce, since the ball must obey both movements, and the particles of the ball cannot follow such mean movement without the position of the axis of rotation changing. I must refrain here from explaining these movements in greater detail and refer anyone who wishes to know their calculation to the aforementioned paper by Mr. Poisson.

It is now easy to apply to the earth what has been said above. While the earth makes one rotation around the sun, it rotates 365 1/4 times around its axis, and no further force is required to maintain the axis in a permanent position and always parallel with itself and therefore to cause the return of the seasons in the same order after the completion of each rotation of the earth around the sun.

However, at the poles the earth has a compressed shape, such as would result if the earth were to be encircled with a kind of ring which was thickest at the equator and decreased on both sides in the direction of the poles in such a way that its thickness disappeared at them. The side of this ring forming one body with the earth, which if turned toward the sun and moon will be more strongly attracted by them than the other side.

These forces of attraction therefore tend to reduce the angle at which the equator of the earth intersects the plane of the earth's orbit and therefore to move the earth's axis nearer to a perpendicular position in relation to the plane of the earth's orbit, just as in the aforedescribed machine the laid-on weight G tends to move the axis of the ball into a position perpendicular in relation to the horizon, which in this case represents the ecliptic. If the earth had no axial rotation, it would be just as impossible for the inclined position of its axis to be maintained, as for the axis of the ball in the machine to be able to remain in an inclined position, if the ball does not rotate, and the weight G acts on its axis. Since, however, the earth rotates around its axis, the angle made by its axis with the plane of the earths orbit remains substantially unchanged, but the axis itself receives a very slow movement, as a result of which it deviates from a parallel position gradually and progressively in a direction which is opposite to the direction of the axial rotation of the earth and its rotation around the sun, as the machine shows.

Due to the slight deviation of the earth from the spherical, the aforementioned increase around its equator it relatively small, and therefore the disturbance of the position of the earth's axis due to the forces of attraction of the sun and moon is also very small, so that after about 72 years the earth's axis deviates by 1 degree from the parallel position, and requires a period of more than 25,800 years for one complete rotation. However, between the forces of attraction of the sun and the moon and the earth, and the effect of the weight G simulating them on the machine, there is the difference that the later is invariable, but the former are variable, due to the different positions of the earths axis in relation to the sun and moon, caused by the changing seasons and the high and low state of the moon. As a result, instead of an even movement, an uneven movement is produced, and a slight variation of the earths axis, which can be noticed only by astronomical observations and can not, in any case, have any appreciable influence on the seasons. The only change which can be observed after a large number of years even without astronomical tools and which, therefore did not escape the ancients is that if the deviation of the earth's axis from a parallel position is opposite to the rotary movement of the earth around the sun, the seasons recur earlier than would be the result of the period of rotation of the earth around the sun in relation to the fixed stars. After the lapse of about 12,900 years, therefore, those fixed stars will be observed in the sky at midnight at the period of the longest day which are at present seen in the sky at midnight around the period of the shortest day.

I would also point out in relation to the machine, that if it is imagined standing on a table, the points of rotation ab correspond to the poles of the ecliptic. The ring AB then therefore represents an immobile maximum circle of the celestial sphere (one parallel of latitude) which is perpendicular to the ecliptic and which may be imagined to extent through a fixed star, the ring CD representing the so-called colurus of the equinoxes (?). (translators note: the foregoing bracketed question mark appears in the original German). The point C and the opposite point is the point of the spring and autumn equinox. Lastly, the angle which the axis ef makes with the vertical line is the inclination of the ecliptic. (Translators note: the line of type below the last line of the text on page 65 of the original German indicates that the paper was published in the German Annals of Physics "in the year 1818").

*Foucault Gyroscope. France, from 1852.
Brass, bronze, steel. Height 1m.25.

Very rare. Three columns joined together at their base and their summit support a tube in which hangs a plumb line necessary to put the device in horizontal position.

On the vertical axis rests a rotating support which holds four large horizontal wheels acting as bearing for the axis of the bronze rotor.

The graceful columns remind one of the style of industrial models of the mid 19th Century. This gyroscope must thus be of the period of its invention by Foucault.

The axis of the core, run up to high speed, appears to describe a cone of revolution about an axis fixed in space. The movement seen is due to the observer, who is displaced by motion in the opposite direction (that of the earth) about a straight line parallel to the earth's axis.

By placing horizontally the circle on which rests the gyroscope spindle, this oscillates from side to side of the true meridian, etc...

Modern applications of this device are numerous: gyroscopic compass for marine and aerial navigation, anti-roll device for ships, artificial horizon, turn indicator, automatic pilot for aircraft and guided missiles.

*Translation of the French original.

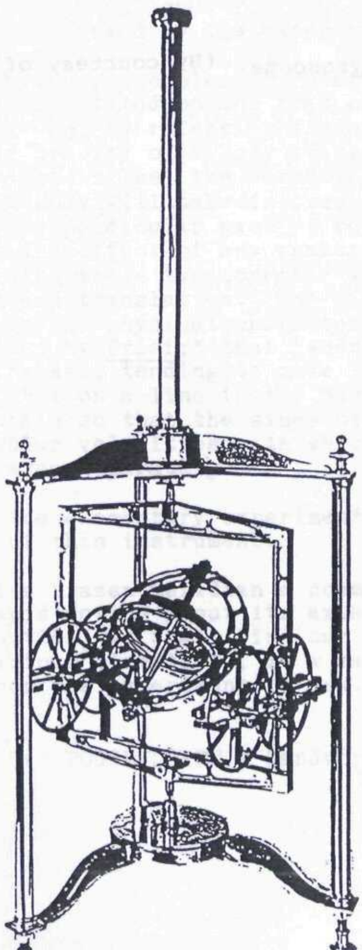


Figure 2.1.3 Gyroscope de Foucault by courtesy of Alain Brieux

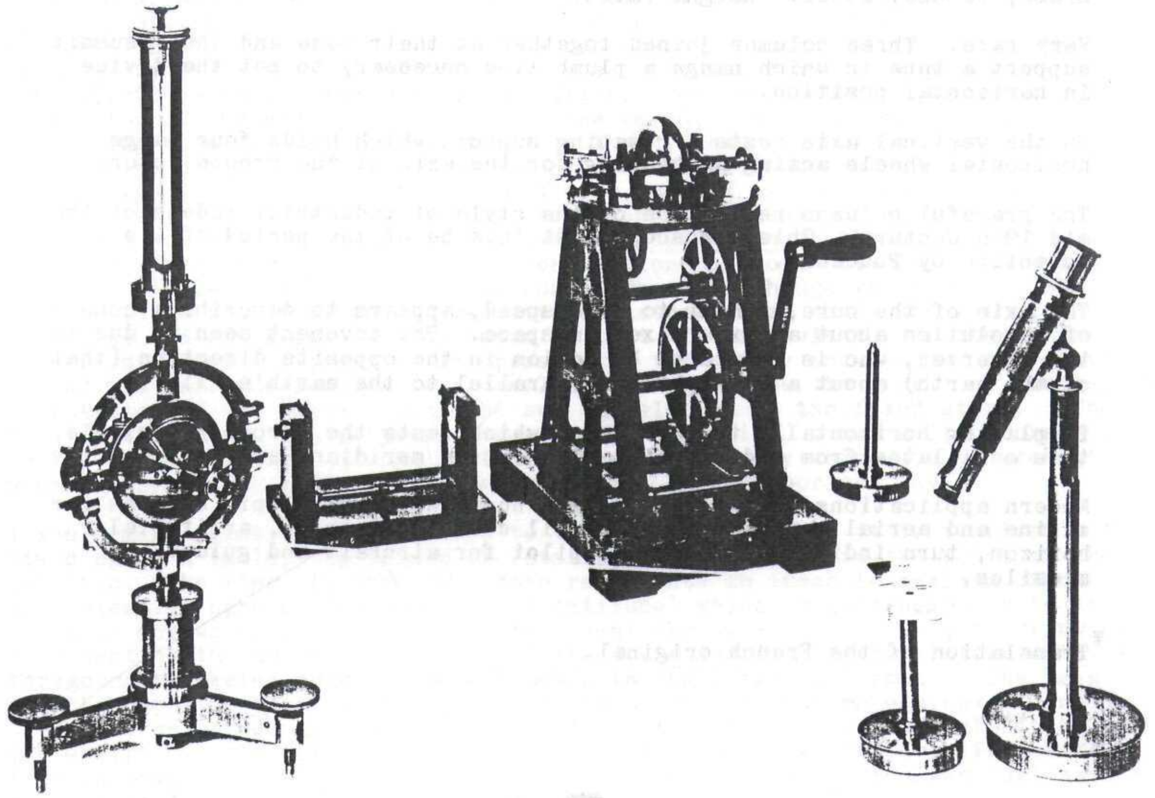


Figure 2.1.4 Foucault gyroscope. (By courtesy of the Science Museum London)

2.2 JOHNSON'S ROTASCOPE

Taken from the Journal of the Franklin Institute, Vol. VIII - No. 6. December 1831.

'Description of an apparatus, called the Rotascope, for exhibiting several phenomena, and illustrating certain laws of rotary motion. By WALTER R. JOHNSON. - (With a Plate.)

Our common books of mechanics generally contain concise accounts of the doctrines of rotary motion, limited for the most part, however, to the consideration of central forces, the centre of precession and of gyration, and the centre of spontaneous rotation, to which may be added, the laws of oscillatory motion.

The forces tending to change the position of the axis of rotation are generally either wholly omitted, or if concisely stated in an abstract form, are apparently regarded as incapable of experimental illustrations. The whirling table of Mr. Ferguson is an ingenious apparatus for exhibiting the amount and direction of the several forces exerted by a body in its own fixed plane of revolution. But that instrument makes no provision for the phenomena above referred to.

When we consider that the extensive diffusion of a branch of knowledge, often depends on the facility with which its elements can be made apparent to the understanding, we are at no loss in estimating the practical value of philosophical instruments, whether intended for demonstration or for research. Of this truth the machine of Attwood may be taken as an illustration. This machine gives a most elegant and satisfactory exhibition of the principles of uniform, accelerated, and retarded motions, as dependant on the force of gravity.

All the motions in the machine may be so slow as to reduce the resistance of the air to an unimportant element, and the friction and inertia of the parts being separately determined and allowed for, the theoretical laws of motion are seen to be perfectly confirmed by the experiments.

As to the manner in which the principles of rotation have generally been explained, it may be briefly stated on the plan of what are called rectangular co-ordinates. As, by referring the effect of any force applied opposite to the centre of gravity of a body at rest, to three lines mutually crossing each other at right angles, the resulting direction which the centre of gravity of that body will take in free space, is inferred; so, by a consideration of three perpendicular axes of revolution within the body itself, we may determine the effect of any number of forces tending to produce rotation. Combining these two together we have the resultant motions both of rotation and translation. One of the most important propositions pertaining to the physical character of the subject, is that discovered and demonstrated by Frisi;* that "when a body revolves on an axis, and a force is impressed, tending to make it revolve on another, it will revolve on neither, but on a line in the same plane with them, dividing the angle which they contain so that the sines of the parts are in the inverse ratio of the angular velocities with which the body would have revolved about the said axes separately."

The following are among the elementary experiments and observations which led to the construction of this instrument.

1. When we take up by its brazen meridian a common artificial globe, and, having given it a rapid motion about its axis, attempt to move the poles from their position, we shall find our efforts resisted, or the globe impelled in various directions, in a manner which will generally surprise those to whom the experiment is new. If the globe be held by

* PAOLO FRIZI (1728-1784) See POGGENDORFF's Handwörterbuch - Vol I (1863) p.806

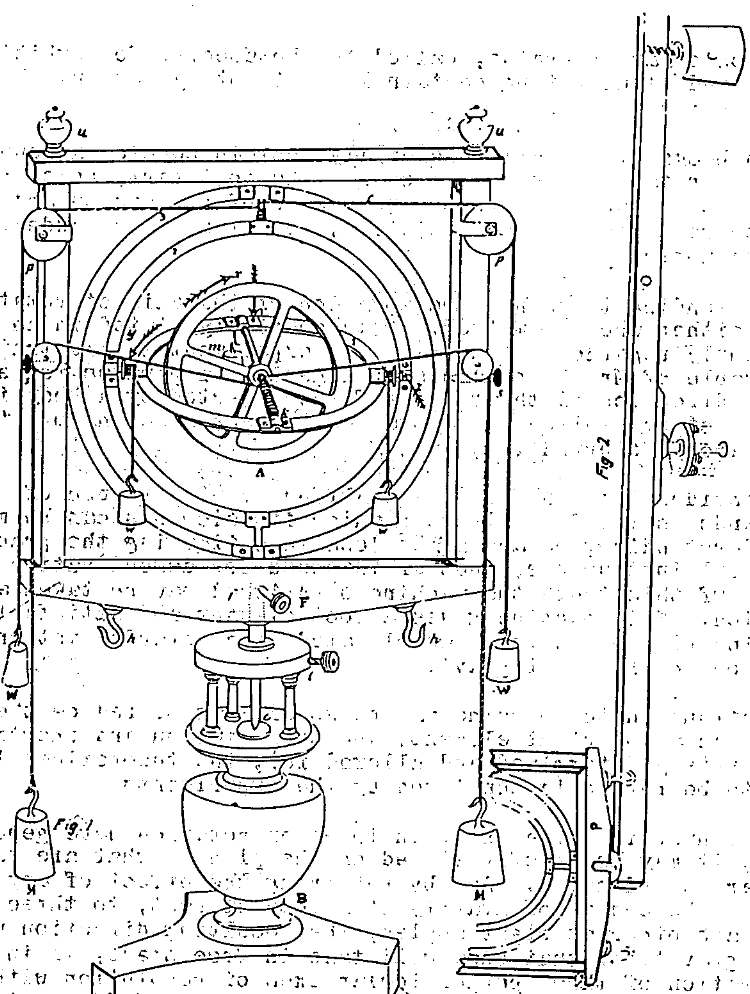


Figure 2.3.1 Rotascope by W.R.JOHNSON

the meridian, at points over the equatorial circle of the sphere, and the axis placed nearly vertical, and in this state of things the revolving globe be carried alternately from the right to left, and the reverse, by the extended arms as in a small orbit, or portion of an orbit, the tendency of the sphere to change the position of its axis will be felt in one or both directions of the movement.

2. There is in use a small apparatus for striking fire, composed of a semicylindrical box of tinned iron, a few inches in length, at one end of which is a small cavity for receiving the tinder, and above it is mounted, on an axis, a disk of steel to strike, when in rapid motion, upon a flint, held just above the tinder. The steel disk is put in motion by the friction of a string drawn briskly over a pulley on the axis. If, when the wheel in this apparatus, (which is about two inches in diameter,) is revolving vertically, we hold the whole loosely in the hand extended, and carry the latter alternately right and left before the body, so as to cause the wheel to describe a horizontal curve, to which the direction of its axis at the commencement of the motion is a tangent, we shall perceive a strong tendency in the wheel to leave the vertical and assume the horizontal position.**

3. A small apparatus, said to have been devised by the celebrated Laplace to illustrate the precession of the equinoxes, has been made in France, and imitated by an ingenious mechanic of Philadelphia. It is formed of two concentric rings revolving on axes at right angles to each other. Within the inner ring is a small spheroid, loaded at one of its poles in such a manner as to produce a rotation in the axis of the inner ring when the spheroid is caused to revolve with rapidity. The chief parts of the rotascope had been devised and construted before I had an opportunity of seeing the above described apparatus.

4. A number of ingenious experiments were some time go contrived and executed by Mr Rufus Tyler, a skilful mechanician of Philadelphia, with an apparatus resembling, in some respects, the common top; included in a ring, and placed on a whirling table.

In that arrangement his experiments coincided to a certain extent with some of those which are presented with the rotascope on the orbit-rod.

There was wanting, however, the means of developing and exhibiting the causes which produce the changes which are actually seen to take place. This end is most important in whatever concerns the principles of mechanics. It is what constitutes the great beauty of Attwood's experiments, that the action of gravity is made to coincide, in principle, with its actual operation when unrestrained; while, at the same time, the bodies submitted to its action, move with velocities which can be readily followed by the eye.

** This neat little experiment had been made and was first communicated to me by Mr. William Mason, who, to render it the more striking, had mounted the whole box on a pointed axis passing longitudinally through the centre of gravity.

The following description refers to the accompanying plate (Fig 2.3.1):-

A, is a fly wheel, about eight inches in diameter, formed in such a manner as to receive but slight resistance from the air. It is supported on the centre of a perfectly cylindrical axis about 3/8 of an inch in diameter, terminated by cones to serve as pivot points, on which the wheel runs. The wheel is of brass, the axis of steel, one part, from the wheel towards the pivot, being polished, the other bronzed, for more readily distinguishing the changes of position. The wheel and its axis weigh about 2 lb. 11 oz.

B, is the base or tripod of mahogany, which sustains the instrument.

F, is a wooden frame containing the principal moving parts of the apparatus.

1,2,3, are concentric metallic rings, each about 3/4 of an inch in breadth, and about 2/10 of an inch in thickness. The exterior one, (3,) being about fifteen inches exterior diameter, is sustained in its place by the screws s,s, which have their ends conically excavated to receive the pivot. The axis of the next ring, (2,) is at right angles to that of 3, and again the axis of 1, is at right angles to that of 2, and the axis of the wheel A, to that of the ring 1.

The centre of gravity of the wheel is likewise that of the whole system, and the axis of motion of each ring passes through that centre.

e, is a pivot to the vertical shaft e,f, upon which the frame F is supported, and upon which it may revolve. The axis of this shaft likewise passes through the centre of the wheel A.

f, is a socket and cone furnished with a tightening screw.

t, is a thumb-screw to fasten and hold the axis e,f, whenever it becomes necessary to prevent the horizontal motion of the frame F.

p,p, are two pulleys attached to the two upright pieces of the frame by metallic bands, and held fast, at any convenient height, on these supports by a screw on the back side; by taking out the screws s, s, the pulleys may be carried below the axis of the outer ring.

u,u, are nuts to keep in place the upper piece of the frame, and having a hole drilled through their heads to receive cords by which the whole frame may be suspended from the ceiling; and h,h, are two hooks for a similar purpose, and likewise for suspending other weights when not in use.

M,M, are weights acting as moving forces, to set the wheel A in motion. For these weights, the hand of the experimenter may, in many cases, be conveniently substituted, especially where it is not important to know the precise velocity attained. The cord is attached to the axis by means of the small projecting conical knob k, to which the centre of the cord in connected by doubling it, applying the pin to the double part, and then setting the wheel in motion to wind up the cord, so that one end will be drawn off from above, the other from below, and both tend to turn the wheel in the same direction.

c,c, are two cords connected with the axis of the ring 2, and passing over the pulleys p,p.

W,W, are weights attached to these cords.

z, indicates the direction in which those weights tend to turn the ring 2 about its axis. This direction is reversed by winding up the cord in the opposite direction.

w,w, are weights applied to pulleys firmly connected with the axis of circle 1, and tending to produce a rotation in the direction opposite to that indicated by v.

m, is a weight suspended or otherwise attached to the same circle, and tending to produce rotation in the direction of v.

x, denotes the direction in which the wheel will move when actuated by the forces M,M, as here represented; but by turning the wheel in the opposite direction, when the cord is applied over the knobs, it will be put in motion in the opposite direction, and the ring 2, will also move opposite to z, by the force of m.

O, is a bar of mahogany, called the orbit-rod, six feet in length, with a socket, by means of which it may, when the frame F is removed, be placed on the pivot e, and made to revolve. In this case the frame containing the wheel is to be set, or suspended, at one end, while at the other is suspended the weight C, which exactly counterpoises the frame and its appurtenances. This weight is placed below the bar in order to bring the centre of gravity as low as practicable, and produce a more stable equilibrium.

The following directions and cautions in using the rotascope will be found useful to those who may not be familiar with its action.

In winding up the moving cord around the axis of the wheel, it is necessary to keep the two ends as near to each other as practicable without having one overlay, or actually rub against, the other, and to have them wound from beginning to end of the spiral, parallel to each other, without crossing, as the latter will materially obstruct the uncoiling when the force is applied, and endanger the breaking of the cord.

Care should be taken that the uncoiling be made in such a position of the rings that the moving cord will free itself immediately from all contact with the wheel, at the instant it leaves the shaft.

The cords applied to the several pulleys on the first and second rings, should be kept closely wound round their respective pulleys when not wanted for immediate use, as they may otherwise become entangled in the wheel and obstruct its motion, or essentially endanger the accuracy and safety of the whole instrument.

In using the orbit-rod, the weights should be attached first, then the frame, F, put in its place, and finally, set upon the pivot e, when the base, B, will sustain the whole. The revolution should begin with a slow motion, and increase in velocity - all shocks and sudden changes should be avoided.

When it becomes necessary to add any weights to the rings or other parts of the apparatus while on the orbit-rod, an equal weight should be added to the counterpoise to avoid lateral pressure on the pivot e.

When the elementary particles are placed on the axis, the changes of position of the rings should be made gradually, to avoid violent blows of the particles upon the ring 1; otherwise, they may bruise its edge, and be thrown off with violence.

When the wheel is to be stopped, it is most convenient and safe to do it by applying a moderate friction with the fingers to the axis.

To set the wheel in motion, apply the cord round the axis of the wheel, doubling it for that purpose, and putting the fold at the centre over the small pin near the end of the axis. Having wound up the cord, take one end in each hand, and draw the two ends apart with suitable force in directions at right angles to the axis, and as nearly as may be in the plane of the first circle, as well as parallel to the wheel itself.

The following are among the experiments which may be performed by the aid of the rotascope.

1. Take the wheel and its supporting circle from the frame. Connect with the ring, at a point opposite to the axis of the wheel, a wooden rod of sufficient strength to bear the weight when held horizontally, and from nine to twelve inches in length. Attach the end of this rod, remote from the wheel, to a cord suspended from the ceiling. Set the wheel in rapid motion, and then bring it up so that the rod shall be horizontal. Then suddenly abandoning it with the hand, the cord will sustain it, but instead of hanging vertically down, the axis of the wheel and rod, which may be regarded as its prolongation, will be kept for some time horizontal; but though thus suspended, as if by some mysterious agency, they constantly perform a circuit which has an imaginary vertical drawn from the point of suspension, for its axis. If the velocity of the horizontal revolution be diminished, the sustaining rod will incline downwards more rapidly than when left to itself, until at length it reaches the position of rest. But if the velocity of revolution be augmented by any external force, the wheel and ring will rise in opposition to gravity until the rim of the wheel strikes the suspending cord. The wooden rod will then have come to a position nearly vertical, sustaining the wheel and ring at its upper end, but still continuing the horizontal motion. This paradoxical appearance, would continue the longer by having a delicate metallic swivel link in some part of the cord, which would prevent the twist that otherwise soon opposes the horizontal motion to such an extent as to depress the rod in the course of a few minutes. It will be seen that the revolution in a horiztonal direction, being the resultant of gravity, combined with the rotary motion of the wheel, must become more rapid in proportion as the velocity of the latter on its axis is diminished: because the force of gravity is then a greater component in the combined forces which act upon the system.

2. Having replaced the wheel and ring, in their connexion with the frame, set the latter on its pivot upon the base, B. Make the circle, or ring, 3, fast in a vertical position, apply cords to the pulley on the axis of ring 2, bringing the pulleys p,p, to a proper elevation; make them fast, and pass those cords over them to sustain weights. Having given the wheel a rapid motion, take hold of one of the urns u, u, and cause the whole frame to revolve horizontally on its pivot. As the persistency of the wheel in the plane of its motion prevents the ring 2 from revolving, the motion of the frame will gradually wind up the cords about the pulley. At the same time, however, the ring 1 will gradually change its plane, and bring the wheel to a position to obey the action of the weights. The portion of cord which had been previously wound about the pulley will then be uncoiled, and a considerable momentum communicated to the system composed of the rings 1 and 2, which will, if the tightening screw, t, be made fast, again wind up the cord in the opposite direction about the pulley. As soon as the said rings, however, are again deprived of their momentum by the action of the weights, the latter will again tend to produce a rotation in the ring 2, which will be opposed by the persistency of the wheel. If, at this moment, the pivot be released from the screw, the whole system, composed of the two weights, the frame, and ring 3, will be made to revolve by the gravity of the weights, while the ring 2 remains pertinaciously fixed in its position, until the wheel has had time again to invert its axis. This time will be greater or less according to the greater or less velocity of rotation in the wheel, compared with the size of the weights hanging over the pulleys.

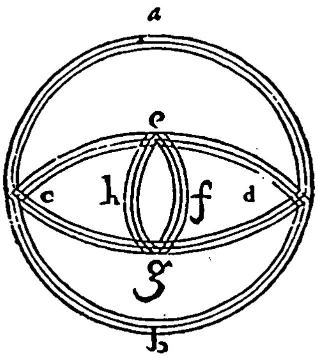
3. Set, or suspend, the frame on the orbit rod. Give the wheel a moderate velocity of rotation, and set the whole in motion upon the pivot e, all the circles being free to move on their respective axes. In whatever direction the wheel revolves, with respect to the plane of the orbit, at the commencement of the orbicular revolution, it will soon be observed to conform in direction to the latter. If this be reversed, that will soon be reversed also.

4. Repeat the third experiment with only the addition of a weight of eight ounces attached to the second circle, opposite to the axis of the first. The effort of the wheel to take and maintain in its rotation the direction of this orbicular motion, will be sufficient to keep the weight elevated nearly to a level with the centre of the wheel.'

2.3 GYROSCOPE SUSPENSION

The gyroscope in many of its most sophisticated forms still relies upon the cardanic suspension and it is a pleasure to remind the reader that we are indebted to Girolamo Cardano (1501-1576) the Italian mathematician who invented the cardanic suspension, a form of support in which an instrument is hung on gimbals, so as to allow free movement in all directions. For interest Cardano's original work is given below:-

De Armillarum inſtrumento.



COnſtat ex circulis tribus inſtrumentum armillis ſimile, quorum ſuperiores ſunt duplicati, & polis ſecundus primo fixis infigitur. Vt ſit A B circulus primus, cui infixi ſint ad rectos polorum CD poli interioris, & vterque duplicatus, vel vt media pars circumagi poſſit ſub eiſdem polis, quia inferior; vel quia ex dimidio ſtabilis præter polos, ex dimidio mobilis. Tertius autem in medio ſecundi, ita vt circumagi poſſint poli eius ex E in C, & C in G, & G in D. Et rurſus polis quaſi nullis ex E in F, & F in G, & G in H. Et vt lateat prorſus coniunctio, adeò vt annexus alter alteri videatur. Tale inſtrumentum vidi apud virum Maximiliani Cæſaris, Mathematicum Medicum & Philoſophum inſignem, Ioannem Sagerum Giſenhaigen Vratiſleuienſem, quanquam neque ipſe docuerit, quomodo inſertus eſſet; neque ego interrógauerim. Ergo fieri poteſt, vt circulus inferior D E C G circumuertatur, ſuperiore immobili: atque ita poli ferentur per E C G D: ſed tunc neceſſaria erit cauitas infra circulum ſecundum, per quam feratur. Sed ſi circulus E C G D integer ſit, fieri non poteſt vt ferantur poli niſi cum circulo, cui infixi ſint: hic autem eſt pars circuli prædicti media, aut etiam inferior. Erunt ergo tres modi. Inferior autem circulus, cum in ſeipſo reuoluitur, poterit manentibus quidem polis circumduci à lateribus, fixo manente medio: nam ſi medius transferatur cui polus infixus eſt, exibit polus circumductus circulum E C G D ſecundum latitudinem: aut transferetur polus per cauitatem. At tunc non erit circulus F G H E ſolidus. Cum ergo voluerimus circulos ambos eſſe ſolidos, relinquentur duo modi tantum, vt pars media circuli ſecundi, cui infixi ſint circuli, circumuoluatur, & extremæ inferioris partes, ſeu latera: aut vt pars inferior ſecundi circuli intruſa ſuperiori, & in qua ſint poli fixi eodem modo ſub ſuperiore circumducatur, atque eo modo totus circulus E F G H circumagatur per E D G C. Ipſe verò circulus E D G C in ſeipſo vt prius manente fixa parte media, in qua ſunt poli infixi, circumducatur lateribus ſuis. Commune autem eſt ambobus, vt poli ſint infixi vtriſque circulis ſecundo & tertio, & quòd latera inferioris manente medio circumagantur. In ſecundo tantum differunt; cum vel media pars manentibus lateribus, vel inferior ſuperiore fixa circumduci poſſit.

3. EARLY APPEAL TO GYROSCOPIC FORCES AND THE EARLY GYROSCOPE

3.1 BOOMERANGS AND SLINGS

The earliest appeal by man to gyroscopic forces takes us into prehistory. The use of a missile delivered with deadly effect by David's hand against Goliath is described in 1 Samuel 17 40.49.

> 40. And he took his staff in his hand, and chose him five smooth stones out of the brook, and put them in a shepherd's bag which he had, even in a scrip; and his sling was in his hand and he drew near to the Philistine.
>
> 49. And David put his hand in his bag and took thence a stone, and slang it, and smote the Philistine in his forehead, that the stone sunk into his forehead; and he fell upon his face to the earth.

Figure 3.1.1
From Brandl E.J. (1973) Australian aboriginal paintings in Western and Central Arnhemland.

At what date it entered the head of some sagacious specimen of homo sapiens to decide upon a missile that would perform its sombre duty and return to the hand of the thrower is beyond recall. Archaeologists now put the genesis of the boomerang at upward of 16,000 years ago.

The earliest find is that of a mammoth tusk boomerang from a cave in Oblazowa Rock in South Poland reported by Powel Valde-Nowak et al. (1987).°* Not dissimilar boomerangs came from Cape York Peninsula + and Wyrie Swamp ° in Australia. The boomerang from Wyrie swamp is made from the wood of Casuarina stricta.

°* Pawel Valde - Nowak Adam Nadachowski and Mieczyslaw Wolsan. Upper Palaeolithic boomerang made of a mammoth tusk in South Poland. Nature 329 (1987)= pp 436-438.

\+ Floud. J.M. Tresize P. A boomerang from Cape York Peninsula. Australian Archaeology. 13 (1981) p 95-96.

° Luebbers. R.A. Ancient boomerangs discovered in South Australia. Nature 253 (1975) p39.

Another European discovery is that of an Iron Age boomerang of oak c300 BC found in the Netherlands.# Fig 3.1.2. Details of gyroscopic forces involved in the flight of a boomerang are given by Jacques Thomas© and by H. Peter*°

A full mathematical analysis of the dynamics of a boomerang in English is late in its arrival (1968) and is due to Dr N. NPEYE°° of the Institute de Mathematicque in Liege Belgium. I give his analysis below with acknowledgement to Zeitschrift für Angewandte Mathematik and Mechanik. It is instructive to compare Dr Npeye's analysis with the extensive analysis in German of Werner Stille in c1872.***

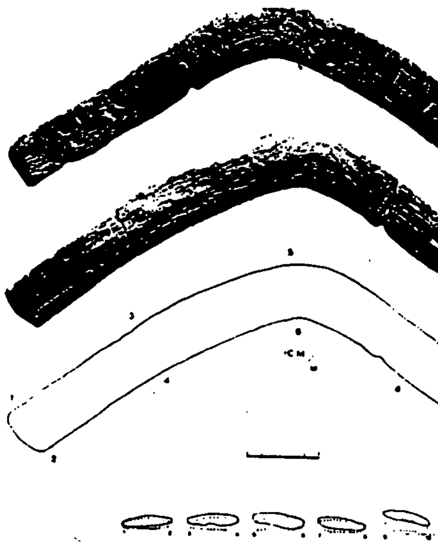


Figure 3.1.2 Iron age boomerang

Photograph by courtesy of Antiquity

Hess F. Antiquity 47. (1973) p303.

© Jacques Thomas. Why boomerangs boomerang (and killing sticks don't.) New Scientist 99 (1983) pp. 838-843.

*° Peter. H. Wesen und Bedeutung des Bumerangs. in. Veroffentlic hungen zum Archiv für Volkerkunde. 2 W. Braumuller. Vienna (1986) pp.289 + plates. Note: The plates show aboriginal manufacture of a high quality boomerang.

°° Npeye. N. The Mathematical Description of the Return Motion of Boomerang - type Projectiles. Zeitschrift für Angewandte Mathematik und Mechanik 48 (1968) pp. T272-T273.

*** Werner Stille. Versuche und Rechnungen zur Bestimmung der Bahnen des Bumerangs Annalen der Physik und Chemie.Ed. Poggendorff. Band. CXLVII Vol. 223 NF 147 (1872) pp. 1-21.

Motion of an Rotating Disk, in a Uniform Gravitational Field, Submitted to a Constant Normal Force.

We shall assume that the total aerodynamic force is the sum of two terms F and T, where T is the airscrew thrust. In the following, the expression "aerodynamic force" will only refer to the component F.

The simplest case we may think of is the one where the aerodynamic force is negligible compared to the thrust and to the gravitational force g, which are both assumed to be of the same order of magnitude. This should be a rough first approximation for a few tenths centimeters diameter symmetrical air screw mounted on a thin ring. The width of the blades measuring about two centimeters, the area under air action is very small. These conditions together with a relatively small translational velocity (about 10 m/sec) enable us to assume that both drag and lift are much smaller than the airscrew thrust.

The moment of external forces about the centre of inertia is zero, since the thrust acts at the centre of the disk. The rotational motion, in a first approximation, simply reduced to an EULER-POINSOT* motion. Furthermore, owing to the constant spin, the thrust, which has been assumed to be a function of the spin only, is constant too. The solution of the rotational motion in terms of standard EULERian angles, is

(1) $\theta = \text{Cle}: \quad \dot{\phi} = \text{Cle}.$

For simplicity the initial conditions are chosen such that the constant direction of the momentum is vertical. Assume that $q_o = 0$ the frequency of the steady precession is given by

(2) $$\dot{\phi} = V = P_o\sqrt{1 + \frac{2n^2}{P_o}}$$

where the subscript 0 refers to initial value and n denotes the spin $n = r_o$. If $\chi_i (i = 1,2,3)$ are the components of the position-vector of the centre of inertia along the unit vectors of a fixed coordinate system $0\ x_1\ x_2\ x_3$ (with $0x_3$ -axis vertical, positive downward), the equation of the translation may be written as follows:

(3) $\ddot{x}_1 = - T \sin\theta \sin\phi$

(4) $\ddot{x}_2 = + T \sin\theta \cos\phi$

(5) $\ddot{x}_3 = - T \cos\theta + g$

The initial conditions are, at t = 0

(6) $x_1 = x_2 = 0; \quad x_3 = x_{30}; \quad \dot{x}_1 = \dot{x}_{10}; \quad \dot{x}_2 = 0; \quad \dot{x}_3 = - \dot{x}_{30}; \quad \dot{\phi} = 0.$

Straight integration of the system (3)-(4) yields the following expressions for the velocity yaw angle χ* and the pitch angle γ** [5],

(7) $$\tan\chi = \frac{\sin vt}{\cos vt - (1 - K_1)}$$

(8) $$\tan\gamma = \frac{\frac{gt}{u}\left(1 - \frac{T}{g}\cos\theta\right) - K_1}{\sqrt{1 + (1 - K_1)^2 - 2(1 - K_1)\cos vt}}$$

where $u = \frac{T \sin\theta}{v}$, and K_i (i = 1,2,3) are the dimensionless initial velocity: $K_i = \chi_{io}/u$. It may be pointed out that when $K_1 = 1$, the horizontal projection of the trajectory is described at a constant velocity, namely u.

* See Leimanis. E. (1965).

Clearly the disk will describe a return motion if during the first half period of the precession, tan χ passes by infinity and zero.

Now

(9) $\tan \chi = 0$ when $vt\,(o) = \pi$,

(10) $\tan \chi = \infty$ when $vt\,(\infty) \doteq \arccos\,(1 - K_1)$

and is a monotonously increasing function of time if $K_1 < 2$. Thus for a return motion to be possible, the following conditions must be fulfilled.

(11) a) $K_1 < 2$,

(12) b) $T/g \leqq 1$.

Besides these conditions if t_1, is such that $\chi_1(t_1) = 0$, it may be required that the disk does not reach the ground before this moment; t_2 is the solution of the equation

(13) $vt_1 = \arcsin\,[(1 - K_1)vt_1]$

In fact a good return motion must still be such that the projectile comes back as near to the thrower as possible.

3.2 PULVERISING MILL

An early mechanical device making use of gyroscopic forces in heavy equipment is that of the pulverising mill or grinding mill - (Fig 3.2.1).

Other types of grinding machines in which gyroscopic forces are used to augment the centrifugal forces are described in extenso by GRAMMEL. R. (1917)°.

There is no detailed account in the English tongue; but the subject is taken in part by FERRY. E.S. (1932)+ and by ARNOLD R.N. and MAUNDER L. (1961).#

Ferry gives a full calculation for the forces exhibited by what he calls the Griffin Pulverising Mill** which I give below:-

The roll of a Griffin pulverising mill weighs 880 lb. and is 8 in. thick. The diameter of the upper face is somewhat greater than that of the lower face and the mean diameter is 23 in. The roll is fastened rigidly on the end of a shaft having a diameter of 5.75 in. and mass of 600 lb. The length of the shaft from the point of suspension to the upper face of the roll is 6 ft. The roll moves around the inside wall of a pulverizing ring having a diameter of 40 in. Find the force with which the roll presses against the pulverising ring when the roll shaft is making 165 r.p.m.

Solution. The total force against the inside of the ring is the sum of the centrifugal force due to the roll and axle rotating about the vertical axis with angular speed ω_z, and the force due to the gyroscopic torque developed when the roll and axle are caused to spin with angular speed ω_s, and also to rotate about the vertical axis with angular speed ω_z. The centrifugal force due to the rotating roll and shaft is

$$F_c = m_r \omega_z^2 l_1 + {}^1/_2 m_s \omega_z^2 l_2 = \omega_z^2 (m_r l_1 + {}^1/_2 m_s l_2) \tag{1}$$

and the gyroscopic torque acting on the roll and shaft is

$$LB = K_s(\omega_s + \omega_z \cos\theta)\omega_z \sin\theta - KA\omega_z^2 \sin\theta \cos\theta \tag{2}$$

where

m_r = mass of roll = $\frac{880 \text{ lb}}{32.1}$ = 27.4 slugs

m_s = mass of shaft = $\frac{600 \text{ lb}}{32.1}$ = 18.7 slugs

θ = deflection of spin-axle from the vertical = $\sin^{-1} \left(\frac{8.5}{72}\right) = 6° \ 40'$

$\sin\theta = \tan\theta = 0.12$; $\cos\theta = 0.99$

° Entitled, Kurvenkreisel und Kollergang.

\+ See pages 92-94 (Chapter 2 art 54)

\# See Pages 171-176.

** The Griffin Pulverizing mill as described by Ferry is from my researches that of BRADLEY P.B. (1894). See U.K. Patent Specification 19824 of 1894.

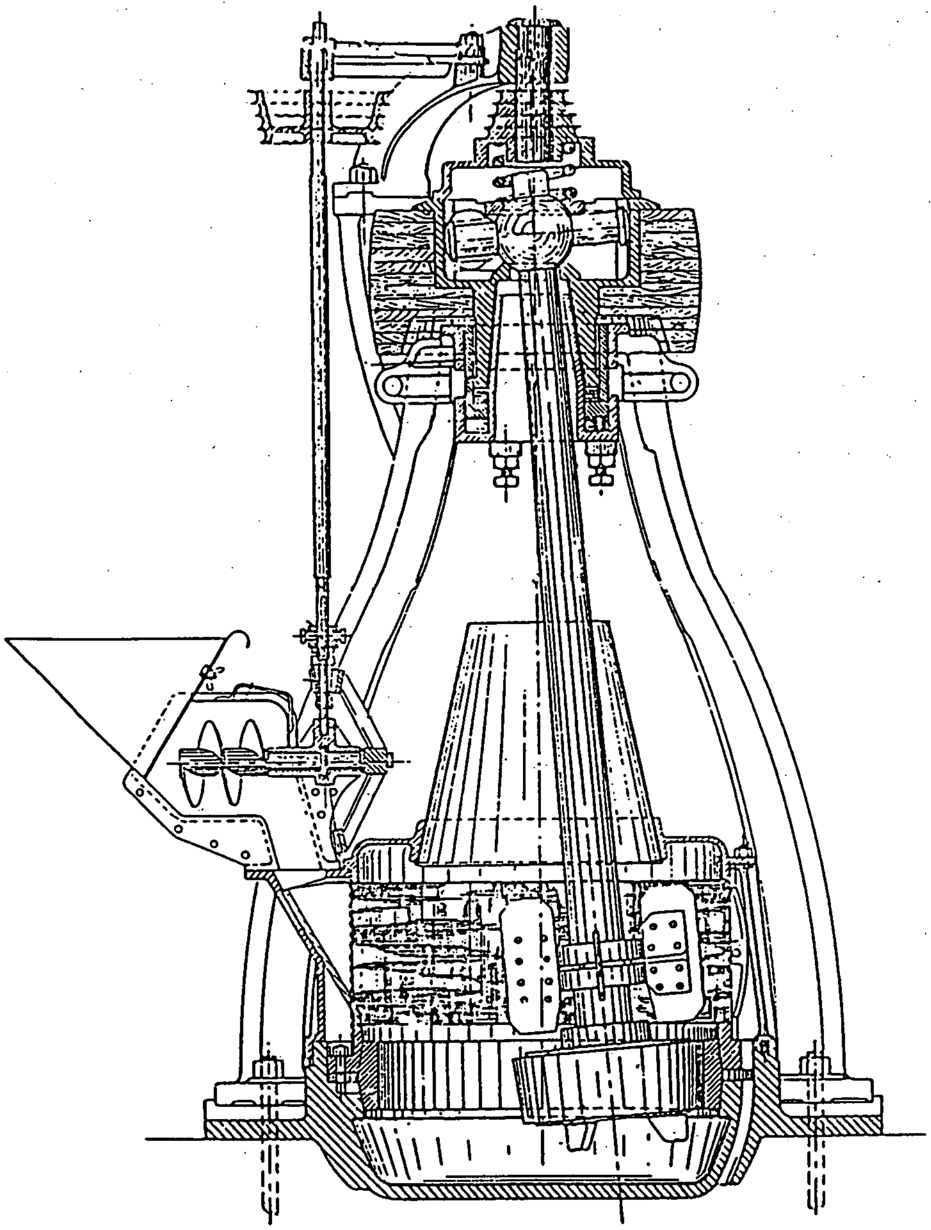


Figure 3.2.1 Pulverizing Mill of Bradley P. B.
G.B Patent Specification 19824 of 1894

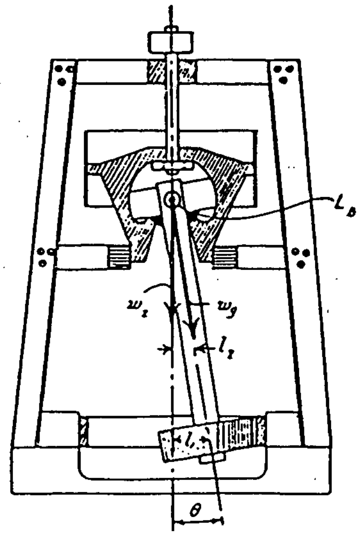


Figure 3.2.2

l_1 = distance of centre of mass of the roll from the vertical line through the point of suspension of the shaft
= 6.33 sin θ ft = 0.75 ft.

l_2 = distance of centre of mass of shaft from the vertical line through the point of suspension = $\frac{0.5(8.5)}{12}$ ft.
= 0.35 ft.

l_s = length of shaft = 6ft.

l_r = length of roll = $\frac{8}{12}$ ft = 0.66 ft

d_s = diameter of the shaft $\frac{5.75}{12}$ ft = 0.48 ft.

d_r = diameter of roll $\frac{23}{12}$ ft = 1.9 ft.

ω_s = angular speed of the shaft about its axis = $\left[\frac{165}{60}.2\pi\right]$ = 17.2 rad. per sec.

ω_z = angular speed of roll and shaft about a vertical axis through the point of suspension.

While the roll goes once around the inside of the ring, it rotates about the axis of the shaft.

$$\left[\frac{40}{23} - 1\right] \text{ times} = 0.74 \text{ time}$$

Therefore,

$$\omega_z = -\frac{17.2 \text{ rad. per sec.}}{0.74} = -23 \text{ rad. per sec}$$

The negative sign is used to indicate that ω_z is in the sense opposite that of the spin-velocity ω_s.

The moment of inertia of the roll and shaft relative to the spin-axle OC,

$$K_s = 1/8\ m_s d_s^2 + 1/8\ m_r d_r^2 = 1/8\ (m_s d_s^2 + m_r d_r^2)$$

$$= 1/8\ (18.7 \times 0.23 + 27.4 \times 3.6) = 12.9 \text{ slug-ft.}^2$$

$$K_A = m_s\left[\frac{d_s^2}{16} + \frac{l_s^2}{3}\right] + m_r\left[\frac{d_r^2}{16} + \frac{l_r^2}{3} + 36\right] = 1222 \text{ slug-ft.}^2$$

On substituting in (1) the values now obtained we find for the cėntrifugal force

$$F_c = 529\left(27.4 \times 0.75 + \left(\frac{18.7 \times 0.35}{2}\right)\right) = 12{,}590 \text{ lb-ft.}$$

and from (2), we find the value of the gyroscopic torque acting upon the shaft and roll to be

$$L_B = 12.9\ (17.2 - 23 \times 0.99)(-23 \times 0.12) - 1222 \times 529 \times 0.12 \times 0.99$$
$$= -76{,}600 \text{ lb.wt.ft.}$$

The gyroscopic torque acting upon the pulverizing ring is equal and opposite to this. The force perpendicular to the axle radially outward from the center of the pulverizing bowl produced by this gyroscopic torque equals the torque divided by the distance from the point of support to the center of the roll, or

$$F_g = \frac{76{,}600 \text{ lb.wt.ft.}}{6.33 \text{ ft.}} = 12{,}100 \text{ lb.wt.}$$

The sum of the horizontal components of the centrifugal and the gyrostatic forces is

$$(12{,}590 + 12{,}100) \cos 6°40' \text{ lb.wt.} = 24{,}400 \text{ lb.wt.}$$

When the roll shaft is deflected from the vertical through an angle θ, there is a horizontal force pulling it back, equal to

$$(880 + 1/2\ 600) \tan\theta = 1180(0.12) = 142 \text{ lb.wt.}$$

Consequently, the total force with which the roll pushes horizontally against the pulverizing ring is

$$(24{,}400 - 142) \text{ lb.wt.} = 24{,}258 \text{ lb.wt.}$$

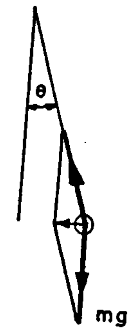


Figure 3.2.3

3.3 EARLY GYROSCOPIC APPLICATIONS AND THE WORK OF E.A. SPERRY

Following the discovery of the gyroscope a number of fine philosophical instruments were produced c.1860 as seen from the brass gyroscope of J.B. Dancer (Fig 3.3.1) and the polytrope of G.E. Sire of 1862 (Fig 3.3.2).

In 1883 we have a full description by Plucker, in German, of an overhung gyroscope and we see the Obry gyroscope of 1894 incorporated with devastating effect into the Whitehouse sea torpedo (Fig 3.3.3). In a short time the modern world of gyroscopic control and guidance is introduced by the work of Elmer Sperry and Lawrence Sperry of New York (Fig 3.3.4) in the USA and by Anschutz at Neumühlem near Kiel in Germany (Fig 3.3.5).

It was Max Schuler working with and for Anschutz who showed that a period of 84.4 minutes of time*, the oscillation time of a simple pendulum the length of which is equal to the radius of the Earth will allow a suitable tuning that is today exhibited in many gyroscopic devices such as for example the fine inertial platform of Societé Commercial ECA of 1954 (Fig 3.3.6).

* This calculation was first performed by Huygens in his Discours de la Cause de la pesanteur. Leyden (1690). The modern calculation using a mean radius/R of the Earth of 6,367,447m and g as 9.780310m.s^{-2} gives the period T=2 R/g as 84.4957 minutes of time.

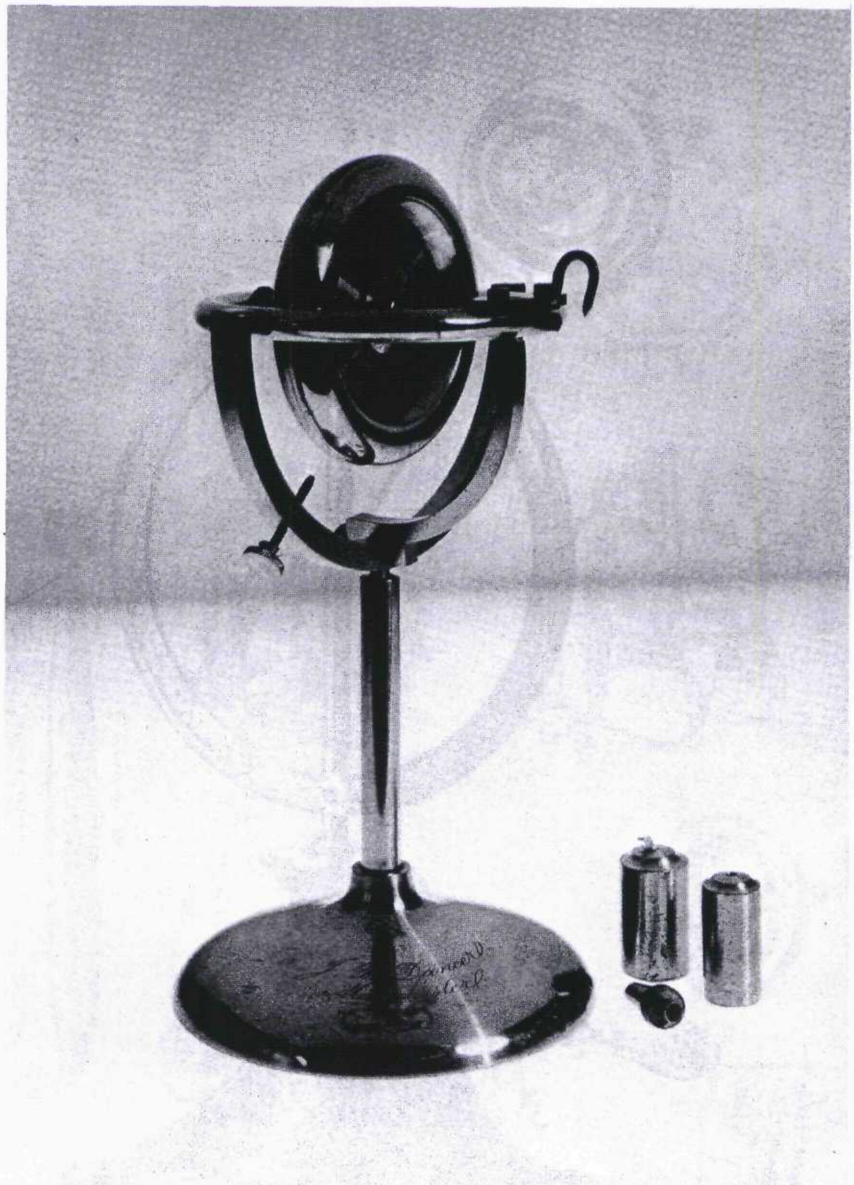


Figure 3.3.1

Brass Gyroscope, signed: J.B. Dancer Manchester. An early example of a gyroscope by John Benjamin Dancer (1812-87).
By courtesy of Museum of the History of Science.
Oxford University.

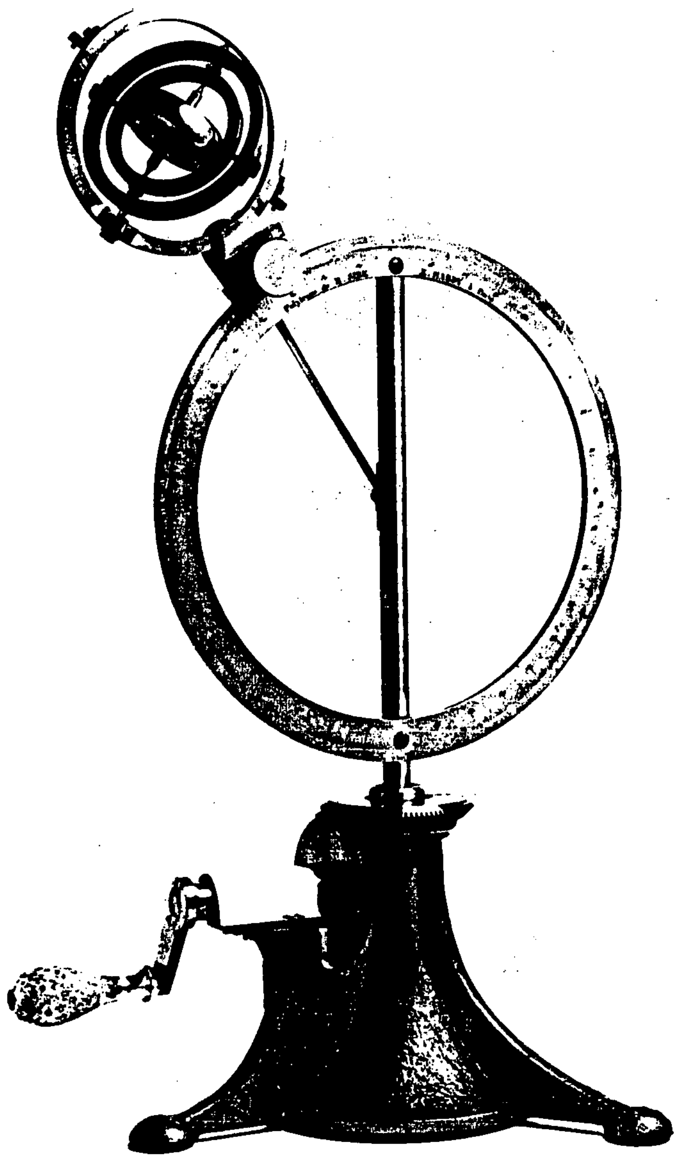


Figure 3.3.2

A development of the gyroscope, signed: Polytrope de M. Sire E. Hardy a Paris. The brass ring represents a meridian on the Earth, and the vertical axis represents the polar axis. This demonstration piece was purchased in 1861 by the Teyler's Museum for Dfl, 234. Undoubtedly a model proposed by G.E. Sire to instrument maker E. Hardy. Paris. c.1860.

By courtesy of the Teyler's Museum, The Netherlands.

For description see W.J. Sears. Lieut. U.S. Navy. U.S. Naval Institute. Vol. 29 No. 1.

Engineering 66 July 15 (1898) pp 89-91.

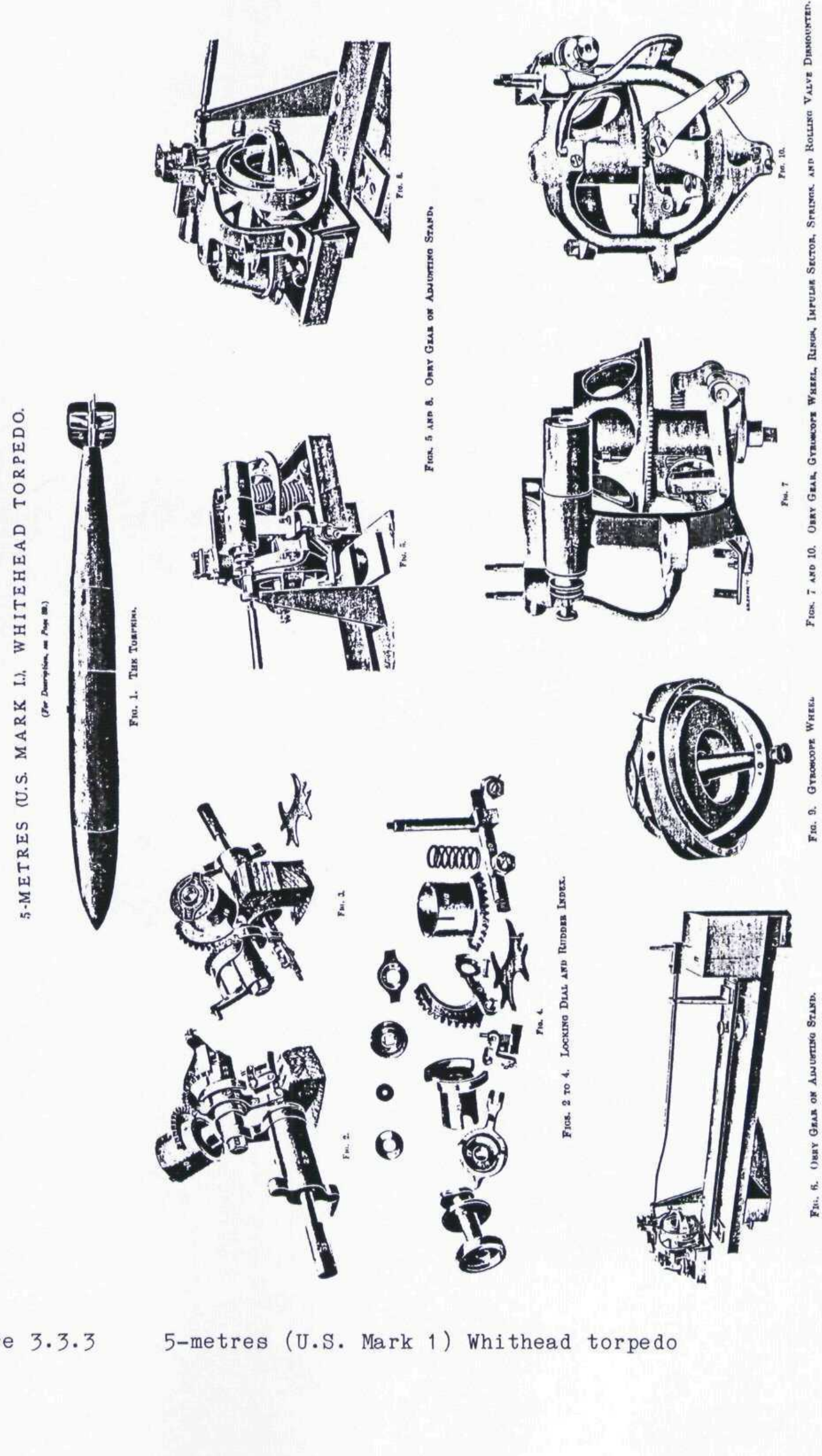


Figure 3.3.3   5-metres (U.S. Mark 1) Whithead torpedo

E. A. SPERRY.
GYROSCOPIC APPARATUS.
APPLICATION FILED JULY 11, 1912.

1,186,856. Patented June 13, 1916.
5 SHEETS—SHEET 1.

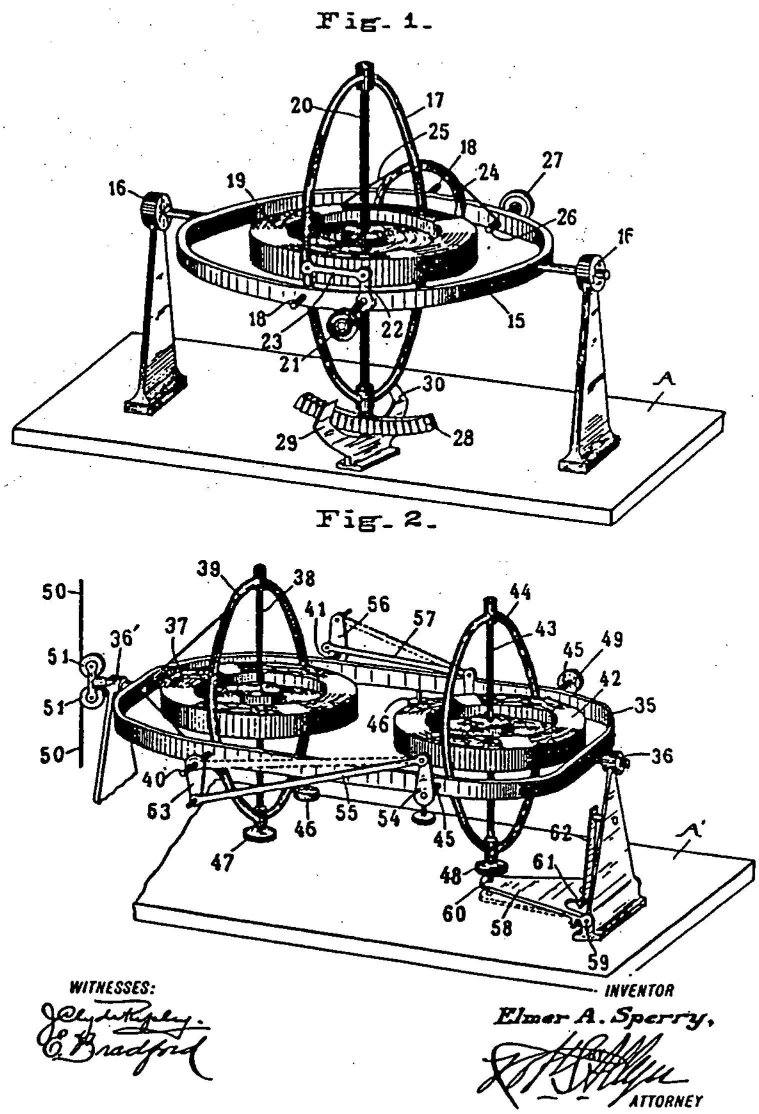


Figure 3.3.4a E.A. Sperry. Gyroscopic apparatus

U.S. PATENT SPECIFICATIONS TO E.A. SPERRY RELATING TO THE GYROSCOPE

1915	1150311	Ship' gyroscope.
1916	1186856	Gyroscopic apparatus.
1917	1232619	Ship stabilizer and rolling apparatus.
1917	1236993	Gyroscopic stabilizer.
1917	1238503*	Automatic gun pointing
1917	1242065	Ship's gyroscope compass set.
1918	1255480	Gyroscopic navigation apparatus.
1918	1274622	Speed and direction indicator for aircraft.
1918	1279471	Gyroscopic compass.
1919	1296440	Repeater system for gyro-compass.
1919	1301014	Electric drive for gyroscope.
1919	1312084+	Torpedo gyroscope.
1919	1312085	Stabilizing gyroscope.
1919	1314157	Gyroscopic apparatus.
1919	1318302	Rotor for gyroscopic stabilizer.
1920	1342397	Controlling mechanism for ship's gyroscopes.
1920	1356505	System of gun fire control
1920	1358258	Stabilizing gyroscope.
1920	1360694	Navigational apparatus.
1921	1368226	Aeroplane stabilizer.
1921	1378291	Driving and governing means for torpedoes.
1921	1393845	Gyroscope (bearings).
1921	1399032	Gyroscope roll and pitch record.
1922	1418335	Automatic pilot for aeroplanes.
1922	1421854	Gyroscopic apparatus for torpedoes.
1922	1426336	Rotor for gyroscopes.
1922	1426339	Wire wound gyro rotor.
1923	1445805	Gyroscopic navigational apparatus.
1923	1446276	Gyroscopic apparatus for torpedoes.
1923	1452482	Rolling of ships.
1923	1452484	Gunfire control for battleships.
1924	1483992	Spinning-up gyro.
1924	1499321	Gyro-Compass.
1925	1522924	Position indicator.
1925	1527932	Alarm device for gyro-compass.
1925	1558514	Multiple gyro ship stabilizer.
1925	1560435	Apparatus for testing gyro compass.
1925	1563934	Gyroscopic inclinometer for aeroplanes.
1927	1626123	Gyroscopic relay transmitter.
1927	1651845	Gyroscopic pendulum.
1928	1688559	Gyroscopic line of sight stabilizer.
1929	1704489	Locking device for gyroscopes.
1930	1778734	Stabilization device.
1931	1788807	Gyro control.
1931	1800365	Means for preventing pitching of ships.
1931	1812994	Bore hole indicator.
1932	1843959	Track record system.
1932	1867334	Automatic steering mechanism for dirigible aircraft.
1932	1880994	Indicator for aircraft.

* With Fiske. B.A.

\+ With Meitner. E.

In this 75th anniversary year of Sperry Corporation we are proud to join the United States Postal Service in honoring Elmer and Lawrence Sperry.

This limited-edition first day cover envelope was postmarked on the day the stamp was issued. The ceremony took place at the Cradle of Aviation Museum, which is located at the former site of Mitchel Field, in Garden City, Long Island. Mitchel Field was where much of the Sperrys' pioneering work in aviation took place.

Born in 1860, Elmer Sperry was a nationally known inventor by the time he founded the Sperry Gyroscope Company at Brooklyn, N.Y., in April 1910 to research practical uses for a device that, up until that time, had been little more than a child's toy—the gyroscope.

Lawrence Sperry joined his father in the business as an inventor in his own right and as test pilot.

In 1911 Sperry introduced the first shipboard gyrocompass, designed to replace the magnetic compass that gave unreliable readings on the U.S. Navy's all-metal ships. From those beginnings, the Sperrys moved into the aviation industry and developed the first true flight instruments and such other "firsts" as the aircraft autopilot, turn indicator, artificial horizon, retractable landing gear, and American-designed parachute.

All of us at Sperry Corporation are honored to continue in the great tradition of our founder and his son —providing high quality, innovative solutions through technology.

Gerald G. Probst
Chairman and Chief Executive Officer

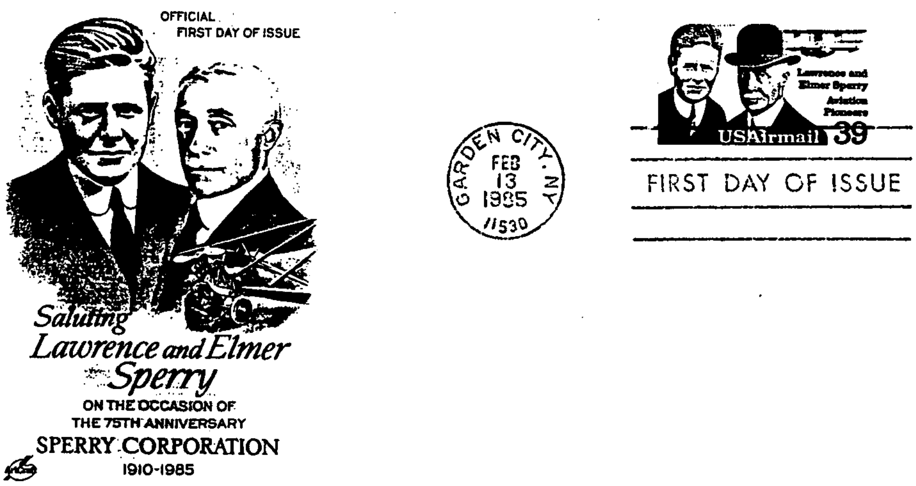


Figure 3.3.4b

Fig.1.

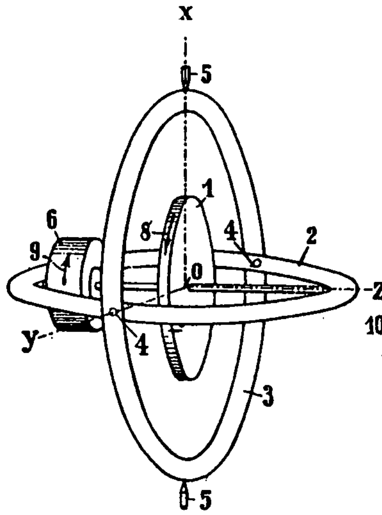


Fig.2.

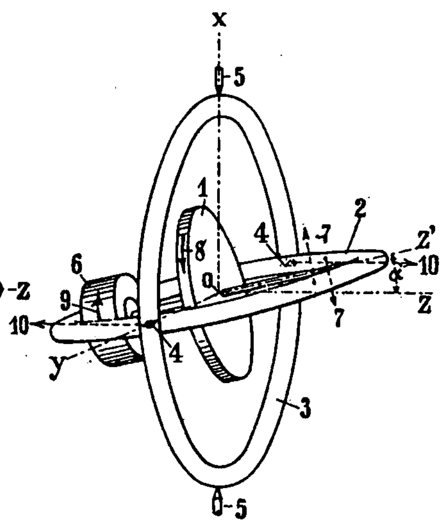
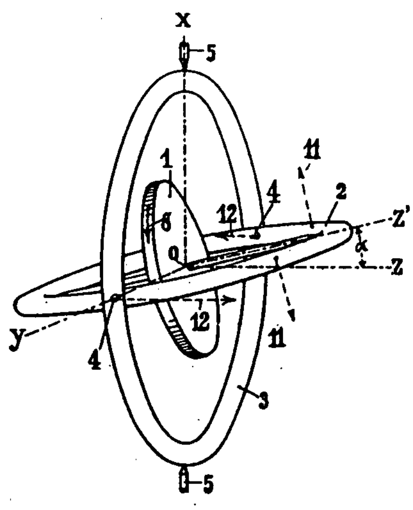


Fig.3.



332. Gyroscopes. ANSCHÜTZ & Co., Heikendorfer Weg, Neumühlem, near Kiel, Germany. Jan. 4. [*Convention date, Jan. 5, 1911.*] *Classes* 9 (ii) *and* 97 (iii).]

Relates to gyroscopes for steering torpedoes, of the kind in which the rotor 1 is started by a spring and then driven by an electric motor 6, the stationary part of which is carried by the inner gimbal ring 2. According to the invention, the electric motor at first accelerates the speed imparted by the spring in order to reduce any initial elevation of the inner gimbal ring and to slow down the precessional movement.

FIG.I.

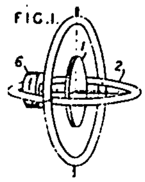


Figure 3.3.5

Anschutz and Co's Specification No. 332
4th Jan, 1912.

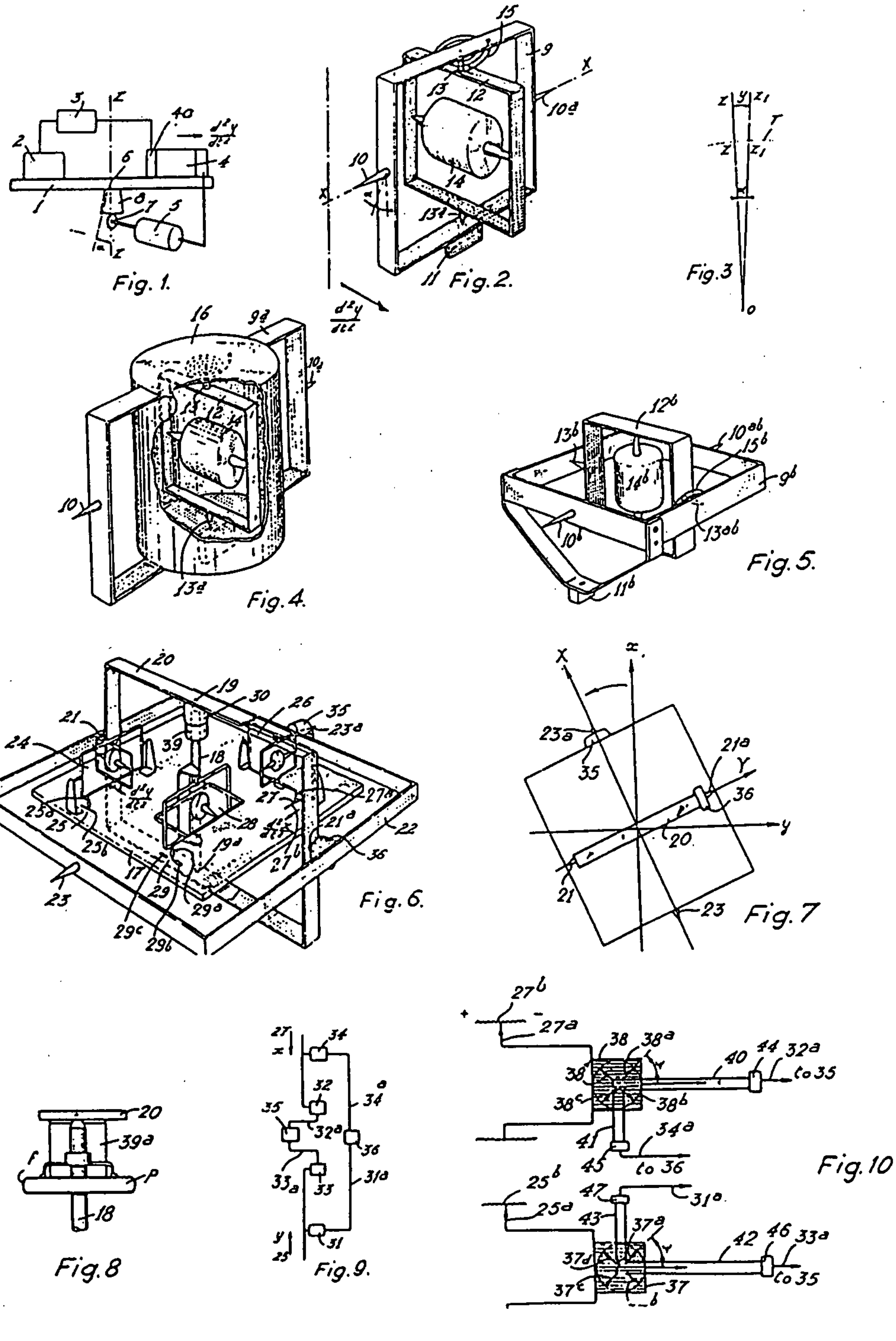


Figure 3.3.6 Inertial platform of Société Commercial ECA.

4. THE DIFFERENTIATING GYROSCOPE

A rate gyroscope is one that is constrained to a single degree of freedom (its spin axis) and with a limited rotation about its torque axis or axis of modes. (axis Oy. Fig. 4.1). From Fig. 4.1, it can be seen that the outer gimbal frame is rigidly fastened to a platform that is able to turn about a substantially vertical axis Ox, through the centre of gravity (o) of the gyroscope rotor (1). Such a rotation is a forced precession of the spin axis Oz about the input axis Ox, and this induces a gyroscopic reaction moment about the torque axis Oy that is resisted by the spring (3). The gyroscopic reaction moment is proportonal to the angular velocity of rotation of the platform and the moment of the spring is proportional to the angle of turn of the frame. Hence the angle of turn of the frame is proportional to the angular rotational velocity of the instrument casing. A damper (4) is used to deaden the oscillation of the frame (2). A more accurate explanation of the modus operandi is due to FIRDLENDER & KOSLOV (1961) when the instrument casing rotates about the measuring axis the axis of rotation of the frame is acted on by the reaction forces of the bearings F1 and F2. The direction of the moment L_1, due to these forces coincides with the direction of rotation of the casing. Under the influence of the moment L_1 the gyroscope frame continues turning about the axis Oy in a direction such that the vector of the angular momentum is aligned with the vector of the external moment L_1 and therefore with the vector cas of the rotational velocity of the casing. The turning of the frame produces a tightening of the spring (3) that causes a moment in the spring Ls directed along the frame axis Oy. This moment strives to turn the frame. Its action, however, in accord with the properties of the gyroscope, leads to an angular velocity of precession pr the direction of which coincides with the direction of the angular rotation of velocity of the casing. At first, when the angle of rotation of the frame is small and the velocity pr is small the reaction forces in the bearings B_1 and B_2 are retained, causing a further turning of the frame about its axis. The frame keeps rotating until it has turned to an angle at which the moment of the spring Ls creates an angular velocity of precession pr equal to the angular rotational velocity of the casing cas. In this case the reaction forces in the bearings B_1 and B_2 vanish and rotation of the frame about the axis Oy is due only to the moment of the spring Ls and the casing makes no more contribution. The damper ensures that the oscillations of the frame are deadened during transition from one steady-state position of the frame to another.

The angular velocity of precession pr is proportional to the moment of the spring, which in turn is proportional to the angle of rotation of the frame. Since in the steady state pr = cas, the angle of rotation of the frame is proportional to the angular rotational velocity of the casing cas about the axis Ox. Clearly, the direction of rotation of the frame is determined by the direction of rotation of the casing.

The rate gyroscope reading can be taken visually from a pointer (5) or from a pick-off (6). It can be shown that if the input of a rate gyroscope is an angle due to the applied moment, then the resultant output is an angular velocity that is proportional to the input angle, that involves the differentiation of an angle; hence its name. More explicitly FIRDLENDER and KOSLOV (1961) derive the equation of motion for the frame of a rate gyroscope and its transfer function which it is valuable to follow. They consider the equation of motion with respect to the axis. Oy, the one that describes the turning of the frame. (See Fig. 4.1).

Let. $$J_e \frac{dq}{dt} - Hp = Ly$$

$$q = \frac{dB}{dt} + \frac{dBtr}{dt} \qquad (1)$$

$$p = \omega_{cas} \cos\beta + \omega_1 \sin\beta$$

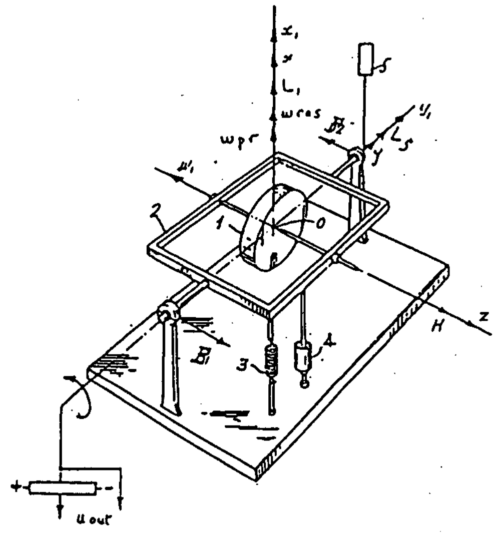


Figure 4.1

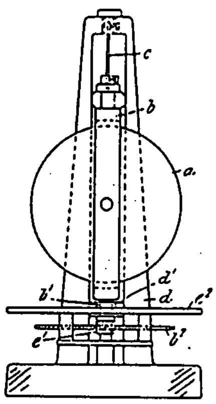


Figure 4.2.1

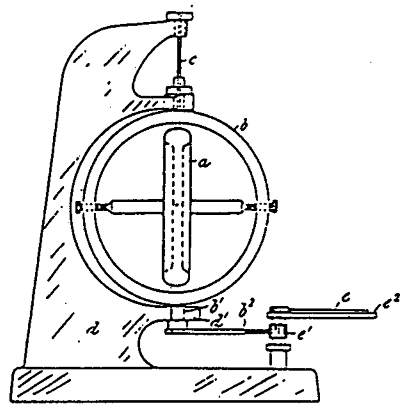


Figure 4.2.2

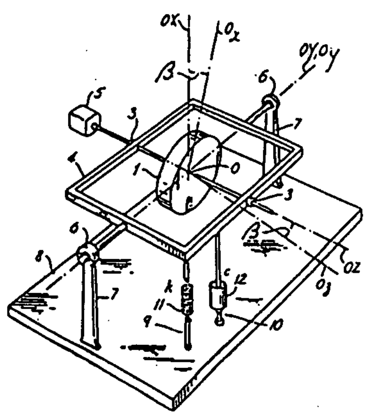


Figure 4.3

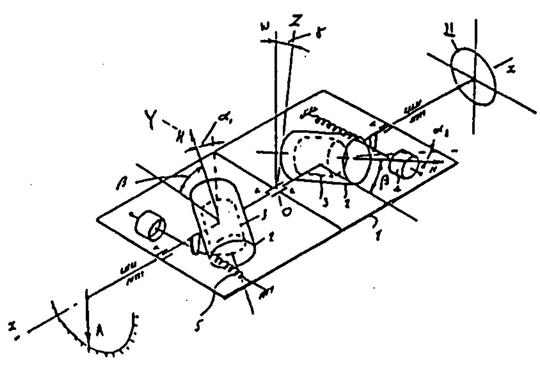


Figure 4.4

where p and q are projections of the angular rotational velocity of the gyroscope with respect to the axes. Ox and Oy, respectively;

J_e is the equatorial moment of inertia of the frame plus rotor with respect to the axis Oy;

β is the angle of rotation of the frame with respect to the casing;

ωcas is the angular rate of turn of the casing with respect to the measuring axis Oz;

ω_1 is the angular velocity of rotation of the casing with respect to the axis perpendicular to the frame axis and measuring axis. Ox_1;

$L\beta$ is the external moment acting on the gyroscope frame;

$\frac{d\beta tr}{dt}$ is the angular velocity of the casing about the axis. Oy (transfer).

The external moment is composed of the moment of the spring L_2, proportional to the angle of turn of the frame with respect to the casing, the moment of the damper L_d, proportional to the angular rate of the turn of the frame with respect to the casing, and a random or indefinite moment L_r. The random moment Lr is formed by the residual imbalance of the frame with respect to its axis, the load imposed on the frame by the current lead-in wires and to the effect of friction in the frame bearings.

Thus $L_\beta = L_s + L_d + L_r$

and $L_\beta = -c\beta - K_d \frac{d\beta}{dt} + L_r$ (2)

where c is the rigidity factor of the spring and k_d is the damping factor.

Substituting the expressions for the external moment (2) into (1) and dividing the whole equation by J_e, we get

$$\frac{d^2\beta}{dt^2} + \frac{k_d}{J_e}\frac{d\beta}{dt} + \frac{c}{J_e}\beta = \frac{H}{J_e}\omega cas \cos\beta + \frac{H}{J_e}\omega_1 \sin\beta + \frac{L_r}{J_e} - \frac{d^2 \beta t_r}{dt^2} \tag{3}$$

Introducing the new quantities

$$\frac{c}{J_e} = \omega_o^2 \quad \frac{k_d}{J_e} = 2d\omega_o$$

and assuming $\cos\beta$ = unity and
$\sin\beta = \beta$ since the angles of turn of the frame are always made small - not more than 15 degrees of arc, we get the following equation of motion of the frame.

$$\frac{d^2\beta}{dt^2} + 2d\omega_o\frac{d\beta}{dt} + \omega_o^2\beta = k\omega_o^2\omega_{cas} + k\omega_o^2 \beta\omega_1 + \frac{L_r}{J_e} - \frac{d^2\beta t_r}{dt^2}$$

where ω_o is the natural frequency of the gyroscope

and d is the relative damping factor for the gyroscope.

Thus the motion of the frame is described by a second-order differential equation and the transfer function may be obtained by dividing the angle of turn of the frame β by the angular rotational velocity of the casing ω_{cas}.

$$\frac{\beta}{\omega_{cas}} = \frac{k\omega^2{}_o}{p^2 + 2d\omega_o p + \omega_o{}^2}$$

where p is a Carson transform operator.*

A considerable patent literature is devoted to the construction and improvement of the differentiating gyroscope in its more usual form of the rate-of-turn instrument; so valuable and widely used in aircraft.

The basic construction using a mechanical torsion spring (c) Figs. 4.2.1 and 4.2.2 was clearly described by WIMPERIS H.E. and ELPHINSTONE G.K.B. GB Patent Specification 7285 (1910). Much ingenuity has been directed to the constraint via the spring including bimetallic compensation (G.B. 579909) aligned torsion springs (G.B. 587719) springs plus silicone fluid (G.B. 886391) and a torsion bar suspension (G.B. 900138). Some one hundred and forty United Kingdom patent specifications are directed to advances in the differentiating gyroscope. It is not without interest to show that one of the most recent advances in the instrument is a refinement in the basic instrument, and we turn to a disclosure due to NATIONAL RESEARCH DEVELOPMENT CORPN. G.B. 1508302 (1942) for MAUNDER L. and BURDESS J.S. The basic instrument is well shown at Fig. 4.3. It is pointed out that an input angular velocity Ω produces an output angular deflection β where $\beta = \frac{CN\Omega}{K_1}$, where

N is the constant spin velocity of the rotor.

C is the principal inertia of the rotor about its spin axis and K_1 represents the stiffness of the spring connecting the gimbal to the platform; and that sensitivity is made difficult to improve by the practical limits imposed in the ranges of values if K_1, C and N. Thus it is not easy to make the spring very large and thus K very small, similarly one cannot make the velocity N very large or increase the rotor to an infinite size by increasing C. It is then proposed to spin the rotor at an angular velocity n, at a time t such that

$N = N_o + N_1 \cos\phi t$ where

N_o is the mean speed of the rotor and can take any value including zero, N_1 is the maximum value of the fluctuating component of the speed, that is the maximum difference between n and N_o.

ϕ is a constant that determines the frequency at which the speed variation takes place and t is the time. The respective values of N_o and N_1 will in general depend upon the nature of the rotor drive system. Preferably the value ϕ is chosen to be substantially equal to $\sqrt{\frac{K_1}{A_o}}$ where A_o is the sum of the inertias of the rotor and gimbal about the axis of rotation of the gimbal.

For this arrangement the response of the gyroscope or its output angular deflection of the gimbal is given by

$$\beta = \frac{CN_o\Omega}{K_1} + \frac{CN_1\Omega \sin \phi t}{K_2\phi}$$

* Re. Carson transform operator, see: SNEDDON I.N. (1976) p.86

By selecting the component $\frac{CN_1\Omega \sin \Phi t}{K_2\Phi}$ at this response of the output it can be seen that the amplitude of this component may be related directly to the input angular velocity Ω and can also be magnified or attenuated by controlling the values either of N_1 or of the viscous damping parameter K_2. The values of K_2 and N_1 are therefore determined by the required magnitude of $\frac{CN_1\Omega}{K_2\Phi}$.

Preferably the magnitude of the output $\frac{CN_1\Omega \sin\Phi t}{K_2}$ obtained from the proposed

rate gyroscope should be better by a factor $\frac{N_1}{2vN}$

$\underline{v}$ being equal to $\frac{K_2}{2\sqrt{K_1 A_0}}$ than that of an otherwise comparable rate gyroscope in which the rotor spins at a constant spin velocity N.

ROANTREE. J.P. and SAUNDERS. J U.S. 3925642 (1975) also propose a sinusoidal modulation of the speed of the gyroscope rotor.

Undesirable drift errors are introduced from the variation in rigidity of the spring with ageing of the spring material and elastic hysteresis; similarly with torsion elements even though for high radial and axial rigidity they are often used in the form of a cross; (See G.B. 933251) and frictional forces are reduced by the hydrostatic un-loading of bearings. To avoid these problems an electric spring may be used. The signal from the pick-off is fed to a torquer through an amplifier to impose a moment on the frame axis Oy. The moment is proportional to the signal and thus proportional to the angular deviation of the frame. An advantage of a rate gyroscope with an electric spring is that any variation in the pick-off feed voltage does not affect the pick-off output signal. If, for any reason, the pick-off feed voltage decreases, the rate gyroscope signal decreases and this, in turn, leads to a decrease in the moment applied to the frame by the torque device. As a result the gyroscope frame keeps turning until the gyroscopic moment becomes equal to the moment produced by the electric spring. This occurs during the return of the pick-off signal to its previous value. The rate-gyroscope with a mechanical spring is a meter of open type, but a rate-gyroscope with an electrical spring is a meter with feed-back; it is explained very fully by NIKITIN. E.A.and BALASHOVA. A.A. (1969). The complete equations of motion for rate gyros are given by LÜTZKENDORF. R. (1972).

An electrostatic rate gyroscope in which a hollow spherical rotor is electrostatically suspended in the gyroscope housing using an electrostatic capture system to null the gyroscope with respect to the housing is proposed by HOFFMAN et al U.S. 3902374 (1975) and CALLENDER STILES. J. U.S. 4061043 (1977).

An electrostatically-captured rotor rate gyroscope with electrostatic pick-off is proposed by FERRISS. L.S. U.S. 4068533 (1978).

An advanced gyroscope rate-range switching and control system is disclosed by JOHNSON J.S U.S. 4179087 (1979).

A superior gyroscopic rotor suspension in which the rotor is suspended by a radial flexure support member having a first spring centering gradient characteristic and by an axial flexure support means having first and second portions with second and third spring centering gradient characteristics respectively, in which the first and second spring centering gradient characteristics tend to be compensated by the third spring centering characteristic to minimize the elastic restraint of the suspension is due to QUERMANN. T.R. U.S. 3529477 (1970) which offers an advance over the rate of turn gyroscope of WING. W.G. U.S. 2719291 (1955).

A basic difficulty with a rate of turn indicator is that it often has variable angles of inclination with respect to the axis of revolution, as experienced when the aeroplane banks in a turn. If by α and ϕ we denote the angle of deviation of the figure axis from the equilibrium position and the angle of bank, and by K we denote the rigidity of the spring then the angular velocity ω of the turn then $K\alpha = H\omega \cos(\phi - \alpha)$

Thus the deviation α is not exactly proportional to the angular velocity ω but is somewhat dependent upon the angle of bank. This problem is explained in mathematically clear terms by BENDIX AVIATION CORPN. G.B. 562886 (1944) and solved by opposing the precession of the gyroscope by a force that is a function of the tangent of the angle through which the gyroscope precesses.

Two earlier constructions using a pair of gyroscopic rotors are to be found in the work of MINORSKY. N. G.B. 137060 (1921) and FLEMING. P.L. disclosed by SCOPHONY LTD G.B. 627123 (1949).

Another construction proposing a more sophisticated answer to the problem is the system of MAKSIMOV. V.V. (1963). He makes use of a pair of canted rotors and two degrees of freedom. The MAKSIMOV construction is shown in Fig. 4.4. The rotors (3) are identically inclined and mounted in gyroscopic housings (2) and each unit is supported by bearings (aa) in a frame (1). Both housings with the gyroscopic rotors are connected with the frame (1) through the springs (5) and buffers or dampers (4). The frame (1) has an axis of rotation on XX that coincides with the axes a a of the gyroscopes and is connected with the base of the system through the buffer or damper.

When the system turns with an angular velocity ω, the vector of which occupies a random position in the plane YOZ both inclined gyroscopes will revolve on the axes (a a) and turn by an angle of α_1 and α_2 determined by values of the gyroscopic movements and the rigidity of the springs (5).

After establishing the angles α_1 and α_2 the gyroscopic moments become equal to the moments of the springs, and

$$h\,\alpha_1 = H\omega \cos(\alpha_1 + \beta + \gamma)$$

and

$$h\,\alpha_2 = H\omega \cos(\alpha_2 + \beta - \gamma) \qquad (1)$$

in which equations h is the rigidity of the springs and ω is the angle between the vectors and the axis OZ. Since α_1 and α_2 are small

$$\cos(\alpha_1 + \beta + \gamma) = \cos(\beta + \gamma) - \alpha_1 \sin(\beta + \gamma)$$

and

$$\cos(\alpha_2 + \beta + \gamma) = \cos(\beta + \gamma) - \alpha_2 \sin(\beta + \gamma) \qquad (2)$$

and we obtain using relationships (1)

$$\alpha 1 = \frac{-H\omega \cos(\beta + \gamma)}{h - H\omega \sin(\beta + \gamma)}$$

$$\alpha 2 = \frac{-H\omega \cos(\beta - \gamma)}{h - H\omega \sin(\beta - \gamma)} \qquad (3)$$

MAKSIMOV assumes that the rigidity of the springs is in each case taken so high that the terms that contain the sines can be disregarded, and thus the moments of the springs are:-

$$h\,\alpha_1 = H\omega \cos(\beta + \gamma)$$

$$h\,\alpha_2 = H\omega \cos(\beta - \gamma) \qquad (4)$$

Clearly the frame under the influence of their difference will turn about axis XX.

Since $h\alpha_2 > h\alpha_1$ therefore the motion goes in the direction of the lessening of the angle Υ. When the angle Υ becomes equal to zero the system stops, since -

$$h\alpha_2 = h\alpha_1 = -H\omega \cos\beta \tag{5}$$

The angles of inclination of the gyroscopes on the frame now are proportional to the magnitude of the angular velocity and are equal to

$$\alpha 1 = \alpha 2 = \frac{H\cos\beta}{h}\omega \tag{6}$$

In this condition there is no cosine of the angle Υ, that is to say the inclination of the platform relative to the vector ω is nil.

The value $H\cos\beta$, here, may be termed the working kinetic momentum of each of the gyroscopes in their direct measurement of the angular velocity.

The difference in the moments that bring about the movement of the whole gyro system around the axis XX is equal to

$$M_\beta = - H\omega[\cos(\beta - \Upsilon) - \cos(\beta + \Upsilon)] = - 2 H\omega \sin\beta \sin\Upsilon \tag{7}$$

Assuming the centre of gravity of the whole system lies on the axis XX and that the friction of the bearings is small, the movement is

$$I_o \frac{d^2}{dt^2} = 2H\omega \sin\beta \sin\Upsilon \tag{8}$$

where I_o is the moment of inertia of the whole relative to the axis XX with damping excluded.

By multiplying equation (8) by $\frac{d\Upsilon}{dt}$ and integrating within the limits of 0 and Υ assuming $\Upsilon = 0$ and $\frac{d\Upsilon}{dt} = \dot{\Upsilon}o$ we get.

$$\frac{d\Upsilon}{dt} = \dot{\Upsilon}o\sqrt{1 - \frac{4H\omega}{Io\dot{\Upsilon}o^2}(1-\cos\Upsilon)} \tag{9}$$

$$t = \frac{1}{\dot{\Upsilon}_o} \int_o^\Upsilon \frac{d\Upsilon}{\sqrt{1 - \frac{4H\omega}{Io\dot{\Upsilon}o^2}(1-\cos\Upsilon)}} \tag{10}$$

The equation (8) is analogous to the equation of the motion of a physical pendulum.

On approaching the position of equilibrium the variable Υ becomes small and

$$\sin\Upsilon = \Upsilon$$

consider equation (8) in the form

$$Io\frac{d^2\Upsilon}{dt^2} + 2 H\omega \sin\beta\,\Upsilon = 0 \tag{11}$$

The solution of (11) is

$$\Upsilon = A\cos\left(\sqrt{\frac{2H\omega\sin\beta}{I_o}}\,t + \Phi\right)$$

where A and Φ are the constants of integration.

This shows that the canted rotor system without friction in the supports XX has non-damped oscillations around the position of equilibrium and will require damping.

5. THE INTEGRATING GYROSCOPE

The working principle of an integrating gyroscope is given by many authors, see for example FIRDLENDER and KOZLOV (1963) and WRIGLEY W (1963 at p.68).

It can readily be shown that the integrating gyroscope measures the angle of turn of its casing. The construction of the integrating gyroscope in its elemental form is not dissimilar to that of the rate gyroscope with the essential difference that there is no elastic counteracting spring. The construction is shown in Fig 5.1. When the instrument casing turns about the measuring axis OX_1, a moment L_1 is imposed upon the axis of rotation of the frame and the direction of the moment coincides with the direction of rotation of the casing ωc. This moment is transmitted to the frame via the reaction forces of the bearings at F_1 and F_2. The moment causes the rotor axis to precess and turn about the frame axis OY_1. The frame continues turning in this direction so that the vector of the angular momentum coincides with the vector of the applied moment L_1, and therefore with the vector of the angular velocity of the casing, thus the direction of turn of the frame is determined by the direction of turn of the casing. When the frame turns the damper (3) creates a moment L_d proportional to the angular velocity of the frame and when acted upon by the damper moment, the gyroscope rotor (1) starts to precess in the same direction in which the casing is turning. This creates an angular rotational velocity of the frame (2) for which the damper moment provides an angular precessional velocity about the axis OY_1, equal to the angular velocity of turn of the casing. Now, the damper moment is proportional to the angular rotational velocity of the frame and to the angular velocity of precession about the axis OY_1, equal to the rotational velocity of the casing; thus the angular rotational velocity of the frame is proportional to that of the casing. Consequently, the angle of turn of the frame is proportional to the angle of turn of the casing and this gives it the title of integrating gyroscope. In the rate gyroscope the angle of turn is proportional to the angular velocity of turn of the casing, but in the integrating gyroscope the angle of turn of the frame is proportional to the angle of turn of the casing, that is to say, to the integral of the angular velocity.

The use of the gyroscope to provide apparatus for the integration of accelerations in a moving body such as on shipboard was well known to HENDERSON Sir J.B. GB 269280 (1929) and some general references may be found also in United Kingdom Patent Specifications.
130095, 559327, 767069, 854393, 924968, 945387, 1021801, 1254385.

A gyroscopic force integrating apparatus suitable for use with a Schuler pendulum is disclosed for BARNES. J.W. by NATIONAL RESEARCH & DEVELOPMENT CORPN GB 698733 (1953). JAROSH. J.J. HASKELL. C.A. and DUNNELL. Jr. W.W. via MASSACHUSETTS INSTITUTE OF TECHNOLOGY GB 753449 (1956)* disclose their now famous single-degree-of-freedom gyroscope that provides via its damping fluid, a high accuracy integrator. The instrument is shown at Fig 5.2 and is described below:-
A case 100 has a single-degree-of-freedom gyroscope consisting of a gyro-rotor 102 which together with a stator 104 constitute a synchronous motor to drive the rotor at a high constant speed about spin axis S. The rotor 102 is mounted in a frame 106 that is rigidly attached to a shaft 110, the axis of which is the output axis O of the gyro. The gyro rotor-stator assembly is enclosed in an inner cylindrical casing 108 also rigidly attached to the shaft 110. Between the casing 108 and the outer case 100 there is a small clearance space 109 of but a few hundredths or but a few thousandths of an inch. This space is filled with a high density, high viscosity fluid to provide a damping means. The ends of the shaft 110 are journalled in bearings 112 fixed in the case 100.

The "input" axis I is perpendicular to both the spin axis S and the output axis O. The unit is sensitive only to motion of the case about the input axis I, which motion, by familiar gyroscope theory, generates an output torque T_p tending to rotate the frame 106 (and hence the shaft 110 and the chamber 108) about the axis O.

$$T_p = H\omega \tag{1}$$

* See also US Patent 2752791.

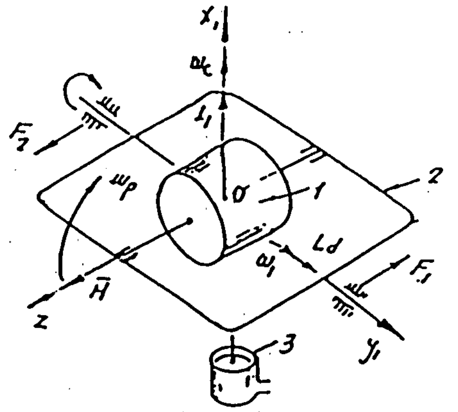


Figure 5.1

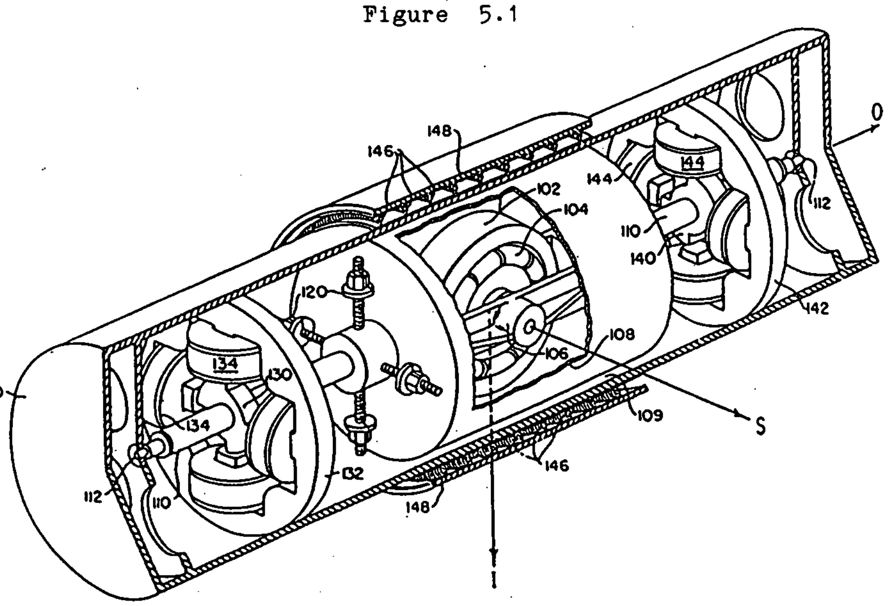


Figure 5.2

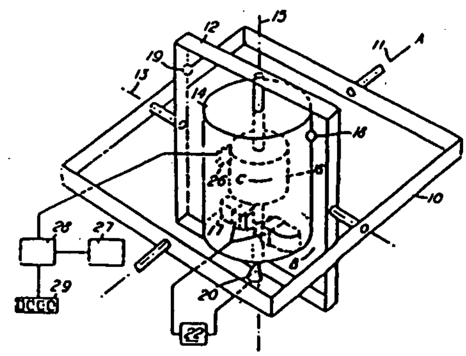


Figure 5.3.1

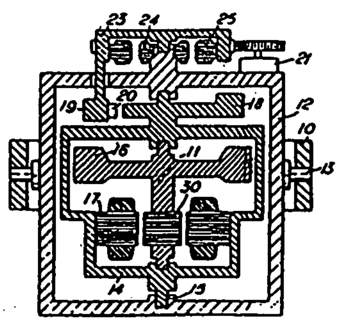


Figure 5.3.2

where H is the angular momentum of the gyro rotor. T_p will be resisted by a damping torque T_d proportional to the velocity of support member rotation:-

$$T_d = C \frac{d\theta}{dt} \qquad (2)$$

where θ is the deflection of the support member and C is the damping coefficient. Equating torques and integrating:-

$$\theta = \frac{H}{C} \omega \ dt \qquad (3)$$

The viscous damping fluid integrates the input angular velocity and causes the output deflection to be proportional to the input deflection.

The fluid completely fills the volume of the case 100 outside the chamber 108; in this way the fluid not only acts as a damper but also as a buoyant medium for the chamber and the entire gyro assembly. Ideally, the force on the bearings due to the weight of the gyro assembly can be completely balanced; in practice, the force is reduced to the order of one tenth of one percent. Consequently the friction in the bearings is greatly lessened and for most applications the use of ordinary jewel or ball bearings will maintain sufficient accuracy in the gyro output. For very precise uses further accuracy may be obtained by the use of pressurized ball bearings or oil bearings.

The flotation action serves another useful function. The gyro assembly is usually fairly heavy, and if the unit is mounted in an aircraft or ship, is subjected to accelerations from motion of the vehicle. The bearings supporting the gyro assembly are necessarily delicate in order to reduce the friction force. Without flotation, these accelerations would cause substantial inertial reaction forces to act on the bearings. The viscous fluid transmits most of the inertial reaction to the case directly, and hence serves as a cushion for the gyro assembly.

Mounted on the shaft 110 is the signal generator rotor 130. The signal generator shown in preferably of the type described in the Mueller U.S. Patent No. 2488734 issued Nov. 22. 1949. The signal generator comprises a rotor 130 and a stator 132; a reference voltage activates the stator windings 134 and an output voltage is read out of them. The output voltage is proportional to the displacement angle of the rotor from its neutral position with respect to the stator. The rotor 130 is connected to the support member 106 by the shaft 110 and the stator 132 to the outer member or case 100. Thus, the signal generator produces an output voltage proportional to the shaft rotation, which is the mechanical output of the gyro. The signal generator converts the gyro output directly into an output voltage which may be amplified for use in succeeding stages, as for example, to activate servo drives. No substantial energy drain need come from the gyro because the gyro output is an extremely small voltage (a few millivolts) and there is practically no resisting torque from the signal generator.

Also provided on the shaft 110 is the torque generator rotor 140. The torque generator is of the same type as the signal generator described above. It comprises a rotor 140, a stator 142 and stator windings 144; its property is that is generates a torque between the rotor and stator proportional to the current input to the windings. Since the stator 142 and its windings 144 are mounted on the case or mount member 100, a torque is generated tending to rotate the support member (the shaft 110 and gyro frame 106) with respect to the case 100. It should be noted that the damping means integrates the torque, denoted T_g. At equilibrium, and without precessional torques

$$T_g = T_d = C \frac{d\theta}{dt} \qquad (4)$$

where T_d is the damping torque, C, the damping coefficient and θ the deflection between inner member and outer member. Integrating:-

$$\theta = \frac{1}{C} T_g \ dt \qquad (5)$$

(Cf. Equation (3)). The superposition of a torque due to input rotation will simply add terms:-

$$\theta = \frac{1}{C} \quad T_g \ dt + \frac{H}{C} \quad \omega dt$$

In this way, the gyro unit can be used as an integrator of very high accuracy.

BODENSEEWERK PERKIN - ELMER & CO GB 888898 (1962) show that control techniques may be formed by electrical means to provide differentials or integrals of electrical voltages derived from the precessional movements of a gyroscope.

The simple flow meter type of integrator using a cyclometer in combination with a gyroscope is shown by GENERAL ELECTRIC CO of Schenectady GB 953585(1964)*. In this instrument the total mass flow of gas into a turbine is readily recorded.

Double integrating accelerometers for measuring distance are disclosed by SIEMENS APPARATE UND MASCHINEN Gmbh GB 44805 (1936) for BOYKOW. J.M. and by HOLT. C.R. GB 474718 (1937). The speed is derived from the acceleration by a first integration and then the distance travelled is derived from the speed by a second integration. This double integration is advanced by the accelerometer due to BARNES, J.W. and WHALLEY. R. which is disclosed by FERRANTI. LTD GB 1109615 (1968). See Fig 5.3.1 and 5.3.2.

An outer gimbal 10 has an axis 11, and an inner gimbal 12 on axis 13. Carried in the inner gimbal 12 is a housing 14 pivoted so as to be capable of limited angular movement about an axis 15 perpendicular to the axes 11 and 13. The housing contains a gyro rotor 16 arranged to rotate about the same axis 15 as the housing. The stator 17 of the rotor is fastened to housing 14. A pendulous mass 18 is attached to housing 14 and a balance mass 19 is carried by the inner gimbal 12 to balance the system about axis 11. A pick-off 20 is provided to detect angular movements of housing 14 with respect to the inner gimbal. Signals from the pick-off 20 are fed to a control unit 22 that supplies current to the stator windings 17 of the rotor driving motor. An inductive pick-off 26 is fastened to the housing 14 to measure the speed of rotation of the gyro rotor 16. Signals from the pick-off are compared with those from a reference pulse source 27 by a comparator 28, and the results of the comparison displayed on a digital counter 29. The double integral of the acceleration forces is determined by the reference source 27 and comparator 28 and counter 29. A train of pulses is produced by the pick-off 26, the repetition frequency of the pulse train being dependent upon the speed of rotation of the gyro rotor. A pulse train from the reference pulse source 27 represents the datum speed of the gyro rotor. Thus a comparison of the two pulse trains gives a measure of the difference between the datum speed and the actual speed of the gyro rotor over a period of time. This speed difference in, say revolution per second, gives an indication of the distance covered by the accelerometer in the period of time, since each "difference revolution" represents a fixed distance. Hence the total of the difference revolutions indicates the total distance covered and the counter 29 calibrated directly in terms of distance, though actually measuring difference in speed.

A mechanically integrating rate gyro is due to KNUTSON. R.G. US 2709922 (1955). He uses a convex spherical surface at the end of a shaft attached to a gyroscope gimbal, the convex surface being in contact with a friction wheel. It can readily be shown mathematically that in the steady state the angular displacement of the driven friction wheel with respect to the main frame of the device is proportional to the time integral of the rate-of-turn of the device.

Similarly LAHDE. R.N US 2951377 (1960) provides integral with the rotor a curved engaging surface facing in the direction of a given axis and having a centre of curvature coinciding with the centre of gravity of the rotor. POPE. K.E. US 2819053 (1958) discloses an eddy current cup accelerometer that may be gyroscopically mounted. WILLIAMS.D.D. US 2949780 (1960) (Fig 5.4.1 and 5.4.2) discloses an important contribution to this art in an integrating accelerometer using an axially unbalanced gyroscopic mass; which is improved upon by LASSEN. H.A. US 2968949 (1961).

* See also US Patent Specification 3084560.

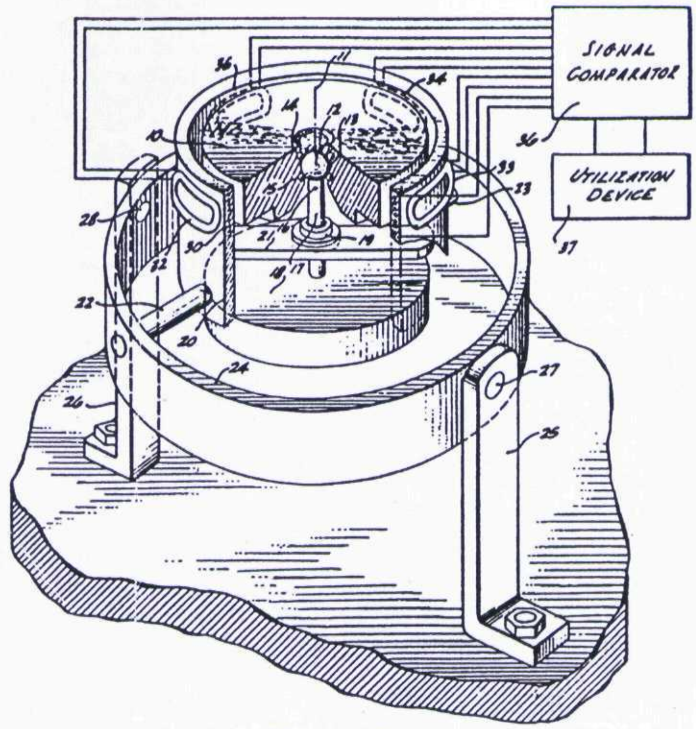


Figure 5.4.1

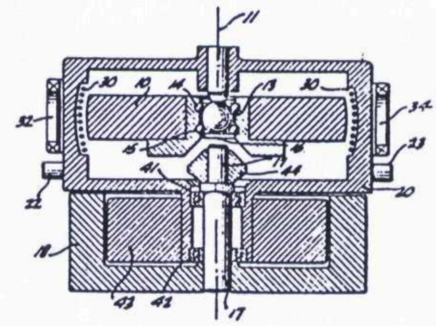


Figure 5.4.2

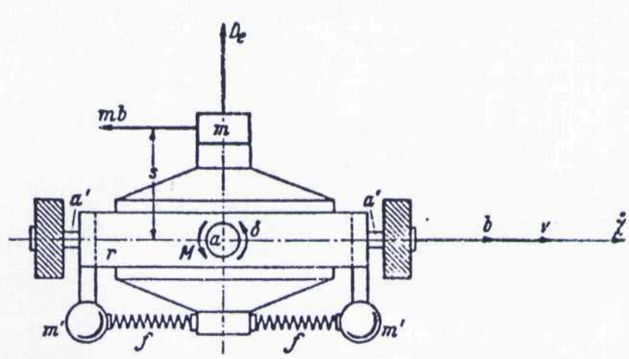


Figure 5.5

A first gyroscopic mass or rotor 10 having an axis of rotation 11 and a centre of rotation 12 is adapted to be rotated about a ball 13 upon which the rotor 10 is mounted through the use of the bearing system 14 and 15. The rotor 10 is so constructed that its centre of mass is noncoincident with its centre of rotation 12. That is, the centre of mass is displaced to a point 16 which lies upon the the axis of rotation 11. The ball joint suspension system for the rotor 10 permits the axis of rotation 11 to precess in response to acceleration applied to the accelerometer normal to the axis of rotation 11.

It can be shown that when an axially unbalanced gyroscopic mass such as the rotor 10 is subjected to accelerations having components normal to its axis of rotation, the amount of precession of the axis of rotation is proportional to the integral of the applied acceleration. The plane of the precession is at right angles to the applied acceleration. All accelerations lying in the plane of the rotor will cause the axis of rotation to be precessed. That the amount of precession is proportional to the integrals of applied accelerations can be shown as follows:

$$\bar{\gamma} = I\omega \frac{d}{dt} \bar{I}_{\downarrow}$$

$$\bar{\gamma} = m\bar{a} \times l\bar{I}_{\downarrow}$$

$$\frac{d}{dt} \bar{I}_{\downarrow} = \frac{m\bar{a} \times l\bar{I}_{\downarrow}}{I\omega}$$

$$\Delta\bar{I}_{\downarrow} \quad \frac{ml}{I_{\downarrow}} \int \bar{a} \times \bar{I}_{\downarrow} \, dt$$

where

γ = torque required to produce an angular velocity of precession

I = moment of inertia about the axis of rotation

ω = angular velocity of the rotor

m = mass of the rotor

$\bar{a}$ = applied acceleration

l = distance between the centre of mass and the centre of rotation

$\bar{I}$ = a unit vector in direction of axis of rotation

$\Delta \bar{I}_{\downarrow}$ = the change in the unit vector $\bar{I}$ due to precession.

It is thus seen that the angle through which the axis of rotation is precessed, which is represented in magnitude and direction by $\Delta\bar{I}$ is proportional to the integral of all accelerations lying in the plane of rotation of the rotor, with the constants of proportionality being easily derived from the physical parameters of the system.

The ball 13 is attached to an axle or shaft 17 which in turn supports a second gyroscopic mass or rotor 18. The shaft 17 and the second rotor 18 are so coupled that the axis of rotation of the second rotor 18 is always coincident with the longitudinal axis of the shaft 17. The gyroscopic action of the second rotor 18 therefore serves to maintain the longitudinal axis of the axle 17 as a fixed spatial reference axis. The second rotor 18 is mounted in a gimbal system which includes an inner gimbal ring or frame 20 constructed from a material having low permeability and which serves to support the shaft 17 by means of a supporting member 21. The supporting member 21 is rididly attached to the inner gimbal frame 20, and a bearing system 19 serves to permit rotation of the shaft 17 within the supporting member 21.

The inner gimbal frame 20 is supported by a first pair of journals or trunnions 22 and 23 which are in turn supported by an outer gimbal ring or frame 24. A pair of supporting arms 25 and 26 support the outer gimbal ring 24 by means of a second pair of journals or trunnions 27 and 28. The second set of trunnions 27 and 28 lie in the same plane as the first set of trunnions 22 and 23 and are at right angles to the first set. The supporting arms 25 and 26 are attached to the vehicle carrying the integrating accelerometer. Each of the trunnions is mounted in a bearing system in a manner which is conventional in the art of gimbal systems, and therefore the bearings are not shown.

As the second rotor 18 rotates it serves to maintain the shaft 17 in a fixed orientation in space and nonsensitive to accelerations applied in the plane of rotation of the rotor 18. Since the supporting member 21 maintains the shaft 17 and the inner gimbal ring 20 in fixed space relationship, the inner gimbal ring 20 is also space stabilized by the rotor 18.

An excellent example of a floated integrating rate gyroscope with temperature compensation and internal balancing after assembly can be found in the work of HORATH. W.* to SOCIÉTÉ DE FABRICATION D'INSTRUMENTS DE MESURE. (S.F.I.M.) French Patent Specification 1224158 (1960), and also in his U.S. Patent Specification 3084559 published in 1963 and G.B. Patent Specification 911913 published in 1962.

Miniature floated rate integrating gyroscopes are discussed by DURKEE. R.P. (1962) and by GENERAL PRECISION INC GB 1056557 (1967).

ANDREICHENKO. K.P. and KOVAL. V.A. (1979) discuss with insight the dynamic errors to be found in floatation integrating gyroscopes.

A sophisticated construction for a dynamic gas-film rate-integrating gyroscope with negligible couloumb friction and bearing noise is described by WARNOCK. L.F US 3339421 (1967) and a rate integrating device including a torquer in open loop configuration for torquing the device to null and having a load connected in a closed-loop configuration to the device for maintaining the device in the null condition is described by LANNI. M.L.; CALAMERA. J., KREBS. L. and O'CONNOR. B.J. US 4005608 (1977). Such a device offers increased accuracy and stability and the electrical means for maintaining the device of null is independent of the dynamic characteristics of the device, such as gimbal moment of inertia and system damping factor.

Inertial space angular velocity integrators, single axis servodriven angular velocity integrators and a three-axis servodriven space integrator are *inter-alia* taken up by DRAPER. C.S. WRIGLEY. W. and GROHE. L.R. (1956) in regard to geometrical stabilization problems on moving bases.

The integration device used to measure the linear velocity of the fluid driven rocket is described by ZEUNERT. G. (1949).

RIVKIN. S.S. (1965) reminds us that integration is a natural property of a gyroscope, since from the law of precession it follows that the three-degree-of-freedom astatic gyroscope is essentially an integrator since its angle of precession is equal to the integral with respect to time of the external torque applied to it. An astatic gyroscope, if we ignore nutation can be considered an integrating network (with transmission factor $K = \frac{1}{H}$ where H is the angular momentum of the gyro) with respect to an input disturbance on the cross axis. The transient response at the output of this network is proportional to the integral in time of the input.

* The importance is shown from the early U.S. translation into English of French 1224158 in National Technical Information Services Report OTS. U.S. 6218919. See also GB 911913 and US 3084559.

RIVKIN sub divides the subject into:

i measurement of angles of turn by integration of angular velocity of an object.

ii measurement of the sum of the angles of turn and the angular velocities of an object and

iii the measurement of linear velocity by means of integrating the linear acceleration.

The first sub-division is catered for by what he calls in translation the hermetic integrating gyroscope described above by JAROSH. J.J. et.al. (1956). The second sub-division requires an integro-differentiating gyroscope and the third a gyroscopic integrator of linear accelerations such as that of ZEUNERT. G. (1949) for example. Another integrating gyroscope is disclosed, and mathematically evaluated by by GRAMMEL. R. (1950).* The device Fig 5.5 uses a three-degree-of-freedom gyroscope with additional mass and compensatory loads. In an outer ring there is a fixed gyro housing r the axis of $\alpha\dot{\chi}$ of which coincides with the longitudinal axis of the vehicle which moves with velocity v and acceleration b.

The cardan ring r can rotate about the axis a'a' which lies in the direction of the velocity v, whereby its angle of deflection χ can be read on a suitable scale. The gyro housing carries an additional mass m at a distance s from the centre of the cardan point and by way of compensation two additional masses m'm' attached eccentrically to the cardan ring r, on to the axis a'a' so that the entire system is balanced by the rotation X. If the instrument is accelerated with the vehicle, there arises in the added mass m and in the compensatory masses m'm' d'Alembert forces mb and m'b opposed to b. While the forces m'b only stress the bearings of the axis a'a' of the cardan ring r, the forces mb acts as a precessional moment

$$M = msb = ms\dot{v}$$

and so rotates the self turning impulse vector D_e and with it the cardan ring r so that

$$D_e\overset{o}{X} = M = ms\dot{v}$$

which an integration leads to

$$X = \frac{mS}{D_e}\, v$$

If the motion of the vehicle is at v = 0, and the pointer starts ot X = 0 the spring constraints f have not been taken into account and the inertia of the system has been disregarded. The angle of deflection X of the cardan ring thus indicates the speed of travel v actually reached with reservations in respect of friction in the bearings.

* See his DER KREISEL 1950 (vol. 2) Die Anwendungen des Kreisels. P. 207

6. FREE GYROSCOPE

Extensive use is made of the astatic or balanced gyroscope that resides in the broad group of the symmetrical gyroscopes.

If no external force moments act on an astatic gyroscope, such a gyroscope may be said to be a free gyroscope. This convention or definition has persisted and we find it cogently expressed in GB Patent Specification No. 892452 to SOCIÉTÉ FRANCAISE D'OPTIQUE ET DE MECANIQUE of 1962.

> "The device provided by the present invention is accordingly characterized by the use of a free gyroscope, that is to say a gyroscope the rotor of which is so mounted on its support that the spin axis of said rotor has its orientation practically uninfluenced by movements of said support in space, the only displacements of said axis being due to a movement of precession, which in linear and can be easily corrected (the errors due to the rotation of earth, to Coriolis acceleration* and to the convergence of the verticals from one point to another being also corrected through known means)."

Clearly the free gyroscope has at least one remarkable property, namely that its kinematic-moment axis retains a fixed direction in inertial space and the magnitude of this moment is constant. This can be shown to be a direct consequence of the law of angular momenta. But it is uncommonly difficult to construct a free gyroscope owing to the inevitable disturbing actions upon it and hence the axis will drift from its original position. The motion of a free gyroscope with respect to the Earth, assuming that the vehicle on which the gyroscope is placed is stationary is well understood. The Earth has two motions, an annual motion in orbit about the Sun and a diurnal motion on its axis. The former is small for a short time period when compared with the latter and the former can be ignored.

The diurnal rotation of the Earth takes place about its axis of rotation, as stated above, which substantially coincides with the polar axis and which may be regarded as fixed in inertial space which is bound to the 'fixed stars' for the comparatively short time for which the free gyroscope is in use.

It will be found that the axis of a free gyroscope will participate in the apparent rotation of the system of the 'fixed stars' about the Earth; and as was initially shown by FOUCAULT, this motion of the gyroscope is a proof of the Earth's rotation.

It is generally agreed that for any initial position of the axis of the free gyroscope, at any latitude, the axis will execute an apparent motion, a regular conical precessional rotation about an axis parallel to the polar axis at a rate equal in magnitude but opposite in sign to that of the Earth's rotation. If these oscillations of the axis of the free gyroscope were to be stopped, then it would set itself parallel to the Earth's polar axis and form an instrument that automatically indicated local latitude; a gyrosextant of little use in such a form.

The free-rotor gyroscope is no longer essentially a rotor wheel in compound gimbals as will be seen from the highly sophisticated free-rotor gyroscope of BULMAN and MAUNDER GB 1522138 (1978). They explain that the true free-rotor gyroscope is one in which the rotor is mounted and driven in such a way that if the body on which the gyroscope is mounted is subjected to a rate of turn about an axis lying normal to the spin axis of the rotor, the resulting movement of the support relative to its rotor gives rise in practice to a minute external torque upon the rotor, and in theory to no such torque at all. The consequence

* Coriolis force, a 'fictitious' force demanded by a change in radial position with no change of angular velocity. See FRANK. N.H. P.161 of SNEDDON'S Encyclopaedic Dictionary of Mathematics for Engineers and Applied Scientists (1976).

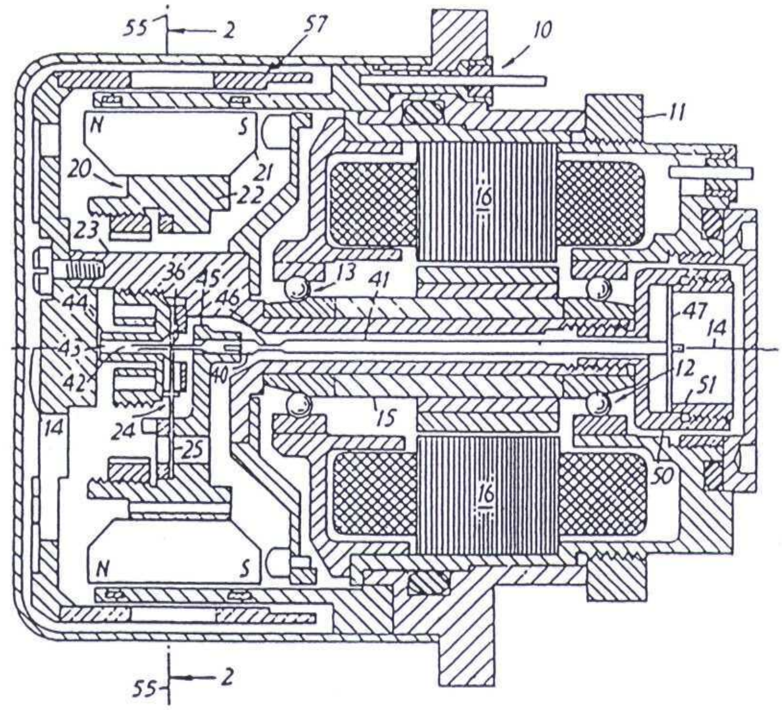


Figure 6.1.1

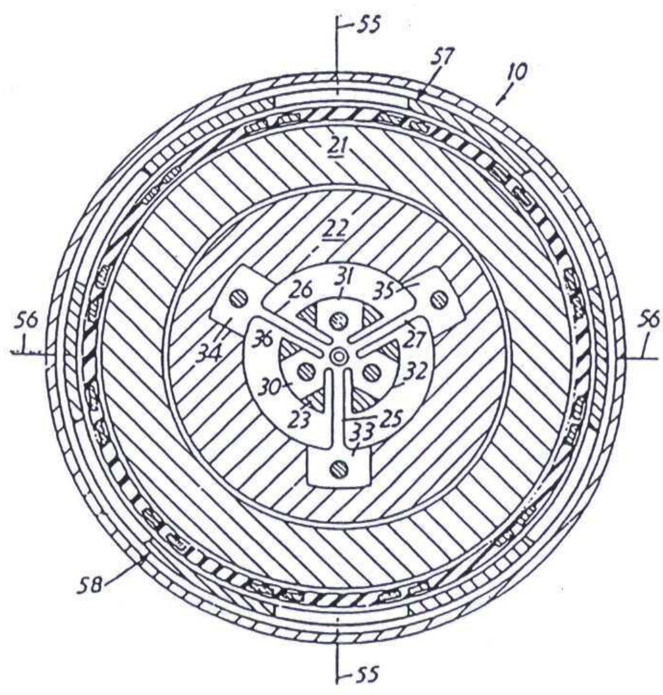


Figure 6.1.2

of such absence of torque is that when the rotor and its housing move relative to each other in response to the rate of turn the angular disposition of that relative movement at once indicates the axis of the turn, and the rotor remains in a plane in space parallel to the one in which if originally lay. Thus the free-rotor gyroscope has properties that readily allow it to be used as the basis for rate of turn indicators, north-seeking instruments, inertial navigation platforms and many other navigation and surveying devices. Unfortunately the necessary freedom of movement of the rotor within its housing can only be achieved by expensive manufacture involving very clean conditions and very close tolerances. The electromagnetic and other drives necessary to spin the rotor without direct physical contact also pose severe difficulties.

Elastically-supported gyroscopes, now well known, are far simpler and cheaper to make and operate than free-rotor gyroscopes. Furthermore, although their indication of some parameters is not as direct and simple to read as that of an accurate free-rotor gyroscope, they readily indicate rate of turn about an axis normal to the spinning axis of their rotor. However, many of the elastically-supported gyroscopes already commonly used have comprised a rotor free to rotate along only one of the axes lying normal to the spin axis of the rotor drive. Should such a gyroscope be subjected to a rate of turn about an axis lying in a plane normal to the drive axis, the rotor cannot take-up a new position in which it lies in a single plane inclined to its original plane; it can only oscillate about its single axis, in a manner that can sometimes be interpreted to indicate the rate and the extent of the turn. In order to be comparable with a free-rotor gyroscope, the rotor of an elastically-supported gyroscope must at least be able to turn about two axes, mutually at right angles and each at right angles to the drive axis.

In another known type of elastically-supported gyroscope in which the rotor can turn about the two axes just described, the rotor is supported from its drive shaft by a Hooke's joint in which the usual pivots have been replaced by torsion springs.

In such a design, however, with an intermediate member between the rotor and the drive shaft, the diametrical support of the rotor necessarily means that the rotor suspension is asymmetrical, that is to say, the relation of the rotor and its suspension to one of the two axes that lie normal to each other and to the spinning axis is different from the setting relative to the other of these two. Studies have suggested that although one or more rotor speeds can be found at which a condition of some resonance is set up and the resulting forces tend to counteract the unwanted torques exerted upon the rotor by its suspension, nevertheless some residual torque must always exist. Two earlier examples assist in the understanding of the problem.

SPERRY RAND CORPORATION GB 1304571 (1973) offer an improved suspension over that first advanced by THE SPERRY CORPORATION GB 722492 (1955). The gyroscopic apparatus has a gyroscopic rotor adapted for spinning about a spin axis by means of a drive shaft, and suspension means for the rotor comprising first flexural support means radially supporting the rotor on the drive shaft for universal tilting about axes perpendicular to the spin axis and having a first spring rate, and second flexural support means axially supporting the rotor on the drive shaft coaxial with the spin axis and having a first portion extending through the first flexural support means for universal tilting of the rotor about axes perpendicular to the spin axis, and a second spring rate, the second flexural support means also having a second portion with a third spring rate, the arrangement being such that the first and second spring rates tend to be compensated by the third spring rate. (Figs 6.1).

In the second example LITTY F.D. US 3211011 (1965) proposes a gyroscopic device that is an advance on that of STILES J.C. US 3077785 (1963).

It comprises a rotatable shaft; a wheel; means coaxially connecting the wheel to the shaft for conjoint rotation in a plane normally perpendicular to the rotational axis of the shaft, the said means enabling deflection of the plane of rotation of the wheel while exerting a restoring force thereon. The device further comprises a plurality of substantially equal masses, means mounting the

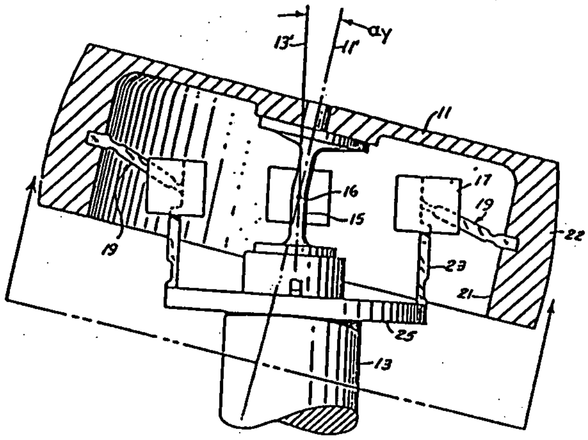


Figure 6.2.1

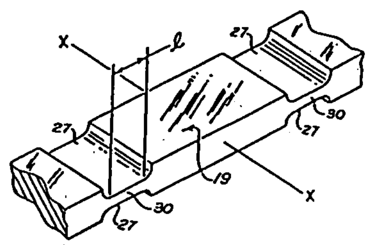


Figure 6.2.2

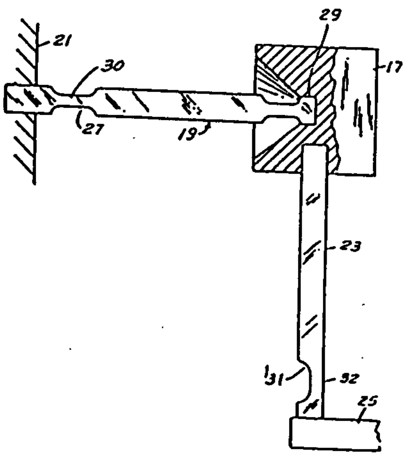


Figure 6.2.4

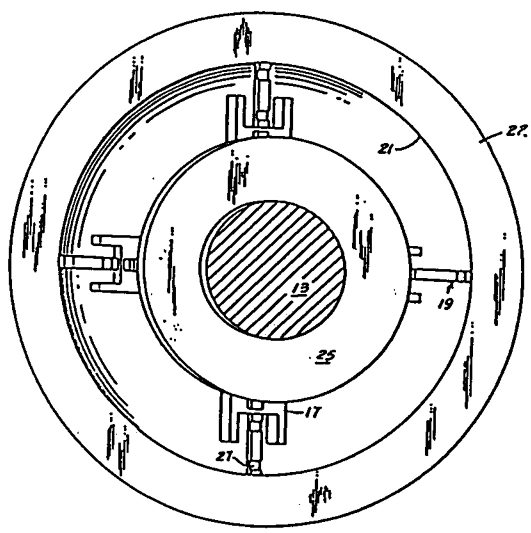


Figure 6.2.3

masses on the shaft for conjoint rotation therewith about the shaft axis and for radial displacement relative thereto, the said masses being equiangularly distributed about the shaft axis at substantially equal radial distances therefrom, and individual radial force-transmitting means connecting each of the said masses to the wheel at respective equiangularly spaced points the locus of which is a circle concentric with the wheel. This legalistic language is clarified by a direct appeal to the clear drawings of Figs 6.2.

The only torque on the wheel 11 resulting from the blocks 17 combined will be about the pivot point 16 in the direction in which the wheel is deflected. The centrifugal forces generated by the masses of the arms 19 will add to the torque applied to the wheel 11 by the blocks 17. The forces resulting from the resistance of the flexible joints of the arms 19 to being bent when the wheel is deflected will subtract from the torque applied by the blocks 17. The centrifugal forces generated by the columns 23 have very little effect and can be neglected. Similarly the columns 23 only flex a small amount in the necks 32 and therefore the resistance of these columns to being bent can be neglected. Thus there are three types of torques introduced by the blocks 17 and the support arms 19 that should be considered: (a) major inertial torques which are supplied by the centrifugal force of the blocks 17, (b) minor inertial torques provided by the inertia of the radial support arms, and (c) static torques due to the bending of the support arms about their flexible joints. The total torque equation taking into account these three components is:

$$T = 2M\omega^2 \frac{a}{c}(a-c)^2 \sin\alpha_y + 2I_1\frac{a}{c}\omega^2 \sin\alpha_y - \frac{EI_2(3a-c)}{l_c}\frac{a}{c}\sin\alpha_y$$

In this formula M is the mass of the blocks 17, ω is the speed of rotation of the unit; a is the radius of the inner wall 21 of the rim of the wheel 11; c is the length of the support arms 19; α_y is the angle of deflection, previously defined; I_1 is the mass moment of inertia of the support arms 19; I_2 is the area moment of inertia of the support arms 19;I_2 is the area moment of inertia of the flexible joints of the support arms 19; E is the elastic modulus of the material of the rms 19; and l is the width of the grooves 27. From this formula it will be apparent that the total torque resulting from the blocks 17 and support arms 19 is linear with deflection through small angles of deflection.

In the course of the novelty investigations carried out by the United States Patent Office, something of these elastically supported designs can be traced back as far as a proposal of <u>ANSCHUTZ-KAEMPFE</u> GB 6359 (1905). See Fig 6.3.

Clearly each of the above examples of <u>SPERRY RAND. CORPN.</u> and <u>LITTY. F.D.</u> shows a design of gyroscope including devices that act so that when the rotor tilts about the drive axis a mechanical compensation is set up so that the undesired spring rate upon the rotor by its deflected elastic suspension is offset and apparently reduced to zero. However, the device of LITTY is such that the way the masses are mounted prevents them from making substantial movements in a direction parallel to the axis of the rotor drive, and the mechanical compensation is said to rely upon the centrifugal forces exerted by masses suspended between the rotor and the driving element within linkages of two struts, these struts lying at right angles to each other. Such an arrangement of struts is difficult to assemble and set-up accurately, in particular because of the toggle action of the strut through which the centrifugal force of each weight is transmitted to the rotor. Moreover each strut is weakened by necked-down portions where it will be subjected to continual bending in opposite senses and thus liable to fatigue. The construction is also subject to additional dynamic effects because of the radial motion of the masses. The radial motion produces accelerations that act tangentially to the arc of spin, and this in turn produces additional unwanted oscillatory forces on the struts. Again the construction of <u>SPERRY RAND CORPN.</u> is totally independent of rotor speed and also relies on a toggle-type action for success.

6359 Gyroscopic apparatus. KAEMPFE, H. ANSCHUTZ, 13, Markt-Platz, Kiel, Germany, March 25. (Date applied for under PATENTS ACT, 1901, March 26, 1904)

Relates to a gyroscope, particularly for use on ships, as a substitute for, or as a means for corroborating or correcting, the magnetic compass, but applicable generally. Spinning bodies 1 are carried by horizontal axles 3, shown arranged vertically in Fig 1 but horizontal when in operation, operated by a motor 2 mounted on a frame 4. This frame carries ball bearings for the axles 3, and turns on spindles 6 mounted in ball bearings carried by a suspended frame 9. The centre of gravity of the frame 4 and the parts carried by it is arranged below the point of intersection of the three axes of rotation, so as to cause the system to be acted on by gravity in such a way that the readings of the instrument are not affected by the earth's rotation, but are affected only by changes in the ship's course. The frame 9 is suspended from a float 10 in a liquid-container 11 supported in gimbals as shown. The float carries a pointer, to indicate the ship's course on a compass card 43, and is connected by a disk 12 and a nave 13 to a pivot 15 turning in ball bearings. The pivot 15 is connected by a spring 21, a piece of flexible shaft, or other elastic means to a pin 19 on the suspended frame 9 so that the gyroscope is not affected by shaking motions of the ship. The end 20 of the pin 19 is allowed a slight lateral play relatively to the annular piece 22. The motor 2 is connected to the axles 3 by spiral springs 24, flexible shafting, magnetic or other couplings which permit a limited relative motion of the coupled parts, or other elastic means. Current is supplied to the motor through a terminal 26, a conductor 27, a brush 28, a ring 29, a conductor 30, a leaft spring 32, a contact-screw 33, a silver wire 34 passing through the hollow axle 6, and a spring contact 35 connected to one brush of the motor. The other brush of the motor is connected to a similar arrangement of conductors or the the framework of the apparatus. The position of the centre of gravity can be adjusted by movable weights 44. Instead of the centre of gravity being arranged below the intersection of the three axes, a spring or an electromagnet may be used. Instead of two spinning bodies with one motor being used, two motors may be used, one on each side of one spinning body, or a single spinning body may be arranged to rotate in a central plane about the housing of a single motor. The apparatus is enclosed in a casing 25. Fig 2 shows a modification in which the whole casing 25 floats in alcohol or other liquid in a vessel 36, a spring 40 being interposed between the casing and a pivot 37 running in a stone 38 at the bottom of a receptacle 39 containing mercury. The electric current enters the apparatus through the pivot 37, and may leave it through a rod or wire 41 dipping into mercury in a channel 42.

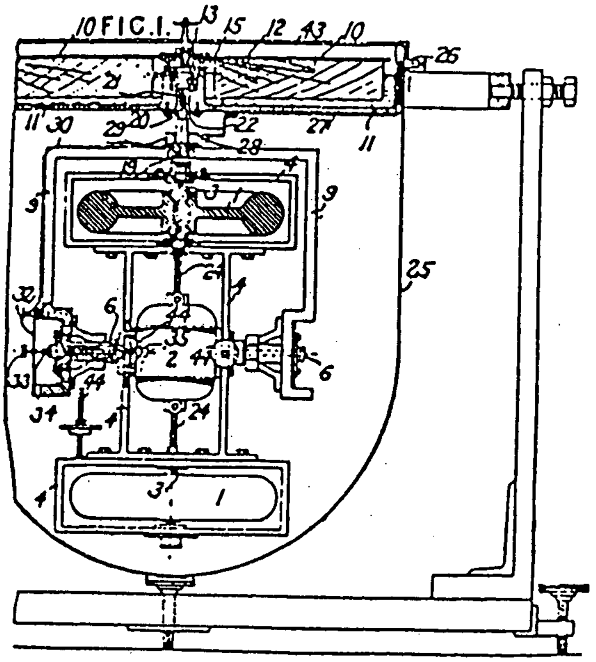


FIG.2.

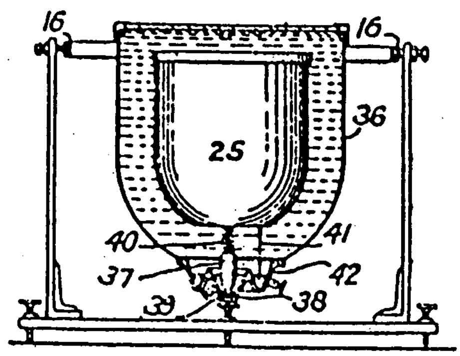


Figure 6.3

It is an old political adage, that freedom is an old Anglo Saxon word for slavery; this is carried into the mechanical and gyroscopic fields with some panache by BULMAN AND MAUNDER in G.B. Patent Specification 1522138 (1978) and disclosed by the National Research Development Corporation. What they propose is shown in Figs 6.4.

Fig 6.4.1 is an axial section through a gyroscope of the invention;
Fig 6.4.2 is a diagrammatic radial section on the line II-II in Fig 6.4.1;
Fig 6.4.3 is a partial and exaggerated diagrammatic view illustrating the behaviour of a vibrating mass in the gyroscope of Fig 6.4.1;

The elastically-supported gyroscope shown in Figs 6.4.1 to 6.4.3 comprises a driving element including a rotor housing 1 attached by screws 2 to a plate 3 carried by a drive shaft 4 driven by a motor shown diagrammatically at 5. A rotor 6 comprises a front part 7 and a rear part 8 held together by screws 9. The rotor is connected to the drive shaft in two ways. Firstly by four mass-carrying structures in the form of elastic radial spokes 10; the radially-inward end 20 of each spoke is clamped to a central pillar 11 of housing 1 by a clamp ring 12 and screws 13, and the radially-outward end of each spoke is clamped to the rotor between parts 7 and 8. The four spokes are formed by stamping blanks from a single sheet of thin metal to leave a cruciform shape; each spoke thus readily flexes in directions lying substantially within planes that include the axis of rotation of shaft 4.

Secondly by a support bar 14 lying along the axis of shaft 4; much of the length of this bar lies within a passage 15 formed within pillar 11, one end of the bar is clamped to a plate 16 anchored to housing 1, and the other end of the bar is clamped to a collet 17 located at the centre of part 7 of rotor 6. If rotor 6 begins to spin seriously out-of-true, the base of rear part 8 makes contact with stop screws 18 mounted in housing 1.

Support bar 14 should be designed to be strong in tension and as strong as possible in compression, so as to give rotor 6 positive axial location within housing 1, but should be as weak as possible in flexure so as to exert the minimum restoring torque upon the rotor whenever, in operation, its spin axis moves out of coincidence with the axis of shaft 4 and intersects it instead. The apparatus is balanced so that the centre of gravity of the rotor lies at point 19, i.e. in the plane of the four spokes 10 and on the driving axis of the shaft 4. Each spoke carries a mass M at a point along its length that is not critical but should be sufficiently close to the pillar 11 that the part of the spoke lying between the mass and the pillar is stiff in torsion. The masses M may well be identical, but for ease of description they are indicated in Fig 6.4.2 as M_1, M_2, M_3, M_4. The three principal axes of the apparatus OX, OY and OZ are shown; OZ coincides with the axis of shaft 4, and OX and OY are mutually at right angles and both at right angles to OZ.

In normal operation of the gyroscope shown in Fig 6.4 the rotor spins in the X-Y plane as shown in Fig 6.4.1 and the spokes 10 and their carried masses M lie in this plane also. Say now the gyroscope is subjected to a rotation about OX. The partial view of Fig 6.4.3 in which deflection of the rotor is exaggeratedly illustrated, shows the effect of this upon the rotor, one of the spokes 10 and its attached weight (M_1) at an instant during such rotation. In particular the weight M_1 has moved, in a direction substantially parallel to OZ, to a position defined by Z_1 and S_1. At this instant the part of the rotor to which the spoke is attached lies in the positive quadrant of the OYZ plane, and the greater part of the length of the spoke lies in a curve instead of on direct radium between the origin O and the rotor. Parameters r_1, s_1 and z_1 are marked on the Figure. Similar parameters but with the subscripts 2, 3 and 4 will apply at the same instant to the other three spokes and their attached weights, which likewise will not coincide with their respective direct radii. However, the relative positions of the other three spokes and their respective radii will of course be different, on account of the asymmetric position that the rotor has taken up.

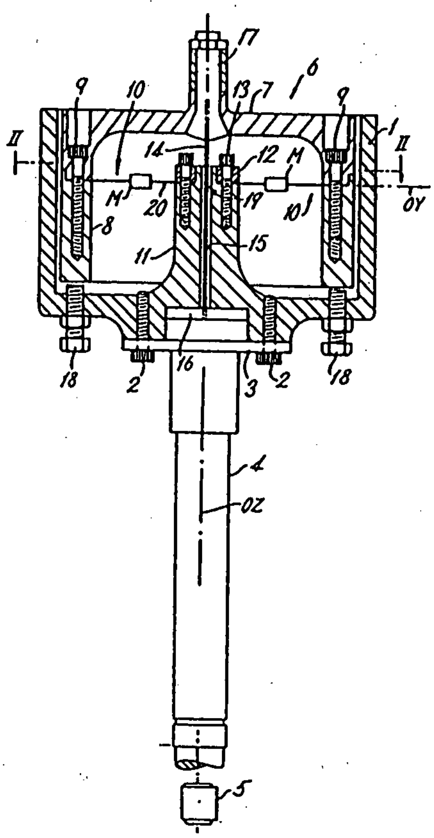


Figure 6.4.1

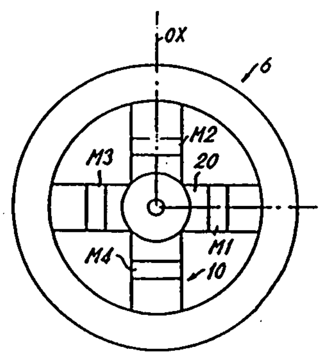


Figure 6.4.2

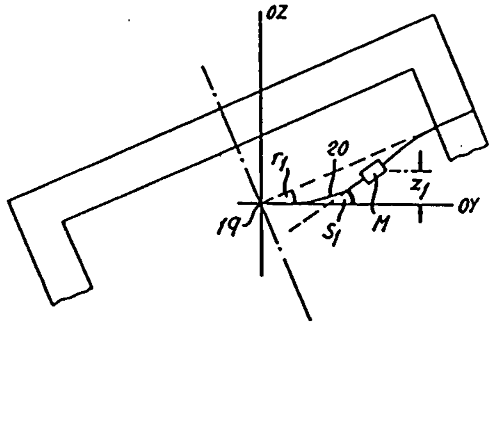


Figure 6.4.3

In this situation, the externally applied torques upon the rotor can be written

$$K_z z_1 + K_s s_1 - 2K_r r_1 - K_b r_1 - K_t r_1 - K_z z_3 + K_s s_3 \qquad \text{(i)}$$

about OX, and

$$-K_z z_2 + K_s s_2 - 2K_r r_2 - K_b r_2 - K_t r_2 + K_z z_4 + K_s s_4 \qquad \text{(ii)}$$

about OY, where K_z, K_r and K_s are spring constants for a single spoke, 1/2 K_t is the torsional stiffness of the outer part of each spoke (typically less than the torsional stiffness of the inner part), and K_b is the stiffness of the centre support bar.

If the external torques upon the rotor are to sum to zero, then equations (i) and (ii) can be re-written as

$$r_1(2K_r + K_b + K_t) = K_z z_1 + K_s s_1 - K_z z_3 + K_s s_3 \qquad \text{(iii)}$$

$$r_2(2K_r + K_b + K_t) = -K_z z_2 + K_s s_2 + K_z z_4 + K_s s_4 \qquad \text{(iv)}$$

For the condition in which the spin axis of the rotor remains parallel to its original setting it will be apparent that r_1 may be written as (H)cos nt and r_2 as -(H) sin nt, (H) representing a deflection of the gyroscope about axis OX, and n being the spin velocity of the rotor about OZ.

The relationship given by equations (iii) and (iv) must be satisfied if the rotor is to behave as a free rotor, and the gyroscope can thus act as a free rotor gyroscope. The equations of motion of the masses M_1 to M_4 must therefore be found and compared with equations (i) to (iv) to see if they can be made compatible.

Let M_1 have principal axes O'u O'v and O'w along which lie the principal moments of inertia a, b and c respectively. Also when $s_1 = 0 = z_1$, let O'u, O'v and O'w be aligned in the directions OX, OY and OZ respectively, thus defining the position of O'. The angular velocities of the mass about its principal axes can now be written as:

s_1 about O'u
$n.\sin.s_1$ about O'v
and
$n.\cos.s_1$ about O'w

and the inertial torque of the mass about O'u is:-

$$a\dot{s}_1 - (b-c)n^2\cos.s_1.\sin.s_1$$

If small angles of deflection are assumed such that $\cos.s_1$ is nearly equal to unity and $\sin.s_1$ to s_1, this torque may be written

$$a\dot{s}_1 + (c-b)n^2, s_1 \qquad \text{(v)}$$

The external torque on mass M_1 can be written as

$$K_m z.z_1 - K_{ms}.s_1 + Kmr.r_1 \qquad \text{(vi)}$$

where $K_m Z$, K_{mr} and K_{ms} will be explained but are related to the bending moments and shear forces at the ends of the spoke, the latter being treated as two springs carrying the mass M between them, and by equating expressions (v) and (vi) and writing the similar equations for M_2, M_3 and M_4, the following expressions are obtained:-

$$K_{mz}z_1 - K_{ms}s_1 + K_{mr}.r_1 = a\dot{s}_1 + (c-b)n^2.s_1 \qquad \text{(vii)}$$

$$-K_{mz}z_2 - K_{ms}.s_2 + K_{mr}.r_2 = a\dot{s}_2 + (c-b)n^2.s_2 \qquad \text{(viii)}$$

$$-K_{mz}z_3-K_{ms}.s_3+K_{mr}.r_1=a\dot{s}_3+(c-b)n^2.s_3 \qquad \text{(ix)}$$

$$K_{mz}z_4-K_{ms}.s_4+K_{mr}.r_2=a\dot{s}_4+(c-b)n^2.s_4 \qquad \text{(x)}$$

By also considering the motion of the masses in the OZ direction we may write:-

$$-K_{pz}.z_1+K_{ps}.s_1+K_{pr}.r_1=m\dot{z}_1 \qquad \text{(xi)}$$

$$-K_{pz}.z_2-K_{ps}.s_2-K_{pr}.r_2=m\dot{z}_2 \qquad \text{(xii)}$$

$$-K_{pz}.z_3-K_{ps}.s_3-K_{pr}.r_1=m\dot{z}_3 \qquad \text{(xiii)}$$

$$-K_{pz}.z_4+K_{ps}.s_4+K_{pr}.r_2=m\dot{z}_4 \qquad \text{(xiv)}$$

Here K_{pz}, K_{pr}, and K_{ps}, which will also shortly be explained, are stiffness constants related to mass force in the OZ direction in the same way that K_{mz} etc. were related to mass torque, and m is the mass of M_1.

Equations (vii) to (x) and (xi) to (xiv) are the equations of motion of the four masses, and if the rotor is to be tuned these must be compatible with equations (iii) and (iv). It may reasonably be assumed from the symmetrical nature of the gyroscope that $s_1 = s_3$, $s_2 = s_4$, $z_1 = -z_3$ and $z_2 = -z_4$, and that s_1, z_1 etc have a sinusoidal motion of frequency n. Equations (iii), (vii) and (xi) may now be combined, writing $s_1 = s_o \cos nt$ and $z_1 = z_o. \cos nt$, to yield:

$$K_{mz}z_o-K_{ms}s_o+ \frac{K_{mr}\ (2K_z.z_o+2K_s.s_o)}{(2K_r+K_b+K_t)} = -an^2.s_o+(c-b)n^2.s_o \qquad \text{(xv)}$$

and

$$-K_{pz}z_o+K_{ps}s_o+ \frac{K_{pr}(2K_zz_o+2K_ss_o)}{(2K_r+K_b+K_t)} = -mn^2z_o \qquad \text{(xvi)}$$

and these equations can be mathematically satisfied if:

$$n^4(a+b-c)\ m+n^2\left[(c-a-b)\ (K_{pz} - \frac{2K_zK_{pr}}{K_{rbt}}) - m\ (K_{ms} - \frac{2K_{mr}K_s}{K_{rbt}})\right]\ldots$$

$$\ldots + (K_{ms} - \frac{2K_{mr}.K_s}{K_{rbt}})(K_{pz} - \frac{2K_z.K_{pr}}{K_{rbt}})-(K_{ps} + \frac{2K_s.K_{pr}}{K_{rbt}})(K_{mz} + \frac{2K_{mr}.K_z}{K_{rbt}}) = 0 \qquad \text{(xvii)}$$

where $K_{rbt} = (2K_r+K_b+K_t)$

In equations (vi) to (xvii) K_z, K_r and K_s are constants defining the torque applied to the rotor by the spoke about the OX axis for unit change in z, r or s respectively: K_{mz}, K_{mr} and K_{ms}, are constants defining the torque transmitted to the mass by the spoke about the O'u axis for unit change in z, r and s respectively, and K_{pz}, K_{pr} and K_{ps} are constants defining the force transmitted to the mass by the spoke along the OZ axis for unit change in z, r and s respectively.

Equation (xvii) results from a consideration of previous equations concerning the behaviour of mass M_1, but exactly the same equation results from like treatment of the corresponding previous equations concerning the behaviour of the other masses. Solution of equation (xvii) thus yields a certain rotor speed n in which the continuously oscillating forces generated by the individual masses M_1 to M_4 yield a steady sum which, in theory, exactly offsets any externally-applied torque to which the rotor is subjected by reason of deflection from its normal spinning position. The rotor will thus behave in a manner comparable to that of a free rotor. The formulae given suggest that in typical apparatus as shown in Figs 6.4 in which the spokes 10 have a width of 3/8" and a total length of 1" and their thickness is of the order of .005 inches, in which the length of their inner part 20 varies in the range 1/8" to 1/4" and the radial length of the part of the weight actually attached to the spoke is 1/8", a tuning condition can be obtained in each case at a shaft speed lying within the range 4,500 - 14,000 r.p.m.

7. VIBRATING GYROSCOPES

7.1 OSCILLOGYRO

The oscillogyro (or gyro-vibrator) is due to JOHN ST LEGER PHILPOT and J.H. MITCHELL GB 599826 (1948) both of R.A.E. Farnborough. The device is clearly shown in Fig 7.1.1, a bar (2) of metal is mounted on a gyro rotor (1) by means of a torsion spring (3) that is fixed at its centre to the bar (2) and at its ends to upstanding lugs (1.1*) on the rotor. Thus the bar (2) can oscillate about the axis of the spring (3) against torsional resistance of the spring while the bar rotates with the rotor about spin axis.XX Capacitors (4.5) (6.7) are balanced against each other in the arms of an alternating current bridge circuit (not shown). The rotor is driven by electric motor (30) and the capacitors are connected to the bridge circuit via slip rings (31).

When the gyro rotor is precessed the bar (2) oscillates. The frequency of the oscillation should ideally be synchronised with that of the rotation by adjusting the thickness of the bar (2) and the stiffness of the torsion spring (3). Any oscillation of the bar will then have an amplitude proportional to the angle of precession and a phase corresponding to the direction of the precession.

The oscillogyro has been described by WHALLEY. R., HOLGATE. M.J. and MAUNDER. L. (1967) and has attracted a wealth of analysis, primarily by ORMANDY. D. and MAUNDER. L. (1967) (1973). Conflicting analyses have been presented by DIAMANTIDES. N.D. (1959) and by SORG. H. (1968).*

An important short note on the device has been given by MAUNDER. L. (1974). MAUNDER has shown that if θ is the deflection of the bar about the hinge axis OY and A, B and C are the principal moments of inertia of the bar about OXYZ (OX longitudinal) and K_1, and K_2 are respectively the torsional spring constant and viscous damping coefficient where n is the spring-rate and Ω is the applied rate of turn; then the equation of motion is

$$B\ddot{\theta} + K_1\dot{\theta} + [K_2 + (C-A)n^2]\theta = -B\Omega \cos nt + (B+C-A)n\Omega \sin nt$$

Further the detailed analysis by ORMANDY & MAUNDER (1973) shows that:

1. The natural frequency is a function of spring rate n that can be made equal to the forcing frequency ($n/2\pi$) so that the device is tuned when

$$K=(A+B-C)\, n^2$$

2. If Ω is constant the steady state output amplitude is proportional to Ω, ie the oscillogyro acts as a rate - measuring instrument within certain limitations.

A modified oscillogyro is due to COMPAGNIE FRANCAISE THOMSON - HOUSTON GB 1023554 (1966) and the Ferranti oscillogyro that can detect rotation as slow as 0.01 degree per hour is the subject of an article by NUTTALL. J.D. (1982).

7.2 VIBRATING RING GYROSCOPE

A vibrating ring gyroscope is disclosed by NEWTON. Jr. R.C US 3307409 (1967) and by DIAMANTIDES. N.D. US 3367194 (1968).

An interesting discussion of the broad concept is given by NEWTON. G.C. (1965) and by STILES. J.C. U.S. 3924475 (1975). They point out that the vibrating ring may conveniently replace the rotating wheel of the conventional gyroscope and that it can be mathematically proven that if a ring is set into vibration in the out-of-round mode and then rotated through some angle the vibration pattern, as indicated by the nodes, will rotate exactly four tenths of this angle.

* See also: WELLBURN J. and REID D.A.
Oscillogyro design etc.
Mechanical Technology of Inertial Devices
Institution of Mechanical Engineers (1987) p.17-28

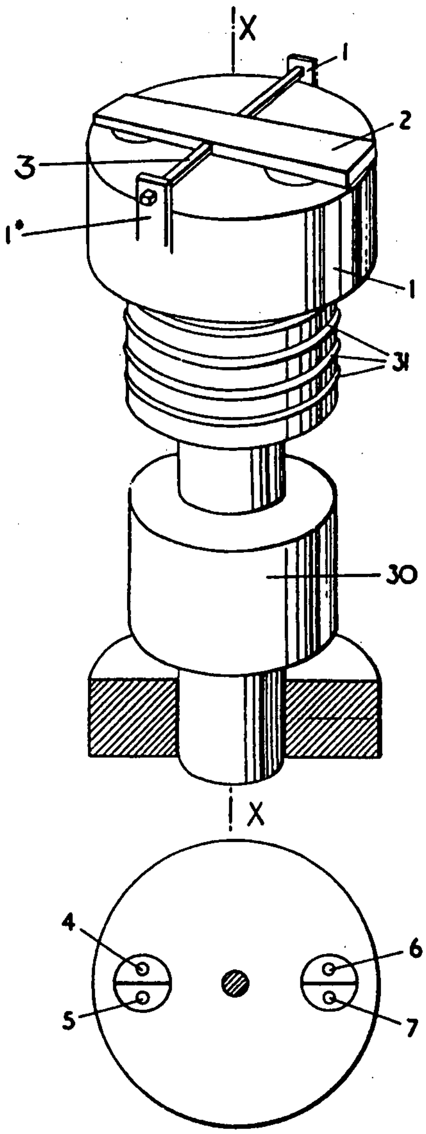


Figure 7.1.1 OSCILLOGYRO

7.3 VIBRATING BELL GYROSCOPE (SONIC GYRO)+

BRYAN. G.H. (1890) remarks upon the vibrations of a bell. He tells us that when the vibrating body is a bell, rotation about its axis will produce an intermediate effect by causing the nodal meridians* to revolve with angular velocity less than that of the body, and depending in each case on the mode of vibration considered. This phenomenon, appears to be new, yet nothing is easier than to verify it experimentally. If we select a wine-glass which when struck gives, under ordinary circumstances, a pure and continuous tone, we shall on twisting it round hear beats, thus showing that the nodal meridians do not remain fixed in space. And if the observer will turn himself rapidly round, holding the vibrating glass all the time, beats will again be heard, showing that the nodal meridians do not rotate with the same angular velocity as the glass and observer. If the glass be attached to a revolving turntable it is easy to count the number of beats during a certain number of revolutions of the table, and it will thus be found that the gravest tone gives about 2.4 beats per revolution. As this type of vibration has 4 nodes we should hear four beats per revolution if these nodes were to rotate with the glass, we conclude therefore that the nodal angular velocity is in this case about 3/5 of that of the body.

It may not, perhaps, be out of place to explain from first principles why the nodal meridians revolve less rapidly than the body. Take the case of a ring or cylinder revolving in the direction indicated by the arrows in Figure 7.3.1, and consider the mode of vibration with four nodes, B,D, F,H. Suppose also that at the instant considered the ring is changing from the elliptic to the circular form indicated in the figure.

Owing to the rotatory motion, the points A,E where the ring is initially most bent will be carried forward and parts initially less bent will be brought to A and E. Similar remarks apply to the points C, G, where the ring is initially least bent. Hence the points of maximum and minimum curvature, and therefore, also, the nodes must be carried round in the same direction as the ring, and cannot remain fixed in space.

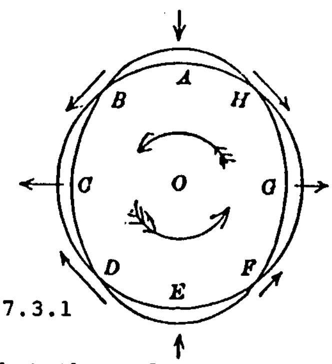


Figure 7.3.1

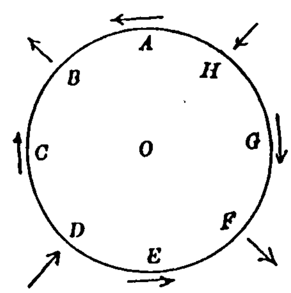


Figure 7.3.2

To show that the nodes do not rotate as if fixed in the ring, let the small arrows in Fig 7.3.1. represent the directions of relative motion of the particles exclusive of the components due to rotation. At A,E, particles are moving towards the centre O. This will of course increase their actual angular velocity and will give them a relative angular acceleration in the direction of rotation, as represented by the arrows at A, E in Fig 7.3.2. At C, G the particles are moving outwards, and this will retard their angular velocity. The particles at B, F are moving with greater total angular velocity than the rest; this will increase their "centrifugal force" and give them a relative acceleration outwards. Those at D, H are moving with the least total angular velocity, and the diminution in centrifugal force will give them a relative acceleration inwards. Hence the rotatory motion of the mass will give rise to relative accelerations of the particles in directions represented by the arrows in Fig 7.3.2. If we compare the arrows in Fig 7.3.1 and Fig 7.3.2 we see at

* That is meridians along which the vibration has no radial component.

\+ See U.S. Patent Specification to Koning, M.G. 4793195. (1988)

once that the effect of these relative accelerations is to cause retrograde motion of the nodes relative to the mass, that is, the nodes will rotate less rapidly than the ring. This explanation is obviously applicable to all the modes of vibration.

A device for detecting rotation appears in some detail from a disclosure due to EMSLIE. A.G. US 3625067 (1971) (See Fig 7.3.3). A bell capable of high Q vibrations is made of a high Q factor aluminium alloy two inches diameter at the lip which is 0.025 inch thick. The frequency of operation is 1600 cycles per second.

A sonic gyroscope of this form including a high Q resonator hemispherically shaped exhibits an elliptical vibration pattern in a plane perpendicular to its polar axis when excited in its lowest order bending mode. Rotation of the resonator about an input axis coinciding with the polar axis causes pattern rotation of about seventy two percent of input rotation relative to inertial space. There are shown to be two principle (normal mode) axes that exist in the resonator, these axes being separated by half a right angle. From certain asymmetrics in the thickness of the resonator the resonant frequency along one of the two normal mode axes is different from the resonant frequency along the other axis. In consequence the elliptical vibration pattern vibrates at one frequency of aligned with one of the axis and at a slightly different frequency when aligned with the other axis. At any other location the vibration pattern will consist of a superposition of these two normal modes of vibration.

To maintain the pattern amplitude the energy lost by the resonator during vibration must be replenished. One approach is given by LYNCH. D.D. US 3719074 (1973), but before we deal with it we should refer to the earlier work of LYNCH. D.D. US 3656354 (1972) in which he discloses a sealed high - Q member having a lip capable of sustaining a vibration pattern having alternate and equi-angularly spaced nodal and anti-nodal regions of radial vibration when the lip is exercised radially. DENIS. R.E. US 3678762 (1972) also discloses a bell, similar to that of LYNCH, made from an alloy having a modulus of elasticity (E) 10.6 x 10^6 p.s.i. and a Q of 3000 in air. The bell when in a high vacuum of 10^{-2} torr increases its Q value by a factor of four to about 12000. The alloys specified are 2024-TR having the composition Al.93.4 Cu 4.5. Mg. 1.5 and Mn.0.6 and EVERDUR having the composition Cu.96 Si.3.0 Mn.1.0 or Cu.91 Al.7.0 Si.2.0.

The sides and the lip have a mean radius of one inch and vary in thickness over the arcuate region from a maximum h_o in the centre to a finite thickness h in agreement with the formula $h=\frac{h_o}{4}(1+\cos\theta)^2$ where θ is the spherical angle subtended from the polar axis through the centre.

The hemispheric resonator of LYNCH (1973) referred to above, is driven by a circular forcer electrode and damping is overcome by modulating a parameter of the high Q system at twice the natural frequency of the said system. We are indebted to LOPER E.J. in cooperation with LYNCH. D.D. GB 2021266 (1979) for showing that hitherto it has not been recognised that a substantial drift error is introduced unless the two components of pattern vibration are in phase with one another, since otherwise the drive advanced by LYNCH. D.D. (1973) will preferentially drive the components. They point out that it is advantageous to maintain the entire resonator motion at a single frequency and phase regardless of pattern location; and that when the sonic gyro is operated as an integrating gyro, inaccuracies are introduced in the amount of pattern rotation resulting from an input rotation unless the entire resonator is vibrating at a single frequency and phase. To achieve the desideratum they provide a high Q resonator of hemispherical form supported by an inner housing carrying a circular forcer electrode and sixteen discrete forcer electrodes. The orientation of the vibrating pattern in the resonator is electrostatically senses by eight pick-offs located on the inner spherical surface of an outer housing that establish two pick-off axes separated by half a right angle. Signals obtained from the pick-off electrodes are utilized to develop a drive signal to the circular forcer electrode electrostatically to maintain a predetermined amplitude of pattern vibration and to develop drive signals for appropriate groups of discrete forcer electrodes to eliminate any phase error between components of the pattern along the two pick off axes. The hemisphere of LOPER and LYNCH is preferably of fused quartz.

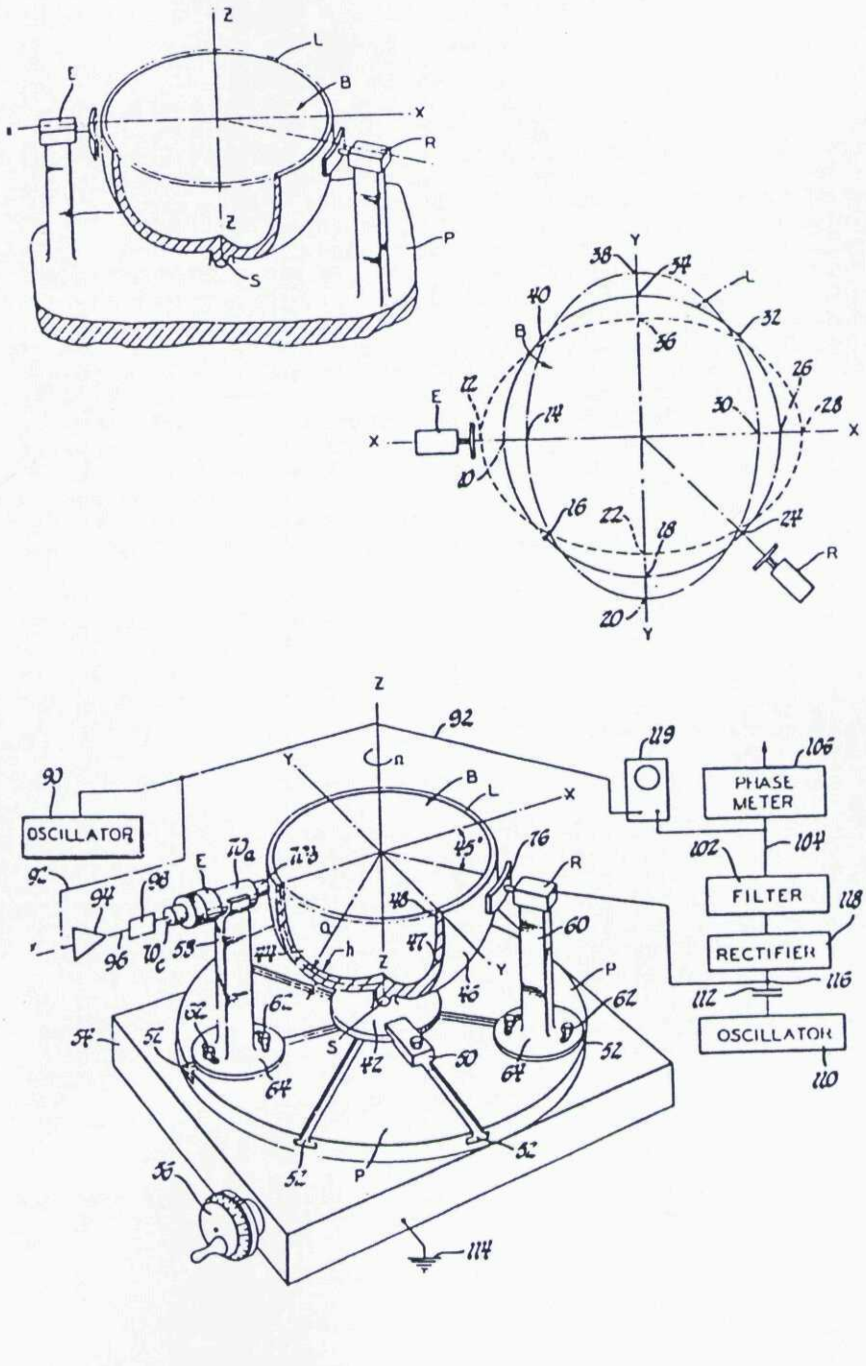


Figure 7.3.3

7.4 SINGLE REED GYROSCOPE

It was well known to BRYAN. G.H. (1890) that "if a straight wire of circular section, clamped symmetrically at one end, be made to rotate slowly about its axis while executing transverse vibrations, - the plane of vibration will remain fixed in space instead of turning with the wire. If the vibrations are audible we shall, therefore, hear a continuous sound".

Gyroscopically mounted vibrating reeds were proposed by HERR US 1728904 (1929) for the indication of the position of an automobile or an aeroplane, and his invention was assigned in part to the LUTEN ENGINEERING C° of INDIANOPOLIS.

LYMAN, J. and NORDEN. E. US 2309853 (1943) proposed a simple single rod free to vibrate at its upper end and supported at its lower end, the vibrations being sustained by an axial cored coil through which alternating current was passed. The device was to be used inter alia as an artificial horizon or a rate-of-turn indicator. The two authors also refer to a turning-fork embodiment discussed below. The operation of the vibrating string gyro is based on the well established principle, referred to above, that a body oscillating in a plane will maintain its plane of vibration fixed in inertial space unless it is subjected to oscillatory forces normal to and in time phase with the velocity vector of the vibrating body. FERRILL. Jr. T.M. US 2466018 (1949) shows a nice instrument forming a stable reference apparatus about a vertical axis in a predetermined relation to the direction of the horizontal component of the Earth's magnetic field, said apparatus using a vibratory strand (13) of nickel or nickel iron alloy, with a longitudinal component drive relying on magnetostrictive phenomena. (Coils 31.32). The instrument is shown in Fig 7.4.1. FERRILL US 2542018 (1951) proposed a compass using a quartz or tungsten strand having a high Q; the strand being mounted for free vibration in an evacuated envelope. The strand is energised to cause it through interaction with the Earth's field to vibrate in a planar mode, thereby to endow the strand with sufficient directional energy as to maintain the plane of vibration substantially fixed in space and uninfluenced by rotation of its support.

BARNABY. R. and REINHARDT. A.E. US 2544646 (1951) offer advances to the design put forward by LYMAN & NORDEN above, they propose the use of independent members sensitive to the elastic guiding restraint and to the elastic tuning restraint. In the same year JOHNSON. M.H US 2546158 (1951)* proposed a gyroscopic device having a wire conductor supported at its ends to permit free vibration, a motor field about said wire and means for supplying alternating current to the wire at a frequency corresponding to the natural period of the wire, to produce vibration of substantially resonance in one plane. An impedence bridge was provided having the wire in one of its branches.

JAOUEN. J. US 2974530 (1961) takes up the broad question of small efficient angular velocity apparatus that rely essentially on Coriolis inertia forces. He uses a vibratory element of ELINVAR, isotropic in character and vibrated by magnetostriction; by isotropic, he means that in each transverse cross section of the element the moments of inertia with respect to any straight line intersecting the centre of inertia of the section and contained within the plane of said section are the same.

MULLINS. Jr W.D. and SCARBOROUGH. W.M. US 3106847 (1963) disclose a gyroscopic apparatus comprising a vibrating electrically conductive string (11) in the form of a gold plated quartz fibre stretched between two opposed diaphragms (12.13) that move the string axially; both diaphragms moving in together and out together thereby forming a dynamically balanced system of high Q. (See Fig 7.4.2).

ERDLEY. H.F. US 3238789 (1966) proposed a vibrating double bar accelerometer having a high mechanical Q as against the generally low Q of the vibrating wire.

* See also EHRAT & MILLIQUET US 2594749 (1952)

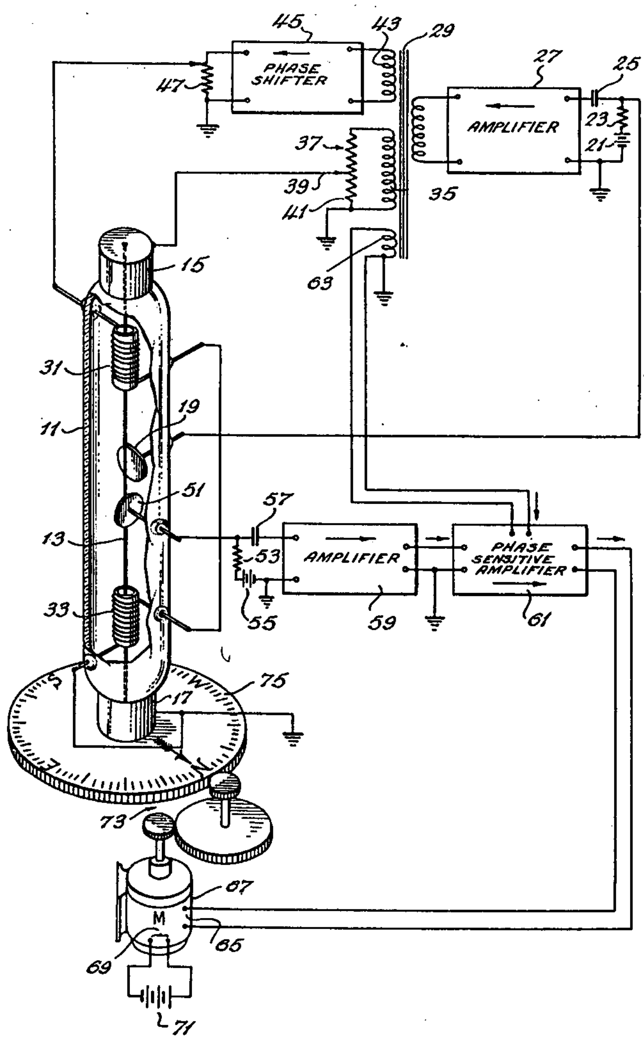


Figure 7.4.1

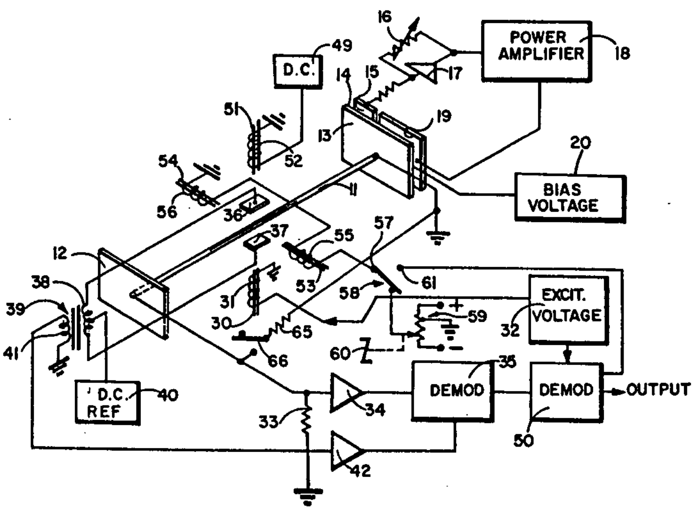


Figure 7.4.2

THOIKOL CHEMICAL CORPN. GB 1063469 (1967) describe a magnetic angular velocity indicating system (MAVIS) in which the wire is made to vibrate in an intense magnetic field and including force-producing means for applying to the wire a periodically recurring uni-directional tensioning force at a frequency that is twice the resonant frequency of the wire in its fundamental mode of transverse vibration.

HONEYWELL. INC. GB 1121750 (1968) GB 1249128 (1971) disclose a vibrating wire gyro in which a taut tungsten wire is stressed to 150,000 lb/sq inch to have a natural frequency of 4560 cycles/second and restrained at three nodal points, the vibration being transmitted to the second nodal point through the intermediate nodal point. We are indebted to COMPAGNIE GENERAL de TELEGRAPHIE SANS FIL (C.F.S.) GB 1130314 (1968) for a dissertation on a more advanced vibrating string gyroscope (See Fig 7.4.3) in which a vibrating electrically conductive string (1) is tensioned between ends (ZZ') of a frame (2) and arranged parallel to the direction OZ. The string is in a magnetic field B_0 and carries a current i_1 at a frequency f_1. At one vibration antinode (position of maximum amplitude) a d.c. inductor (3) provides a magnetic field $\vec{B_0}$ in the direction Oy. This inductor may be, for example, a permanent magnet located at the first vibration antinode. An a.c. inductor (4), supplied with alternating voltage E_2 with a frequency f_2, creates an alternating magnetic field $\vec{B_2}$ in the direction Ox; this is localized, for example, in another vibration antinode of t string. The current i_1 flowing through the string (1) is supplied by a source (5) which supplies the energy necessary for keeping string (1) oscillating at its resonance frequency. To this end, source (5) may comprise a negative impedance - Zc, for example an amplifier with a feedback, so as to meet the conditions for the self-oscillation of the vibrating string (1) by compensating any mechanical damping thereof, and stabilizing means, for example a non-linear amplitude stabilizing network, for keeping the vibration amplitude of the string (1) constant. According to the invention, a circuit (6) provides a selective short-circuit (6) across the string (1), as will be described later. An a.c. ammeter (7) may by provided for measuring the current flowing through circuit (6).

In the absence of any rotation of the reference frame Oxyz tied to the gyrometer, a vertical force F_m is applied to the string (1) which vibrates in the plane xOz. The assembly of string (1) and inductor (3) behaves as an electro-dynamic motor, having between the terminals Z and Z' a total electrical impedance Zc. When the reference frame Oxyz undergoes a rotational movement with an angular velocity component about Oz, the vector product of the relative velocity of the string 1 and the angular velocity component supplies a Coriolis acceleration which results in a force $\vec{F_c}$ added to the force $\vec{F_m}$. As may be seen from Fig 7.4.3B the resulting force $\vec{F_R}$ which is applied to the string (1), is inclined at an angle θ to the axis Ox. Thus the plane of vibration of the string tends to pivot under the action of the angular velocity if F_m and Fc are in phase, or the string tends to describe an elliptic trajectory if F_m and F_c are out of phase. In order to measure this pivotal movement, the invention provides for the analysis of the voltage induced in the string (1) by the inductor (4) which is excited at the frequency f_2.

Fig 7.4.3C shows the trace of the plane of vibration of the string (1). The string intersects at the frequency f_1, the magnetic induction lines $\vec{B_2}$ and the electromotive force induced can be written as follows:

$$e_2 = k_3 \cos. 2\pi f_1 t. \cos 2\pi f_2 t.$$

wherein k_3 is a function of $\sin\theta$.

Similarly, the electromotive force induced by means of the inductor (3) can be written as follows:

$$e_1 = k_1 \cos 2\pi f_1 t,$$

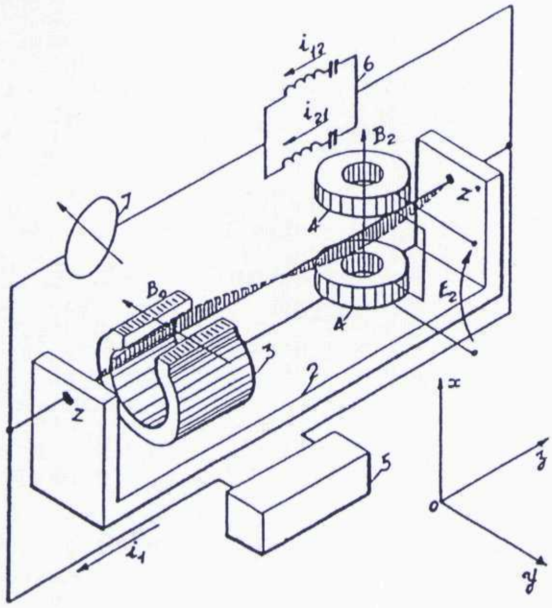


Fig A

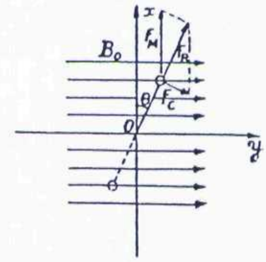


Fig B

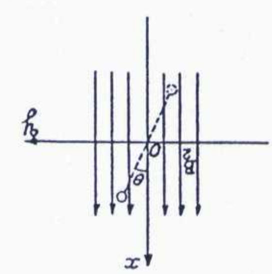


Fig C

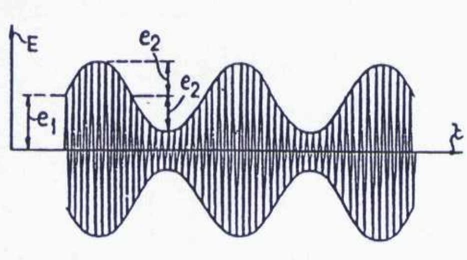


Fig D

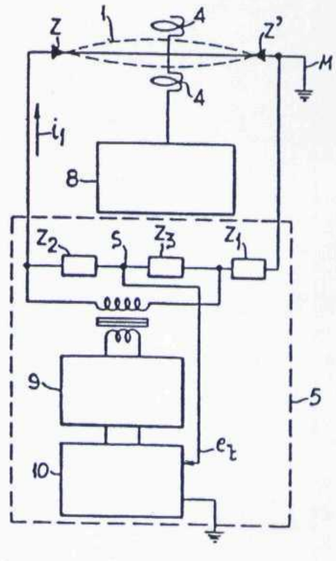


Fig E

Figure 7.4.3

where k_1 is a constant factor. The total electromotive force e_t induced in the string is expressed by the equation:

$$e_t = k_1 \left(1 + \frac{k_3}{k_1} \cos 2\pi f_2\, t\right) \cos 2\pi f_1\, t.$$

The voltage e_t is accordingly an amplitude modulated signal whose modulation envelope has a modulation depth which is a function of the sine of the angle of inclination θ of the plane of vibration. Such a signal is shown in Fig 7.4.3D. By way of example, a vibration frequency f_1 of the order of 2000 c/s may be selected with a frequency f_2 of 200 c/s.

According to the invention, the electromotive force e_2 produces in a measuring branch the currents i_{12} and i_{21}, corresponding, respectively, to the spectrum components f_1+f_2 and f_1-f_2 of the amplitude modulation. An impedance (6), located in this branch, presents an infinite admittance for the frequencies f_1-f_2 and f_1+f_2, and a zero admittance for the frequency f_2. Thus, the short-circuit current flowing through the ammeter (7) causes it to deviate in proportion with the pivoting of the plane of vibration of the vibrating string. The Coriolis force which tends to move the plane of vibration away from the plane xOz is balanced by the force F_m and mainly by the electrodynamic reaction produced in the string (1) by the currents i_{12} and i_{21} in accordance with Lenz's law. In the absence of rotation, these restoring forces tend to maintain the vibration of the string within a well defined plane, parallel to the alternating field B2.

Fig 7.4.3E shows diagrammatically a part of the arrangement of Fig 7.4.3A. A generator (8) supplies the inductor (4) at the frequency f_2. The string (1) forms with impedances Z_1, Z_2 and Z_3 a balanced bridge, which is supplied by a selective amplifier (10) through a non-linear amplitude stabilizing network (9). Between the ground terminal M and the output terminal S, there appears the total voltage e_t proportional to the voltage induced in the vibrating string, and this voltage is fed back to the amplifier (10) which is tuned to the maintenance frequency f_t.

Returning now to the supply circuit (5), when the string vibrates, a component e_1 with the frequency f_1 appears in the induced voltage e_t. It is selectively amplified by the amplifier (10) and clipped by the non-linear network (9) that assures the amplitude stabilization. An actuating current i_1 flows through the vibrating string (1) with such an intensity and phase that the movement of the spring is undamped. The vibrating string gyroscope has simplicity combined with ruggedness and the energy expended is very small, further the useful life is limited only by that of the electronic components since there is no mechanical wear.

Reference may also be made to G.B. Patent Specifications. 647723, 685113, 685369.

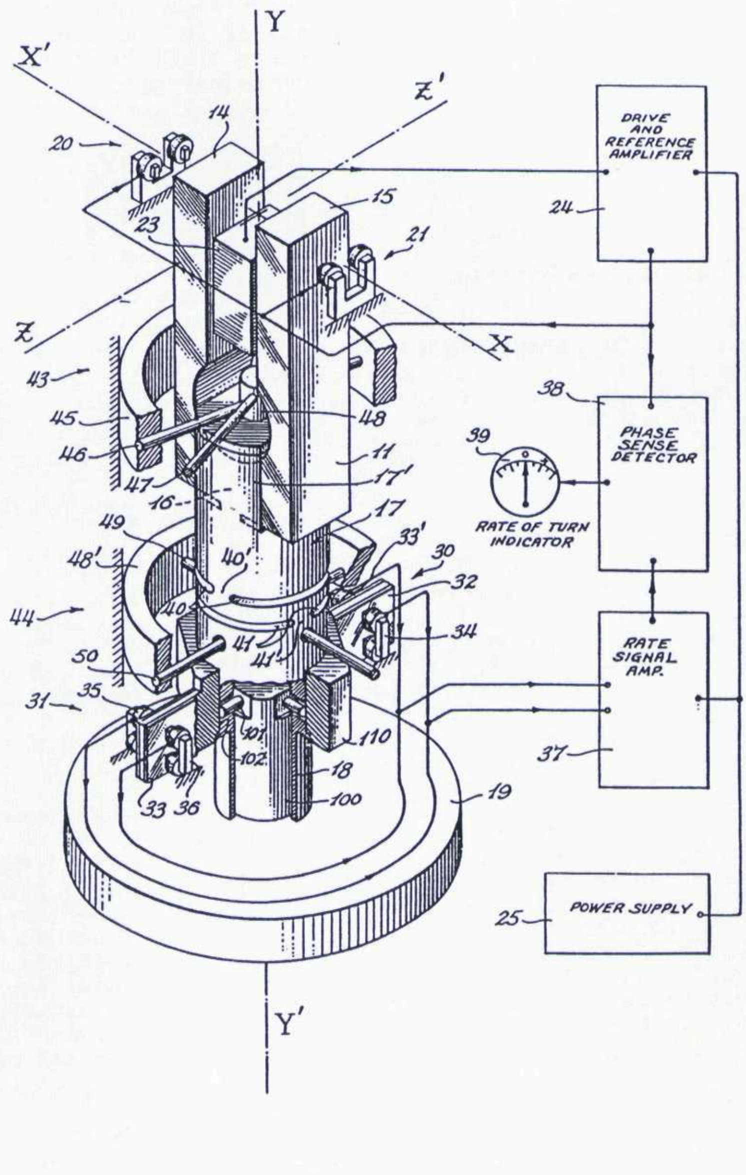

Y
X'
Z'
Z
X
Y'
DRIVE AND REFERENCE AMPLIFIER
24
PHASE SENSE DETECTOR
38
39
RATE OF TURN INDICATOR
RATE SIGNAL AMP.
37
POWER SUPPLY
25
20
14
15
21
23
43
45
46
47
48
11
17'
16
17
49
48'
40'
33'
30
32
44
40
41
41'
34
50
31
35
110
101
102
18
100
19
33
36

Figure 7.5.1

7.5 TUNING FORK GYROSCOPE

The broad concept of using a tuning fork in which the vibrating tines detect Coriolis forces is now known (See PRINGLE. J.W.S. (1948) MEREDITH. F.W. (1949)) to have been exploited continuously by the Diptera of the panorpoid complex from the Permian (230 x 10^6 years ago) to the present day. (See Chapter 1). The use of gyroscopic halteres in the Pipunculidae of the sub-order Brachycera-Cyclorrhapha is thought to give these small flies an opportunity to exhibit with consummate panache their precise hovering, so much admired by entomologists and students of animal flight generally.

LYMAN. J. and NORDEN. E. US 2309853 (1943)* and LYMAN.J. alone GB 601051 (1949) US 2513340 (1950) disclose a tuning fork for use in a vibrating gyroscope and the invention is assigned to SPERRY GYROSCOPE CO of BROOKLYN. Similar early work is disclosed by MEREDITH. F.W.+ assigned to S. SMITH & SONS (MOTOR ACCESSORIES) LTD GB 611005 (1948) who proposed a gyroscopic device for measuring rate-of-turn, including a vibrating tuning fork, the tines of which are driven by an electromagnetic circuit. MORROW. C.T. US 2616681 (1952) proposed a three tine fork responsive to the angular velocity or rate-of-turn of a body and MORROW together with BARNABY & NEVALA. US 2683596 (1954) proposed an electrically oscillated tuning fork connected by a torsional constraining spring to a base adapted to be mounted on a craft, the rate-of-turn of which is to be measured; spurious vibrations of the fork assembly parallel to the turn axis being reduced substantially to zero. See Fig 7.5.1.

The apparatus consists generally of tuning fork element 11 having substantially massive tines 14 and 15 mounted so that the tines will normally vibrate in a plane defined by the axes XX^1, YY^1. The heel portion 16 of the fork is rigidly secured to the top of an intermediate portion or torsion structure 17 at the bottom of which is secured, or formed integrally therewith, a thin cylindrical torsion spring 18, which, in turn, is secured rigidly to a base 19 mounted on a craft or body, the rate of turn of which is to be measured. The torsion structure 17 is preferably constructed of non-magnetic material for the purpose of minimizing stray field coupling between the tine drive field and the torsion pickup to be referred to hereinafter. Adjacent the top or open ends of tines 14 and 15 is a driving means which in the embodiment illustrated may take the form of electromagnetic drive coils 20, 21 adjustably secured to the sidewalls 22 of the apparatus (Not Shown). Between the open ends of tines 14 and 15, there is provided a vibration pickup device 23 which may be of the variable capacitance type. Drive coils 20 and 21 are connected in series and receive their driving energy from the output of a drive and reference amplifier 24. One input of amplifier 24 is connected with a power supply unit 25 and the other input is connected with capacity pickup 23. The pickup 23 excites the amplifier so that the whole system, fork and amplifier, is self-resonant at a frequency preferably corresponding to the natural frequency of vibration of tuning fork 11. The output of the drive amplifier may be limited so that the amplitude of the drive voltage will not build up to destructive values. As the tines 14 and 15 are rapidly vibrated in the plane defined by the axes XX^1, YY^1, and base 19 is rotated about the YY^1 axis, tuning fork 11 can preserve angular momentum only by executing a torsional vibration about the YY^1 axis proportional to the rate of the turning movement of base 19. The magnitude and phase of this torsional vibration are expressed electrically by torsion pick-offs, generally indicated at 30 and 31, and are a measure of the magnitude and direction, respectively, of the rate of turn of base 19 or craft on which the base is mounted. The signal from the pick-offs in the illustrated embodiment of the present invention is supplied to one input of rate signal amplifier 37, also connected with power supply unit 25. The output of rate signal amplifier 37 is applied to a

* The application bears the date 10 April 1941.

\+ The application bears three dates.
27 July 1942, 4 Sept 1942, 10 March 1943.

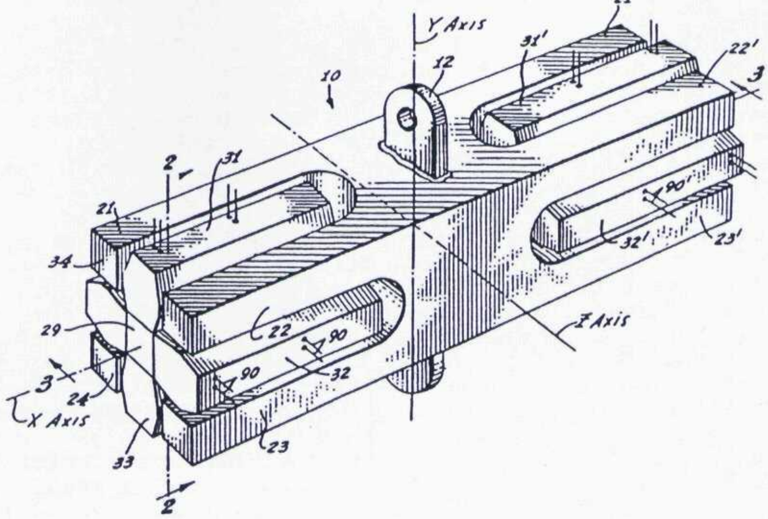
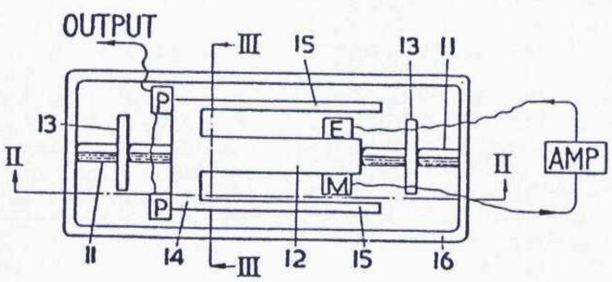
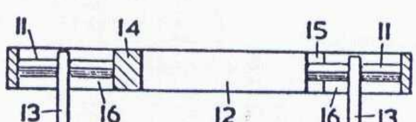
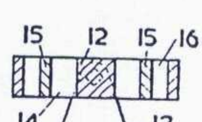


Figure 7.5.2





Section on II II



Section on III III

Figure 7.5.3

phase-sensitive detector 38 in which it is compared with the reference signal from the drive and reference amplifier 24 to thereby obtain a resultant signal of phase and magnitude proportional to the rate of turn of the base member 19. This resultant signal may be connected to a suitable indicating device or meter 39 calibrated to represent a measure of the rate of turn of base 19. The embodiment of the present invention herein illustrated is primarily suited to telemetering applications. However, it should be noted that in some applications of the invention, it may be desirable to use the output signals from the rate signal amplifier to actuate an induction motor, a phase detector, or other converting device in some portion of the system in which the apparatus is applied.

BARNABY & MORROW US 2753173 (1956) show how the tines of the fork may be maintained in continuous vibration. The fork is non-magnetic and a uni-directional magnetic field is produced in the direction in which the tines are adapted to vibrate; further a pole piece in each tine adjacent its tip has two parallel core limbs extending perpendicular to the axis of symmetry of the fork in the plane of vibration of the tines so as to provide two parallel paths for the polarisation flux of the uni-directional field. The length of the gap between two limbs forming one path are different from the length of the gap of the limbs forming the other path in the normal position of the tines. The length of each gap is sufficient to permit vibration of the tines and an energisable winding is mounted on the support in such a manner that it surrounds the path provided by one pair of limbs with its axis lying substantially in that path. An improved fork is due to HOLT W.J. US 2838698 (1958). A number of minor yet important contributions to this art is provided by KAROLUS. A. GB 947310 (1964) GB 1008999 (1965) GB 1009021 (1965) HANSEN. R.A. and HOLT. Jr. W.J. US 3127775 (1964) disclose an eight tine structure and this is shown in Fig 7.5.2.

HUNT. G.H. and HOBBS. A.E.W. (1964-1965) published an account of their development of an accurate tuning-fork gyroscope later to be published by MINISTER OF AVIATION LONDON GB 989101 (1965). They proposed a torsion bar mounting shown in Fig 7.5.3 having two torsion bars (11.11) a highly rigid bar (12) joining them and two support members (13.13) joined to the torsion bars (11.11) at nodal zones intermediate of their lengths. A tuning fork arrangement (14) joined at its base to the bar (12) has its tines (15.15) extending parallel with the bar (12) at equal distances therefrom at opposite sides thereof. Also a rigid rectangular frame (16) is joined to the outer ends of the torsion bars (11.11) at points midway of the lengths of its shorter sides so that it is symmetrical about the axis of the torsion bar mounting. The arrangement is balanced and it has equal natural frequencies of oscillation about the axis of the torsion bar mounting (11.12.13). These natural frequencies of oscillation are the same as the natural frequency of oscillation of the tines. Thus, the bar (12) with the tuning fork (14) and the rigid frame (16) constitute respectively two oscillatory bodies that are resiliently coupled by the torsion bars (11.11), so that the oscillation of the one is accompanied by the oscillation of the other in antiphase, just as oscillation of either tine (15) of the tuning fork (14) is accompanied by oscillation of the other tine (15) in antiphase. This construction was the essential part of the A5 tuning fork gyro of the Royal Aircraft Establishment in 1962 that gave random variations of zero error of less than one degree of arc per hour.

BÖRNER. M. (1966) published a note on the new solid gyroscopes in which a type of vibrating gyroscope is described and since it is so often compared with the tuning-fork gyroscope we chose to take it here. The gyroscope resonator is extremely symmetrical in its geometry and made from a metal with small attenuation of mechanical vibrations. We are indebted to TELEFUNKEN PATENTVERWERTUNGS-GESELLSCHAFT m-b-H. GB 1049794 (1966) for a more detailed description in English. A right cylindrical resonator having axial symmetry is associated with electromechanical means for imparting to the said resonator longitudinal oscillations in the axial direction, wherein the resonator is constructed in such a manner that it can be excited to longitudinal oscillations of the $n\lambda_{L/2}$ resonance in the direction of its longitudinal axis and exhibits, when turned about the said longitudinal axis, an $n\lambda_T$ resonance substantially at the same frequency of the longitudinal oscillations, n being any whole number

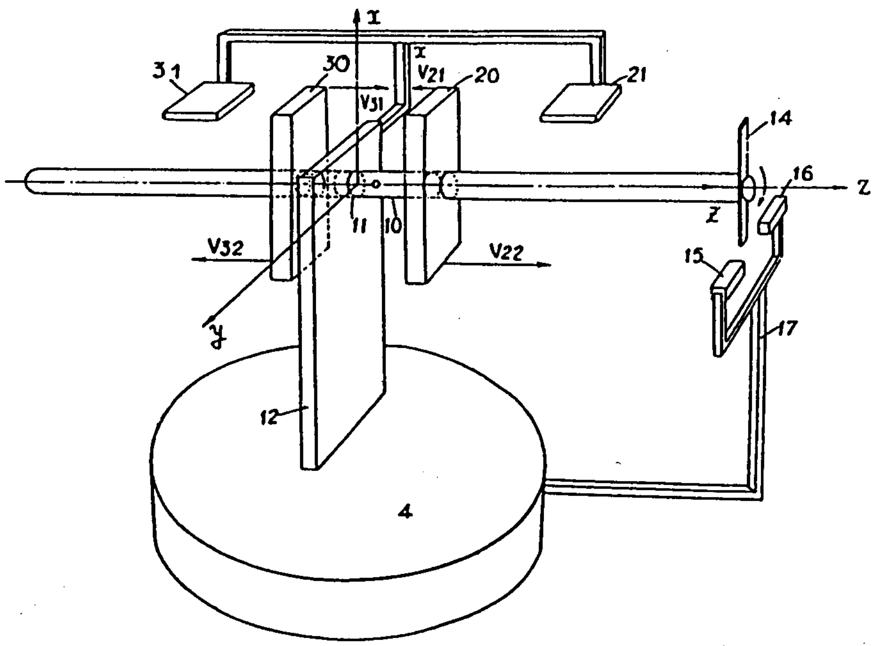


Figure 7.5.4

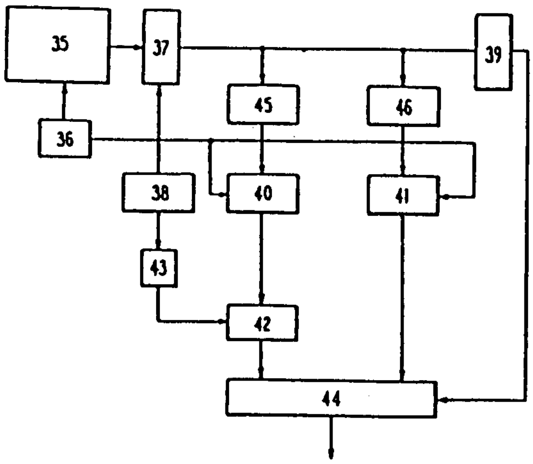


Figure 7.5.5A

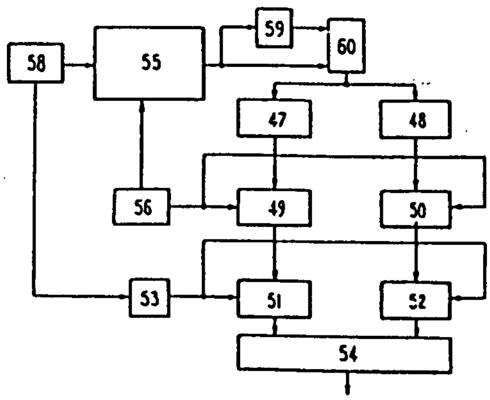


Figure 7.5.5B

and λ_L representing the wavelength of the axial oscillations and λ_T the wavelength of the torsional oscillations. A further refinement of the resonator is shown by TELEFUNKEN PATENTVERWERTUNGS. GB 1102477 (1968). The lack of a centre of symmetry in tuning fork gyroscopes and the removal of the spurious vibrations that result are taken-up by CSF-COMPAGNIE GENERAL DE TELEGRAPHIE SANS FIL GB 1054082 (1967). They propose an unusual construction shown at Fig 7.5.4 on which two indentical prongs (20) and (30) are carried by a rod (10) having a sensitive axis OX.

JAZ. S.A. of Paris GB 1139083 (1969) discloses a tuning fork gyrometer using an H shaped fork.

INTERNATIONAL STANDARD ELECTRIC CORPN GB 1141727 (1969) were aware of the need to reduce random drift which in a tuning fork gyro is seldom better than one tenth of a degree of arc per hour, whereas the theoretical sensitivity is 10-3 degrees of arc per hour or less; their proposal will be explained in some detail below. First we should explain that as the tines of the fork oscillate, the moment of inertia of the fork about the handle axis varies periodically with the frequency of the vibration of the fork and as shown by PERCIVAL. A.L. (1954) and others, this moment may be expressed by

$$I = I_0 + I_1 \sin\omega t$$

where $n = \omega/2\pi$ is the frequency, I_0 is the moment of inertia of the fork when the tines are in the mean position, and I_1, represents the greatest departure from this mean. If a rotation with constant angular velocity Ω about the handle axis is given to the fork, the angular momentum about the axis is given by

$$V = \Omega\ [I_0 + I_1 \sin\omega t]$$

In order to maintain this rotation, the couple applied to the handle of the fork must be $L = \frac{dv}{dt} = I_1\Omega\omega \quad \cos\omega t$

It is this alternating couple that the engineer must detect and measure.

The basis of detection is to support the fork on a torsional spring of such stiffness that its natural frequency on the spring is also n. The fork then develops an oscillatory torsional motion just as though the couple L were being applied to it directly, thereby making use of the large dynamic mangification of the resonant system. What is actually measured, by a suitable pick-up, is the velocity associated with the oscillatory rotation, which is superimposed on the steady applied rotation Ω. Provision is made for electromagnetic damping of the motion, this being necessary when measuring higher angular velocities. The velocity of the oscillatory rotation gives a measure of the angular velocity Ω, and its phase relative to the tine motion, indicates the direction of Ω. The output signals are suitably combined to give a direct reading of angular velocity on a dial. A considerable amount of electronic equipment is, of course, used; but the whole instrument, including amplifiers, although only in the early stages of development, weighs less than 7 lb.

It was later found that the vibration of a tuning-fork is affected by its mounting. Longitudinal inertia forces, related to heel flexibility, mass eccentricity and pendulous motion of the tines cause coupling between fork and mounting and induce first-and second-harmonic longitudinal motion of the fork as a whole. Thus the fundamental frequency of the system depends on the mounting characteristics, a feature that should be avoided if a high performance is consistently to be achieved. Further the primary component for longitudinal unbalance may be eliminated by suitably off-setting the tine mass distributions, and that no simple method of eliminating secondary unbalance is available, and that the second-harmonic longitudinal motion has a negligible effect on the fundamental frequency of vibrations. Clearly it is important in certain instrument applications to ensure that large longitudinal vibrations are not set-up, and hence that the mounting should be designed such that its natural frequency, when carrying the inert fork, is well removed from both fork frequency and twice fork frequency.

To these refinements we may add the finding of INTERNATIONAL STANDARD ELECTRIC CORPN. (1969) refered to above. They were concerned to reduce the random drift effect in rotation rate sensors whereby a double modulation of the gyroscope signal, or a single modulation of the gyroscope signal plus an amplitude modulation of the output signal is effected. When the signal appears in an amplitude form the signal may be written in the form;

$$E = U \cos Rt + a \cos (Rt+h),$$

where R designates the angular velocity of the excitation frequency of the strips of the tuning fork gyroscope, and h designates an arbitrary phase shift existing between the useful signal U and the random signal a.

The vibratory movement of the two blades of the tuning fork is obtained by an electrical excitation supplied by the circuit 36. This angular velocity R is in general the fundamental angular velocity of the two strips of the tuning fork. Is is assumed that the useful signal U to be measured is small as compared to the random signal a. Figs 7.5.5A and 7.5.5B give the block diagrams of the circuits enabling one to cancel the random term. In the circuit of Fig 7.5.5A this cancellation is obtained by modulating, in a circuit 37, the output signal of the gyroscope 35 by a signal of angular velocity r supplied by a circuit 38. The modulated signal Em will be in the form:

$$Em = (1 + \cos rt)\ [U \cos Rt + a \cos (Rt + h)]$$

viz.:

$$Em = U\cos Rt + a\cos(Rt+h) + \frac{U}{2}\cos(R-r)t + \frac{a}{2}\cos[(R-r)t+h] + \frac{U}{2}\cos(R+r)t + \frac{a}{2}\cos[(R+r)t+h]$$

It is assumed that the angular velocity r ranges about one tenth of R.

This modulated signal Em is applied to two filters 45 and 46 which pass respectively only to the components of the signal Em of angular velocities (R-r) and R. The said filtered signal from filters 45 and 46 are applied respectively to phase detectors 40 and 41 the reference signal of which, of angular velocity R, is supplied by the circuit 36. The ouput signals of the said phase detectors 40 and 41 are respectively in the form:

$$\frac{U}{2} \cos rt + \frac{a}{2} \cos (rt-h)$$

and

$$M = U + a \cos h.$$

The output signal from the phase detector circuit 40 is applied to a phase detector 42 which receives a signal of the form sin rt from phase shifter 43 which is obtained from the signal cos rt used for modulating the output signal of the gyroscope. This phase detector 42 supplies an output signal proportional to N = a sin h. The output signals of the phase detectors 42 and 41 as well as the signal proportional to (U+a) which is supplied by an amplitude detector 39, are applied to a computer 44 which carries out the operation $\sqrt{M^2+N^2}$ assuming that U is small compared to a, viz.: the result a+U cos h which, subtracted from the signal U + a, enables one to obtain a signal U(1-cos h) proportional to the useful signal U. Computer 44 is such that it may be designed by one ordinarily skilled in the art.

The same result should be obtained if the excitation signal of the strips were modulated by an electrical signal of the form (1+m cos rt), r being an angular velocity of value much lower than R, (i.e., in a ratio of ten to one, or less) and m a factor lower than unity.

Fig. 7.5.5B gives the block diagram of another circuit arrangement which cancels the random term a in the case of a tuning fork vibratory gyroscope. This device is based on the fact that the frequency spectrum of the vibratory movement of the tuning fork comprises a fundamental frequency and higher frequencies called "partials". A device similar to the double polarization of a ring laser gyroscope may be put into operation in that way, by making provision for instance, for two circuits 58 and 56 which excite, respectively, the tuning fork at the fundamental angular velocity r and at the angular velocity R corresponding for instance, to the second partial, the two angular velocities being such that $\frac{R}{r} \geqslant 10$.

The output signal of the tuning fork gyroscope 55 has the form:

[U cos rt+a cos (rt + h)] [U cos Rt+a cos (Rt + h)].

In order to enable certain simplifications, it is advantageous to obtain the ratio of the output signal of the gyroscope 55 to the random signal a. This ratio is carried out by a circuit 60 receiving on the one hand the output signal of the gyroscope 55, and on the other hand, the signal U+a, which is taken equal to a since U is assumed to be small compared to a, supplied by an amplitude detection circuit 59 which receives also the output signal of the gyroscope 55. The output signal of the circuit 60 may then be written:

$$[U \cos rt + a \cos (rt + h)]\ [\frac{U}{a} \cos Rt + \cos (Rt + h)].$$

By writing explicitly the last formula, a term $\frac{U^2}{a}$ is obtained which may be neglected, so that three terms are available at the angular velocity (R-r):

$$\frac{U}{2} \cos [(R-r)t-h]$$

$$\frac{U}{2} \cos [(R-r)t+h]$$

$$\frac{a}{2} \cos (R-r)t$$

and two terms are available at the angular velocity (R+r):

$$U \cos [(R+r)t+h]$$

$$\frac{a}{2} \cos [(R+r)t+2\ h]$$

which may be separated by filtering in circuits 47 and 48. The output signals at the angular velocity (R-r) of the filter 47 and at the angular velocity (R+r) of the filter 48 are applied respectively to the phase detectors 49 and 50 which receive the reference signal at the angular velocity R supplied by the circuit 56. The output signal of the phase detector 49 comprises the terms:

$$\frac{U}{2} \cos (rt + h) + \frac{U}{2} \cos (rt - h) + \frac{a}{2} \cos rt,$$

and the output signal of the phase detector 50 comprises the terms:

$$U \cos (rt + h) + \frac{a}{2} \cos (rt + 2\ h).$$

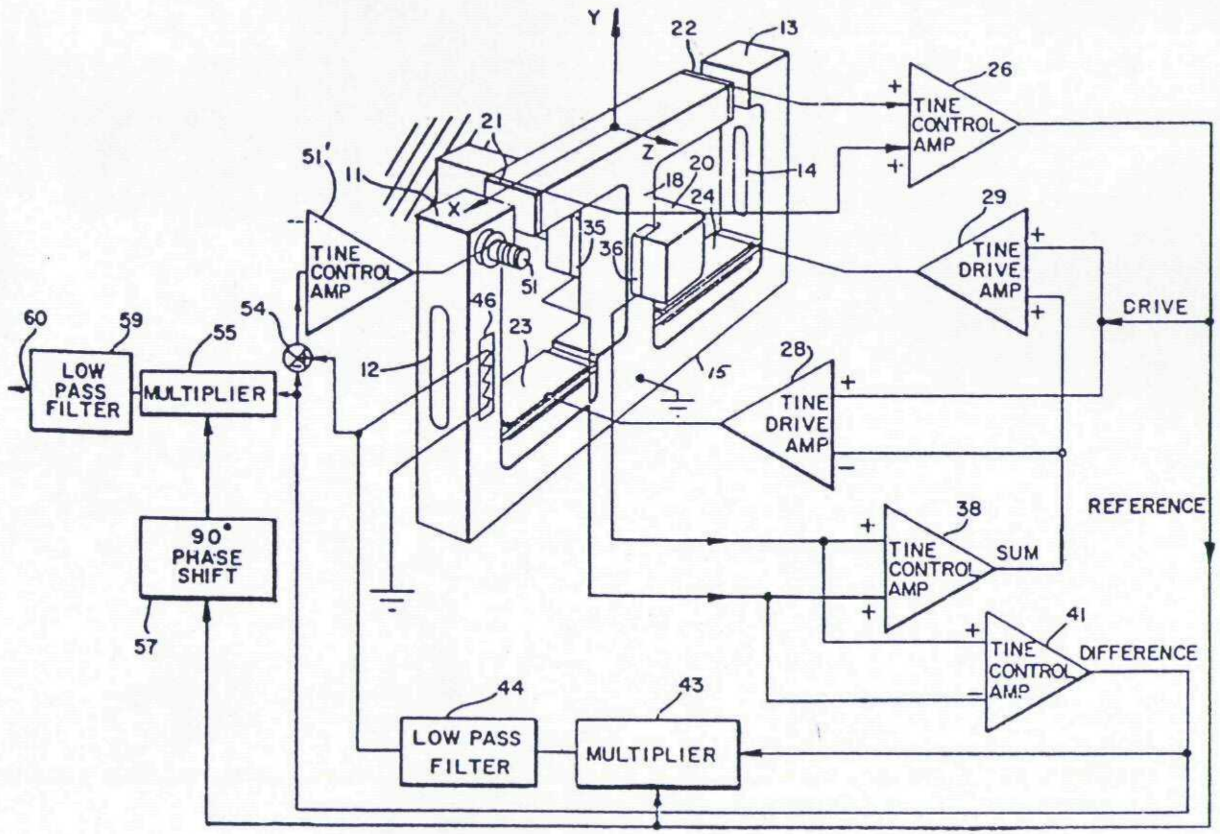

TINE CONTROL AMP
TINE DRIVE AMP
DRIVE
REFERENCE
SUM
DIFFERENCE
LOW PASS FILTER
MULTIPLIER
90° PHASE SHIFT
LOW PASS FILTER
MULTIPLIER

Figure 7.5.6

These signals are applied to two other phase detectors 51 and 52, the admittance Y of which may be written:

$$Y=Yo\ [1 + n \cos (rt + h')].$$

The reference signal for these phase detectors 51 and 52 of the angular velocity r is supplied by the circuit 58 via the phase shifter 53 which shifts the output of circuit 58 by the quantity h'. This value h' is set in such a way that $h' + h = \frac{\pi}{2}$,
where h is a fix phase angle.

The output signal of the phase detector 51 is then:

$$\frac{U}{2} \sin 2\,h + \frac{a}{2} \sin h$$

and the output signal of the phase detector 52 is:

$$-\frac{a}{2} \sin h.$$

It is thus seen that by adding these two signals in a circuit 54, the random term a is cancelled and a signal U sin 2 h is obtained which is proportional to the useful signal U. It is also seen that when the useful term U is zero i.e. when no rotation at all is to be detected, the output signal of the circuit 54 must also be zero. This provides a means for setting the phase shifting circuit 53 to the phase shift h' so that $h' + h = \frac{\pi}{2}$.

Another approach is disclosed by SCHLITT. H.W.E. US 3839915 (1974). (See Fig 7.5.6) who provides a vibratory tuning fork in which oscillatory torques caused by residual misalignments of the tine motions are minimized by one of the tines (11) being selectively distorted by heating (resistor 46) under a servo loop control so as dynamically to minimize such misalignment, to aid the distortion, tine (11) is slotted and tine (13) also to retain symmetry.

Since this loop can operate at a low bandwidth and relative slow response time, e.g. in the order of 1 cycle per second, quite high levels of gain can be employed, i.e. in the order of 10^9. Accordingly, this source of error, which is otherwise difficult to distinguish, can be reduced to a correspondingly low level. By reducing misalignment torques in this way, it is possible to substantially eliminate errors due to the residual cross-coupling between quadrature components which occurs in any practical synchronous detection system.

The Y axis torque signal is also synchronously demodulated to obtain a quadrature component by means of a second multiplier 55, the common mode drive signal being applied as a second input to this second multiplier through a 90° phase shifter 57 so as to obtain an appropriate phase reference signal. The d.c. component of this detection process, obtained by means of a low pass filter 59, is then indicative of the rate of rotation of the tuning fork sensor around the Y axis.

In order to minimize errors due to non-linearities and other sources, the turn rate sensing circuitry is also operated in a feedback mode. For this purpose, an electromagnetic forcer 51 is mounted adjacent the tine 11 for exerting a torque on the fork about the Y axis. By energizing this forcer synchronously with the tine vibration and with an amplitude which varies as a function of the amplitude of the Coriolis-induced torque, a so-called torque-to-null mode of operation can be obtained. In this mode, the fork is torqued about the Y axis to balance the oscillatory Coriolis-indiced torque thereby minimizing or nulling the corresponding component of the differential mode signal developed between detectors 35 and 36. For this purpose, the a.c. difference signal from the amplifier 41 is applied to the electromagnetic forcer 51 through an appropriate amplifier 51'. This loop may advantageously have an a.c. bandwidth in the order of 100 cycles per second. The level of signal required to obtain null is then an accurate measure of externally impressed rotations around the Y axis up to 100 cps.

Preferably, the forcer 51 is also energized with a d.c. component derived from the tine misalignment correcting loop. This d.c. component is summed with the a.c. component, as indicated at 54, prior to the drive amplifier 51. This feedback path aids in minimizing the misalignment, operating with a faster time constant than the heater 46. This d.c. feedbank path through the magnetic forcer 51 also aids stability in the tine alignment loop since the high gain applied requires a relatively high natural frequency for stability. Any residual component in phase with the tine deflection is eliminated from the output signal by the synchronous detection of the quadrature component at the multiplier 55.

In a recapitulatory note we should add that the work of LYMAN, MORROW, BARNABY and NAVALA and others that resulted in the Sperry Gyrotron angular tachometer is described by LYMAN. J (1953) and by BARNABY. R.E. et al. (1953).

We also have a paper by MORROW C.T. (1955) in which is discussed with insight the question of zero signals in the Sperry tuning-fork gyroscope. The mathematical theory of vibratory angular tachometers is taken up with singular erudition by FEARNSIDE. M.A. and BRIGGS. P.A.N. (1957). By use of MATHIEU'S* equation they show how performance depends on the parameters of the system, particularly the resonant frequency and damping factor of the torsion mechanism.

CHATTERTON. J.B. (1955) gives some general comparisons between vibratory and conventional rate gyroscopes and NEWTON. Jr. G. C. US 3241377 (1966) gives a comparison of vibratory and rotating wheel gyroscopic rate indicators and a detailed investigation into the sensitivity of both instruments; for the vibratory instrument he gives

$$S_v \triangleq \frac{(E_2/\Omega)}{\sqrt{R_{m\phi}}} \tag{1}$$

where

E_2 is the amplitude of the open-circuit pickup voltage, Ω is the angular rate of the instrument as a whole around the z-axis, and $R_{m\phi}$ is the resistance of the mechanical system associated with the sensing axis as reflected into the pick-up secondary circuit. For maximum power transfer into the electric circuit, the resistance facing the pickup should be equal to $R_{m\phi}$ if it is assumed that the winding resistance of the pickup secondary is negligible and that the effects of inductances have been removed by tuning.

The tines of the fork are assumed to have a mass m; this mass is considered to be the active mass of the instrument. The amplitude of vibration of the tines is limited by the maximum strain ζ_{mv} that can be permitted in the material used to construct them. In order to account for the speed of response of the instrument, a time constant T is defined as the rate of change of the phase of the modulation envelope of the pickup output signal with respect to frequency for sinusoidal oscillation of the instrument about the z-axis. Under these conditions the sensitivity becomes:-

* More correctly MATHIEU FUNCTION see McLACHLAN. N.W. Theory and Application of Mathieu Functions. Oxford Univ. Press. (1951).

$$S_v = \frac{0.441 r^{1/2} m^{1/2} \left(\frac{E}{P}\right)^{1/2} \zeta_{mv}}{\alpha\phi^{1/2}} \quad (2)$$

Here E is Young's modulus and p is the density of the active material. $(E/P)^{1/2}$ is the velocity of sound for extensional waves. The quantity $\alpha\phi$ is the ratio of the total moment of inertia around the sensing axis to that represented by the tines alone when they are in their neutral positions.

For the rotating wheel rate gyro the wheel is assumed to be a massive rim which is thin compared with its radius R. It is assumed that there is negligible mass in the web of the wheel. Thus the active mass m is the mass of the rim. The speed of response of the instrument is accounted for by a time constant r which is the rate of change of phase of the gimbal tilt angle θ with respect to frequency for sinusoidal variation of the angular rate about the z-axis. r is the negative slope of the phase versus frequency characteristic in the vicinity of zero frequency. It is also assumed that the maximum tilt angles of the gimbal which are encountered are small. Using these assumptions it is possible to arrive at the following expression for sensitivity. α_y is the ratio of the total moment of inertia associated with the gimbal axis to the moment of inertia of the wheel. This inertia ratio will customarily range from three to five. ζ is the damping ratio which characterizes the transient response of the gimbal deflection . This damping ratio normally ranges from 1 down to 3/10. The square root of E/p represents the velocity of sound in the rim material of the wheel. mr represents the maximum strain that is permitted in the rim material. The dimensions of the sensitivity Sr are the same as before and the numerical constant carries no dimensions.

The inductive reactance $X_{m\phi}$ for the rotating wheel instrument is substantially analogous to the resistance $R_{m\phi}$.

In this arrangement the sensitivity becomes:-

$$S_r \triangleq \frac{(E_2/\Omega)}{\sqrt{X_{m\phi}}} \quad (3)$$

where

$$S_r = \frac{0.707 r m^{1/2} \left(\frac{E}{P}\right)^{3/4} \zeta_{ms}^{3/4}}{\zeta_y^{1/2} R^{1/2}} \quad (4)$$

Finally the sensitivity of vibratory gyroscopes to acceleration is the subject of a paper by STRATTON. A. and HUNT.G.W. (1963).

We should remind ourselves of an observation by FIRDLENDER & KOZLOV. (1961) that the foremost difficulty in the tuning fork gyroscope is the measurement of comparatively small angles of twist. For, example, when measuring angular velocities ranging from 0.1 to 50.0 degrees per second at K_{tf} = 0.003 deg/deg/sec, the angles of turn requiring measurement range between 0.003° and 0.15°. This can be done only by means of a capacitor pick-off that requires an amplifier having a high amplification; where K_{tf} is the sensitivity defined as the ratio of the amplitude of the angle of torsional oscillations in the turning fork base to the measured angular velocity.

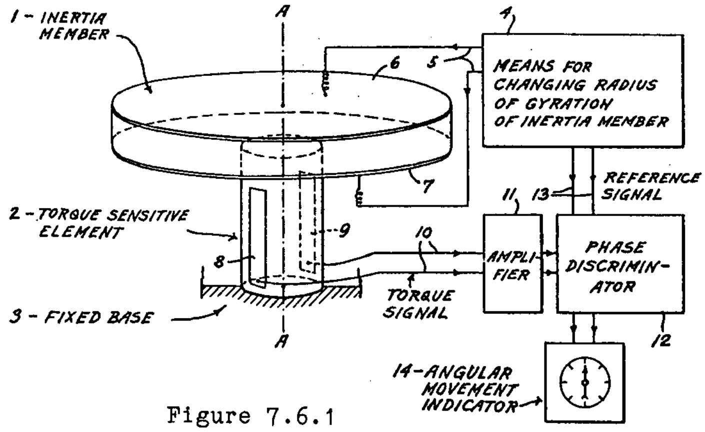


Figure 7.6.1

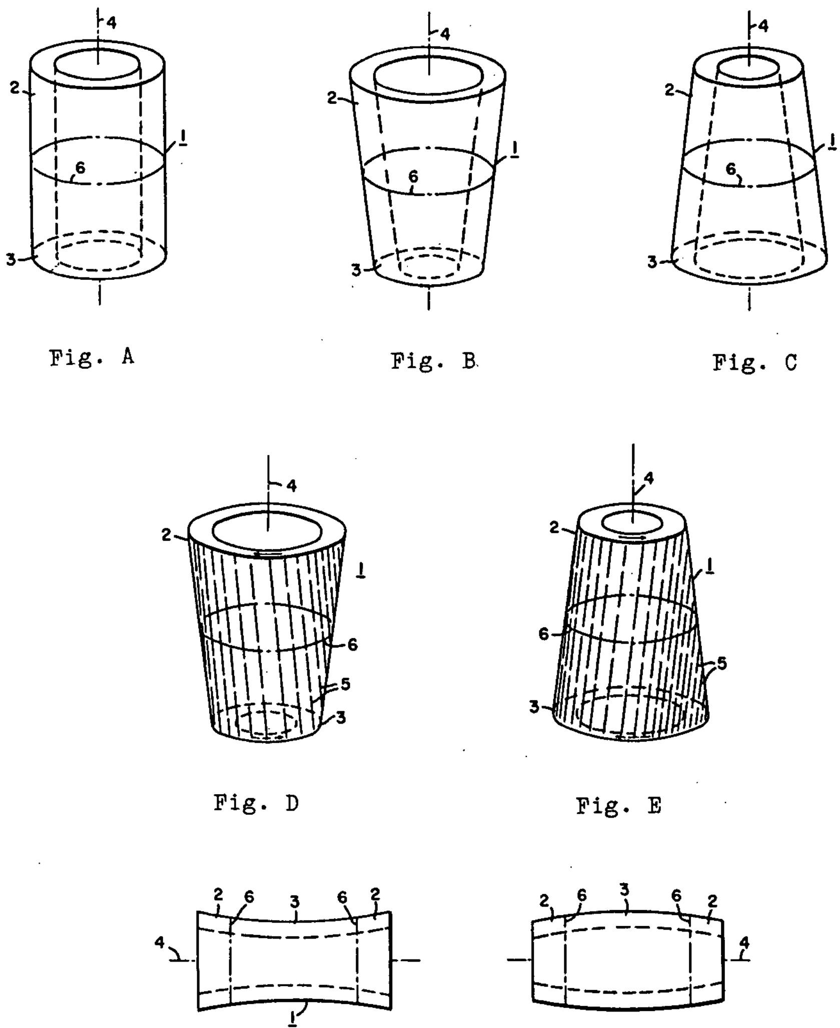


Fig. A

Fig. B

Fig. C

Fig. D

Fig. E

Fig. F

Fig. G

Figure 7.6.2

7.6 CRYSTAL GYROSCOPE

WILEY. C.A. US 2683247 (1954) discloses a space reference device in which a right cylindrical disc like inertia member (See Fig 7.6.1) is made from a piezo-electric material such as barium-titanate mounted on a torque sensitive element in the shape of a shaft of a piezo-electric material such as ammonium di-hydrogen phosphate crystal. Driver means are provided for rapidly changing at high frequency the radius of gyration and the moment of inertia of the said inertia member. When driven by a one hundred kilo-cycle exciter WILEY calls his device a crystal gyro.

For a general discussion of piezo-electric crystal having electrodes covering selected areas and the earlier art, reference may be made to SYKES. R.A. US 2223537 (1940).

JONES. C.H. THOMPSON. J.H. and DOUGLAS. G.R. US 3182512 (1965) show an angular velocity measuring device comprising a vibrating right cyclinder of circular annular section made from a material having piezo-electric properties, such as barium titanate, lead zirconium titanate. The cylinder is vibrated in the radial mode and its various states are well shown in Fig 7.6.2, A,B,C,D,E,F,G, where Fig 7.6.2A is a three dimensional view illustrating the mass distribution of a hollow right cylinder of the self-vibrated mass and signal producing element in its relaxed non-vibrating, non-excited state.

Fig 7.6.2B and 7.6.2C are similar views to A showing with exaggeration the mass distribution of such element during successive half-cycles of vibration in the driven mode.

Fig 7.6.2D and 7.6.2E are similar views to A, B, and C showing with exaggeration the strain reaction in the element during the successive half cycles of the driven mode while the element is being turned about its input axis. (4).

Fig 7.6.2F and 7.6.2G are views in outline of another self-vibrated-mass and signal producing element with exaggeration, the mass distribution while driven at a frequency corresponding to a mode of higher order than that shown in Fig 7.6.2A to 7.6.2E. (The nodal plane is shown at (6)). A full discussion of the problems of solid state vibrating gyroscopes is given by WESTINGHOUSE ELECTRIC CORPN. (1962).

SIMMONS. A.L. BUCKLEY. J.J. US 3408872 (1968) show a vibrating gyroscope including a cylinder of piezo-electric ferromagnetic or magnetostrictive material with masses attached symmetrically about the ends and a voltage for longitudinally polarizing and driving the cylinder in the circumferential mode. The cylinder is rotatably mounted at the mid-point of its longitudinal axis so that as a force is applied the cylinder moves about its mounting point, and a signal is produced indicative of such movement.

8. INTERFEROMETRIC GYROSCOPE

8.1 LASER GYROSCOPE

The laser* gyroscope is strictly a ring laser using techniques explored originally by SAGNAC. G. (1913) and by MICHELSON. A.A. (1904).

The Michelson-Sagnac effect with its experimental and theoretical ramifications can be traced back to BOSCOVICH. R.J. (1766) and the history is ably presented by ZERNIKE. F. (1947) and by POST. E.J. (1967). The Sagnac interferometer is shown at Fig 8.1.1. The light beam from source A is split at B into a beam circulating the loop in a clockwise direction $BB_1B_2B_3B$ and a beam circulating in the same loop in a counter-clockwise direction $BB_3B_2B_1B$. The two beams are reunited at B and interference fringes are observed at C when the whole interferometer with light source and fringe detector is set into rotation with an angular rate of Ω radians/second, a fringe shift ΔZ with respect to the fringe position for the stationary interferometer is observed, which is given by the formula

$$\Delta Z = 4\Omega.A/ \lambda_o c$$

in which A is the area enclosed by the light path. The vacuum wavelength is λ_o and the free-space velocity of light is c. The scalar product $\Omega.A$ denotes that ΔZ is proportional to the cosine of the angle between the axis of rotation and the normal to the optical circuit.

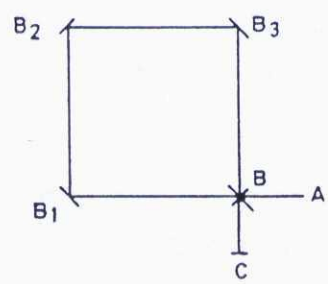


Figure 8.1.1

POST explains that the fringe shift given by the formula above can be doubled by making a comparison between the fringe positions obtained on rotating in opposite directions. Sagnac thus obtained, for the wavelength of indigo mercury light and a loop area A of 866 cm^2 and a fringe shift of 0.07 fringe for a rate of rotation of two revolutions per second. The light source and the fringe shift detection occur on the rotating disc. Sagnac also established that the effect does not defend on the shape of the loop or the centre of rotation. The fringe shift detectability at that time was about 0.01 of a fringe and the precision of Sagnac's experiment very close to its limits of accuracy. It is the invention of the laser that has enabled a working gyroscope of great precision to be made using such an interferometer. A self oscillating version of the Sagnac ring was proposed by ROSENTHAL. A.H. (1962) and this was advanced into a practical device by MACEK & DAVIS (1963). We should not, however, forget to mention a short but perspicacious note by HEER. C.V. (1961) which has an important place in the history of the laser gyroscope. HEER was working for Ohio State University and supported by the National Science Foundation; the note is given below in extenso.

* The laser is a development of the maser, both words being acronyms, the first being the initial letters of light amplification by stimulated emission of radiation, and the second being its equivalent in the microwave part of the electromagnetic spectrum. The first lasers were gas discharge, see LAMB. W.E. (1964). The stimulated optical radiation in ruby being due to MAIMAN. T.H. (1960) of Hughes Research Laboratory at Malibu.

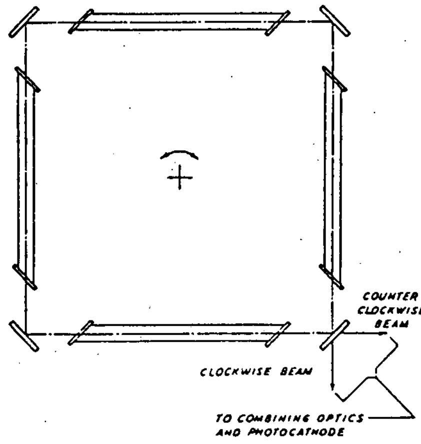


Figure 8.1.2

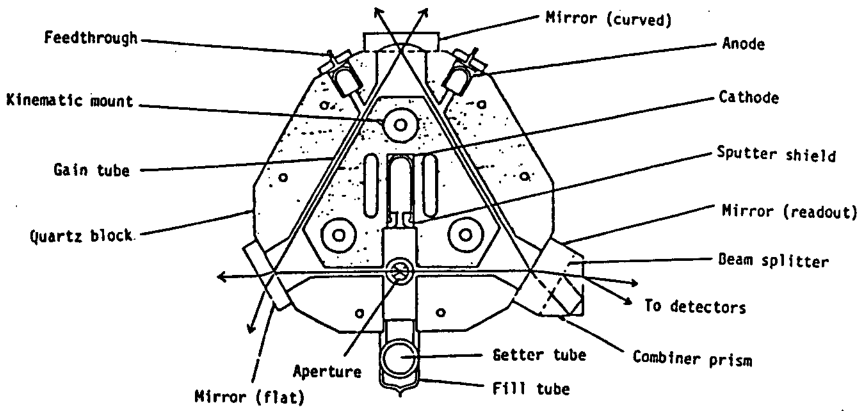


Figure 8.1.3

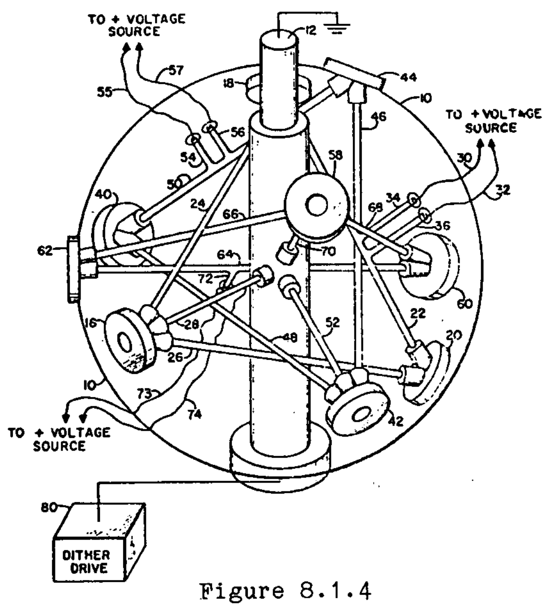


Figure 8.1.4

'Experimental phenomena in a nonpermanent gravitational field, i.e., the field as measured on a platform rotating with angular velocity and similar to that discussed by Michelson* and Sagnac** for light, is considered for resonant cavities for electromagnetic waves and for matter waves. A resonant cavity such as a toroid that is not simply connected has the degeneracy removed by the rotation, and the resonant angular frequency of the wave travelling in the sense of rotation differs by the angular frequency of rotation from the wave counter to the rotation. A "ringing" cavity yields beats at the frequency of rotation. Use of the same metric with the Klein-Gordon equation as with Maxwell's equations for the particle equivalent of the Michelson experiment yields a phase difference between paths BCDB and BDCB on a rotating platform of $\Delta = 8\pi \ A\,(\Omega/c)(v/c) \approx 8\pi \ A\Omega\,/\lambda_C c$. $\lambda c = (h/m_0c)$ is the Compton wavelength and A the area enclosed by the path BCDB. This is similar to the equation used by Michelson with the wavelength of light replaced by the Compton wavelength.'

The idea of using the highly monochromatic light source of a neon-helium gas discharge optical-maser (laser) giving a relative line width of down to about 10^{-14} in the stimulated emission of the 11530 A neon line with a frequency bandwidth of less than 2cps is as stated above to the credit of ROSENTHAL. A.H. (1962). In his work he pays tribute to the Michelson-Sagnac type of interferometer and shows how it may be made self-oscillating. This form of ring laser was later developed, as stated above, by MACEK. W.M. and DAVIS. D.T.M. Jr. (1963) of the Sperry Gyroscope Company using the 1.153μ line of a helium neon gas travelling wave ring laser using a metre square ring resonator. (See Fig 8.1.2). In a short classical paper they explain that four gas tubes were used with Brewster angle windows and four external corner mirrors. Since the clockwise oscillation is obtained by directing the waves around the complete optical ring instead of retroreflecting them, the clockwise and counterclockwise modes are independent. These modes were extracted through a corner mirror, rendered collinear by external combining optics, and mixed on a photocathode. Beat notes showing frequency splitting from 1 kc to 40 kc were observed as the rotation rate of the entire systems was varied. The frequency splitting arises from the removal of the mode degeneracy existing for the two oppositely traveling waves, due to the differential cavity path-length change produced by the rotation. The resulting mode splitting is therefore proportional in frequency to the angular velocity. The limits of rotation detection are currently set by equipment and techniques and are not the physical limits of the rotation rate sensing effect.

An effort is currently in progress for sensing the earth's rotation rate (9.6 deg/h at the latitude of Great Neck). Consideration is also being given to possibilities for detecting the Earth orbital rate around the Sun.

The present experiment improves, by orders of magnitude, the sensitivity of the classical experiments of Michelson-Gale and Sagnac where the effects of rotation on the propagation of light were studied by a modified two-beam interferometer arrangement. The two beams from the beam splitter were made to travel around a closed loop in opposite directions but along identical paths, and then combined to produce an interference fringe pattern. The fringe pattern shift, resulting from the differential path-length changes for the clockwise and counter-clockwise beams caused by rotation of the system, was proportional to the rotation rate. The results were severely limited by fringe shift detection capabilities, thereby necessitating either extremely large optical circuits or high rotation rates. The laser frequencies are determined by the condition that the cavity optical path length must equal an integral number of wavelengths. For the stationary system the paths p are equal; therefore, wavelengths are equal, and modes are degenerate in frequency. For the rotating system the paths are unequal since the wavefronts must now travel $p \pm \Delta p$ to close on themselves; therefore, the wavelengths are unequal, and the mode degeneracy in frequency is lifted. Basically, the improved sensitivity of the present experiment arises

* A. Michelson and H. Gate, Astrophys. J. 61, (1925) p140
** G. Sagnac, J. phys 4, (1914) p177

because optical heterodyning techniques and laser coherence allow the direct measurement of frequency differences to the order of $1:10^{12}$. The path-length effect was shown by Langevin to be of first order and therefore may be treated either classically or relativistically. The cavity pathlength change Δp is given by

$$p = \pm 2\ \omega A/c,$$

where ω is the rotation rate, A the projected area of the optical ring, and c the velocity of light. Also, since the fractional frequency shift $\Delta v/_{v}$ equals the fractional path-length changes $\Delta p/_{p}$, the mode split or frequency difference is given by

$$\Delta v = 4\omega\ A/p\lambda$$

or by

$$\Delta v = \omega l\ /\ \lambda$$

for a square optical circuit of side length l.

Since the effect, as shown by LANGEVIN. P. (1921), is first order in v/c, classical theory will give the correct answer to a first order as shown by DITCHBURN. R.W. (1976), and strictly speaking the special theory* of relativity is not applicable, since the light must be considered on a rotating frame. The only rigorous correct theory is the general theory of relativity. The classical theory shows that if the components of the velocity of one of the mirrors in the direction of the propagation of light is $\underline{v}$, the velocity of light in one direction is (c+v) and in the other (c-v). If the length of the path for each beam is $\underline{S}$ when the mirrors are stationary there is a difference in transit time of

$$\frac{S}{c-v} - \frac{S}{c+v}$$

when the system is rotating. When $\underline{v}$ is small compared with $\underline{c}$, the corresponding path difference is $\Delta S = \frac{2vs}{c}$ and the observed displacement of fringes agrees.

The beat frequency of a ring laser, unlike the fringe shift of the ring interferometer depends on the properties of the co-moving medium traversed by the beams and KHROMYKH. A.M. (1966) has pointed out that dispersion can affect the observed beat frequency. He has derived an expression for the frequency shift of oppositely moving waves in a ring laser in a non-inertial reference system, by taking into account the motion of the dielectric medium filling the resonator. He has also corrected the equations derived in an earlier analysis by HEER. C.V. (1964). The layman is much indebted to three authors, MACEK. W.M. & McCARTNEY. E.J. (1966) and KILLPATRICK. J.E. (1967), all intimately involved with the early researches into the laser gyroscope, for excellent early dissertations on the device.

The configuration of the Honeywell Inc, solid block laser gyroscope from the article by KILLPATRICK is shown at Fig 8.1.3. A three axis laser gyroscope in solid quartz due to <u>LECHEVALIER. R.R.</u> US 3503688 (1970) is shown at Fig 8.1.4. This laser gyroscope is also a product of Honeywell Inc and the most advanced known to ARONOWITZ. F in his detailed and erudite account of the laser gyroscope in the reference work by MONTE ROSS (1971).

* This is comforting since the special theory is open to severe criticism on several grounds. See <u>inter alia</u> DINGLE. H. <u>Science at the Crossroads</u> (1972). ESSEN.L. <u>Einstein's Special Theory of Relativity.</u> Proceedings of the Royal Institution of Great Britain. <u>45</u> (1972) p141-160.

Information concerning the rotation of the laser gyroscope is obtained by monitoring the oppositely directed waves. Figure 8.1.5 (page 12) shows one method of combining the beams according to KILLPATRICK. J. US 3373650 (1968). The beams are made substantially co/linear by a prism to form a fringe pattern. According to ARONOWITZ the fringes are a measure of the instantaneous phase difference between the oppositely directed beams. For the case where the intensities are matched and the beams are nearly colinear (angular divergence of E), the fringe pattern is given by

$$I = I_o \left[1 + \cos\left(\frac{2\pi EX}{\lambda} + \Delta\omega t + \phi\right)\right]$$

where $\Delta\omega$ is the angular beat frequency and ϕ is some arbitrary angle. Thus when the laser is not rotating $\Delta\omega=0$, and the fringe pattern is stationary. When the laser is rotated, the fringe pattern moves at the beat frequency rate. The fringe spacing is given by $\frac{\lambda}{E}$. For a parallel substrate E is given by $E = 2n\theta$ where n is the index of refraction of the prism and ϕ is the deviation of the prism angle from a right angle.

For a prism angle deviation of 15 arc sec and for the 0.633 µm He-Ne transition the fringe spacing is 3mm. This figure is said by HUTCHINGS. T.J. US 4152072 (1979) to be typical.

For a detailed discussion of a high resolution laser gyroscope capable of measuring down to 0.02 arc second of motion one may turn with profit to a discussion by MATTHEWS. J.B. GNESES. M.I. and BERG. D.S. (1978). The limits of sensitivity were explored initially by BRUNET. H. (1965) and taken up by ROZANOV. N.N. (1970) and LANDA. P.S. (1970) (1971).

ARONOWITZ. F. (1971) gives three types of errors that are critical in the design of the laser gyroscope, and since the later literature bears this out it is of value to itemize them; null-shift; lock-in, and mode pulling.

A null-shift error arises when the cavity is anisotropic with respect to radiation travelling in the two directions. This results in the optical paths being different for radiation travelling in both directions and hence the two waves oscillate at different frequencies. Unless the gyroscope is properly designed null shift errors can be very much greater than the input rate.

Lock-in is said to be common to all coupled oscillator systems. (see VANDER POL (1934))+. In the laser gyro there is a mutual coupling between the waves such that at low input rates both waves lock to a common frequency and in this region the gyro is not responsive to input rotations and its ability to act as a rotation sensor vitiated.

Mode-pulling effects are explained by LAMB. W.E. (1964), they arise from dispersion within the medium that is the source of the laser radiation.

Null-shift is taken up by ARONOWITZ. F (1972) and he has been able to show that it is possible to operate a 0.633 um He-Ne laser gyro filled with a single neon isotope to obtain a greater amount of detail from the data and a greater ease of analysis.

ROLAND. J.J. and LAMARRE. J.M. (1973) discuss periodic Faraday bias and lock-in phenomena in the laser gyroscope. Positive scale-factor correction is taken by ARONOWITZ. F. and LIM. W.L. (1977). Power dependant correction to the scale factor is taken by ARONOWITZ. F. KILLPATRICK. J.E. and CALLAGAN. S.P. (1974)

\+ See VANDER.POL.B. The Non-Linear theory of Electric Oscillations. Proc IRE. 22 (1934) p1051.

Multilayer light scattering is taken by SCOTT. M.L. and ELSON. J.M. (1978). Dispersion and gas flow effects are taken by ARONOWITZ. F and LIM. W.L. (1979). Rate biasing by means of a magneto-optic bias mirror using the transverse Kerr effect is taken by McCLURE. R.E. and VAHER. E. (1978) and the body-dithered laser gyroscope is reviewed experimentally and theoretically by HUTCHINGS. T.J. and STJERN. D.C. (1978). We have referred above to the work of LAMB and we should point out that a theoretical model for the behaviour of an optical-maser in which the electromagnetic field is treated classically and the active medium is made up of thermally moving atoms that acquire non-linear electric dipole moments under the action of the field according to the laws of quantum mechanics is very ably presented by him, see LAMB. W.E. Jr. (1964).

It is to the United States specifications that we turn now to close this account of the laser gyroscope.

We point out that an interesting advanced interferometer of the Michelson type with little or no insight into its possible use as a rotation sensor is given by SCOTT. L.B. US 2841049 (1958). A bias system for a laser gyroscope in which the gyroscope is electrically or mechanically oscillated (dithered) is advanced by KILLPATRICK. J.E. US 3373650 (1968) so that the gyroscope is effectively above the threshold rate the majority of the time and the arrangement includes an optical system to compensate for the bias over short time intervals. Over longer time intervals the accumulated bias is relatively negligible. FRIEDLAND. B. US 4132482 (1979) takes the arrangement of KILLPATRICK above, and uses a dynamic feed-back system between the output and the ring laser gyro and the dither rate input.

LECHEVALIER. R.R. US 3503688 (1970) discloses a multiple axis laser angular-rate sensor using a spherical block of quartz (See Fig 8.1.4) the passages being completed by mirrors secured to the block at the ends of the passages to reflect radiation from one passage to another. Angular oscillation applied to the block simultaneously 'dithers' the several passages to prevent lock-in. The laser gyroscope makes uses of work by PODGORSKI. T.J. (See US. 3390606) and has been practically advanced by HONEYWELL. INC. (See ARONOWITZ. F. (1971) p. 195).

BONFILS. G. EP 003086 (1979) for S.F.E.N.A. proposes a process for the attenuation of errors of linearity in a laser gyroscope in ring form having N mirrors so arranged that the laser beams travel in the opposite sense and define two identical polygons in which one (or more) of the mirrors is oscillated, thereby reducing the dither energy expended in the device, a proposal advanced in part by earlier workers such as COCCOLI. J.D. US 3533014 (1970) PODGORSKI. T.J. US 3581227 (1971) and BJORKHOLM. J.E. US 3786368 (1974).

SCHUTT. S.G. EP 0021419 (1981) shows a pneumatically dithered laser gyroscope in which the pressure differential and oscillatory gas flow along the bore of the gain section of the laser induces an oscillatory bias in the gyroscope output; a means previously proposed by TURNER. E.H. US 3466121 (1969) and PODGORSKI. T.J. US 3744908 (1973). TURNER is concerned with non-reciprocal optical devices in which laser light is transmitted through a moving medium, the velocity of which affects the velocity of propagation of the light in accordance with the Fresnel drag effect. TURNER's dissertation is one of considerable importance making reference to the earlier work of FOX. A.G. et al. (1955) and to the important observation on Fresnel drag with the ring laser by MACEK et.al. (1964).

PODORSKI uses a linear induction motor positioned adjacent one leg of the laser gyroscope so as to generate a moving magnetic field in the laser plasma. Such a field moves or pumps the plasma gas so as to present an apparent change in the index of refraction of the gas and an apparent change in the path length to the two oppositely travelling beams. Many proposals have been advanced to prevent lock-in and the reader is directed inter alia to CUTLER. L.S. US 3714607 (1973) GIEVERS. J.G. US 3841758 (1974) and WILBER S.A. US 3846025 (1974).

It has been shown by DORSCHNER. T.A. SMITH. I.W. and STATZ.H. (1978) in an introductory review of the art that conventional laser gyro approaches are based on two counter-rotating optical resonator modes. In principle, these modes split in frequency when the gyro rotates about its sensitive axis. In practice, there is always some scattering of light from one mode into the other which causes the well known lock-in phenomena common to all oscillators. As a result of lock-in the frequencies do not split for low rotation rates, thereby giving a dead-band in gyro output. Lock-in is typically broken only when the rotation rate is larger than 20-2000 degrees per hour. Furthermore, even for rotation rates above lock-in threshold, the output remains a non-linear function of rotation rate. Most approaches try to overcome this problem by applying a bias to the gyro output frequency either by rotating the gyro or by inserting into the cavity a non-reciprocal optical element such as a Faraday cell. To avoid stringent stability requirements, the bias is usually intentionally reversed exactly half the time by either mechanical or electro-optical (Faraday) dithering. The drawback to the dither scheme lies in the fact that the gyro passes through the lock-in region twice per dither cycle. Upon leaving lock-in, the gyro has lost its memory of phase and thus an error of a fractional count accumulates per cycle. These errors add randomly, giving a "random-walk" cumulative output angular error that increases as the square root of elasped time. Such angular random walk, a phenomenon not observed in spinning mass gyroscopes, appears to be an unavoidable source of error in dithered ring laser gyros. This error source is particularly serious for short measurement times, where the uncertainty corresponds to a large error in apparent rotation rate.

The four-frequency, or multi-oscillator ring laser gyroscope was invented by DOYLE. B. US 3468608 (1969) who uses a first and second pair of counter-rotating beams co-acting with a transparent spinning disc of fused quartz to introduce a Fresnel drag effect and this was advanced by ANDRINGA. K. US 3741657 (1973), US 3854819 (1974), US 3937578 (1976) as a means of circumventing the lock-in problem. This approach may be described as two independent laser gyros operating in a single stable resonator cavity, sharing a common optical path, but statically biased in opposite senses by the same passive bias element. In the differential output of these two gyros, the bias then cancels, while any rotation-generated signals add, thereby avoiding the usual problems due to drifts in the bias and giving a sensitivity twice that of a single two-frequency gyro. Because the bias need not be dithered, the gyro never passes through lock-in. Hence there are no dither-induced errors to limit instrument performance. For this reason the four-frequency gyro is intrinsically a low-noise instrument, and is well suited for applications requiring rapid updating of position and/or high resolution.

A differential laser gyro systems due to YNTEMA. G.B. et al US 3862803 (1975) employs both polarization anisotropy and directional anisotropy in the optical path so as to provide two laser gyros operating in the same cavity with cavity modes of different frequencies in the form of mutually opposite polarization in each of the clockwise and counter clockwise directions. Two different species of lasing medium may be utilized so as to reduce interference and source depletion in supplying the four distinct effective frequencies across a wide range of operating conditions. Parameters are adjusted to maintain substantially the same change in oscillation frequency for variations in cavity tuning, and other parameters for each of the four frequencies. The differential laser gyro of YNTEMA etal has the advantage that bias switching asymmetries are obviated since only a single polarity of bias is utilized, and under ideal conditions any extraneous magnetic effect (such as the Earth's magnetic field) operates equally on both polarizations, and is therefore substantially cancelled.

Other gyros may not use bias polarity reversal and cancellation, but rather simply rely on subtraction of the known nominal bias magnitude from the result obtained. As is known, the gain versus frequency characteristic of the laser gain medium always has associated therewith an index of refraction, or dispersion characteristic, which provides a slightly different optical length through the laser gain medium for waves of different optical frequencies; e.g. for clockwise vs counterclockwise waves when they are biased to different frequencies as above. The effect is opposite in dependence upon the polarity of the bias (that is, whether the clockwise or counterclockwise wave is operating at a higher or lower frequency, and therefore at a higher or lower point on the dispersion characteristic). In gyros where bias is reversed for bias cancellation purposes, this difference in the optical length is cancelled along with the applied bias. However, in gyros which do not employ bias reversal, and in the differential laser gyro wherein bias cancellation is automatically effected by the combination of outputs from opposite polarizations, the variations which can occur in the effective optical length for the counter-rotating waves are not cancelled. These variations result from the fact that, due to minor shifts in the optical length of the total optical cavity of the laser gyro, the absolute frequency of both counter-rotating waves may increase together, or decrease together, and since the dispersion characteristic is nonlinear characteristic, this can cause the additional variation of optical length within the gain medium to vary as between the two waves, thereby providing the same effect as the change in the rotational rate of the gyro, which introduces significant errors in the rate sensed. Similar errors can also result from fluctuations or perturbations of the gain medium itself.

In a dispersion compensated laser gyro according to FERRAR. C.M. US 3973851 (1976) these difficulties are overcome by an axial magnetic field applied to the laser gain medium of a laser gyro which provides, through the Zeeman Effect, a pair of gain versus frequency profiles relating to respective counter-rotating waves (clockwise and counterclockwise) with their frequencies of maximum gain displaced from one another by an amount which is proportional to the applied magnetic field, and which is substantially equal to the frequency difference caused by an applied directional bias, such that the separation between the frequency of each wave and the frequency of maximum gain of the related gain profile is the same for both waves. This causes the counter-rotating waves to operate on points of equal phase (or index of refraction) on the corresponding dispersion characteristic curve associated with the related gain versus frequency profile. Under these conditions, even if the effective optical length of the gain medium changes in response, for example, to externally induced perturbations of the medium or of the resonant laser frequency, the change is essentially the same for each of the counter-rotating waves, and is thereby substantially cancelled when the wave frequencies are differenced to obtain a gyro output. For example, when cavity length drift causes increases or decreases in both frequencies, thereby causing the waves to intersect different portions of the related dispersion characteristic curve, which are typically not linear or stable, the pair of dispersion characteristics will have similar nonlinearity and instability, and errors resulting therefrom will be substantially cancelled.

DORSCHNER et al (1978) explain that the four frequency differential requires a minimum of solid matter in the optical path and to remove devices such as Faraday rotators they propose a magnetic mirror from which reflections give rise to a non-reciprocal polarisation rotation by virtue of the magneto-kerr effect (See KERR. J. (1877)). Unfortunately most magnetic materials are strongly absorbing at optical wavelengths, and to overcome this drawback they produce a magnetic substrate mirror using manganese bismuth films.

SMITH. I.W. Jr and DORSCHNER. T.A. US 4110045 (1978) propose a four frequency laser gyro in which electromagnetic field distribution rotating means includes a catoptric arrangement that reduces the loss scatter and linear birefringence associated with a ring resonator when included in such a gyroscope.

Four mode laser gyroscopes are proposed by LITTON SYSTEMS INC. Fr 2430598 (1980) in which reflecting mirror sunfaces are mounted on chamfered surfaces of a quartz block; and by BRESMAN. J.M. and COOK. H.J. US 4198163 (1980) in which two colinear beams each including all four modes are taken to identical assemblies of polarization conversion and discriminatory means including heterodyning detectors. SANDERS. V.E. US 4123162 (1978) advances the work of ANDRINGA. K. US 3937578 (1967) and discloses a four mode ring laser gyroscope in which its four modes are at different frequencies combined and applied to a single photodiode. The output from the photodiode is applied to a nonlinear device and the resultant sum and difference frequencies are applied to (1) a laser cavity length control circuit (2) a rotation rate detection circuit, and (3) a rotation direction determination circuit. The cavity length control circuit operates by the determination of full modulation of one of the beat frequencies by a second. The direction of determination circuit utilizes the phase of an a.c. dithering power supply that varies the anode to cathode plasma current of the laser gyroscope.

DORSMAN. A.K. US 4099876 (1978) and SHUTT. S.G. US 4113387 (1978) disclose laser optics adjustment means comprising a mirror, within the laser optics of the gyro, that is mounted upon a three terminal duo-mode bimorph device for duo-mode modulation of the said optics.

SMITH. I.W. and DORSCHNER. T.A. for RAYTHEON COMPANY GB 1550578 (1979) point to the highly successful laser gyroscope of ANDRINGA. K. US 3741657 (1973), US 3854819 (1974) using circular polarization for each of its four beams and it is essential to extract a portion of each beam circulating within the laser cavity to produce two output signals each one of which represents the difference in frequency between wave pairs having the same sense of circular polarization. In order to accomplish this purpose it is desirable at some point within the output structure to combine these two beams in such a manner as to produce two new beams, each including waves having the same sense of polarization. this is an onerous task, and the known structures are often bulky and optically misaligned and they do not separate successfully, the polarization states resulting in spurious crosstalk at the detector output.

Three output optics structures are proposed to overcome the above disadvantages and these are shown in Figs 8.1.6A, B and C. The four electromagnetic waves f_1-f_4 are such that f_1 and f_4 circulate in the clockwise direction and f_2f_3 in the counter clockwise direction. At various points in the structure L and R signify Left and Right hand circular polarization; H and V. horizontal and vertical linear polariztion. Polarization states labelled + and - signify linear polarization oriented at plus and minus some angle to the horizontal, typically 30 to 45 degrees of arc.

For relevant earlier art see:

MACEK. W.M. US 3382760 (1968) PODGORSKI. T.J. US 3390606 (1968) and ANDRINGA. K. US 4006989 (1977).

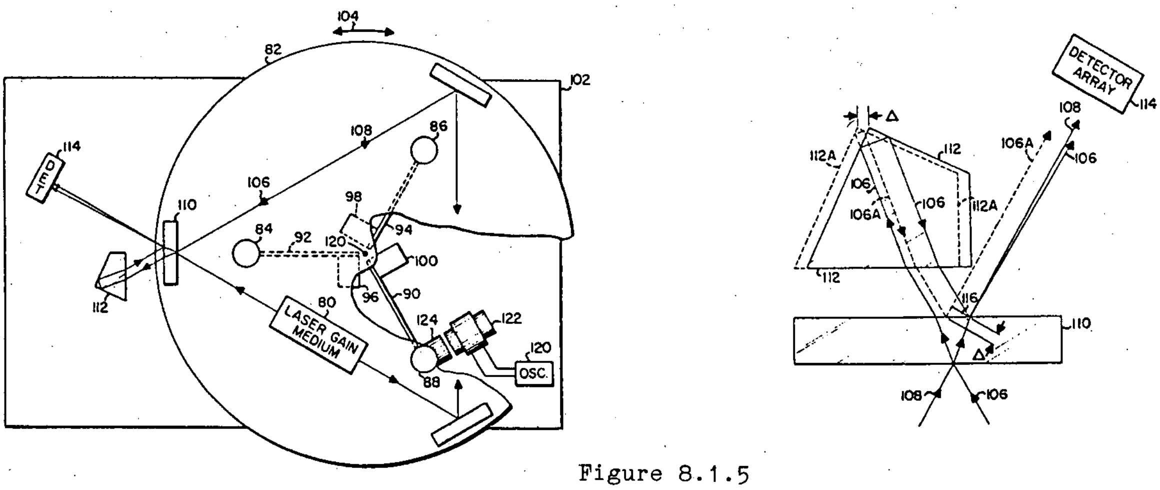


Figure 8.1.5

A: $(f_2Lf_3R)_{ccw}(f_1Lf_4R)_{cw}$
B: f_2Lf_3R
C: f_2Vf_3H
D: $f_2Lf_3Rf_1Lf_4R$
E: $f_2Rf_3Lf_1Rf_4L$
F: $f_2Hf_3Vf_1Hf_4V$
G: f_3Vf_4V
H: $(f_1Rf_4L)_{cw}(f_2Rf_3L)_{ccw}$
I: f_1Rf_4L
J: $f_1Hf_4Vf_2Hf_3V$
K: f_1Hf_3H

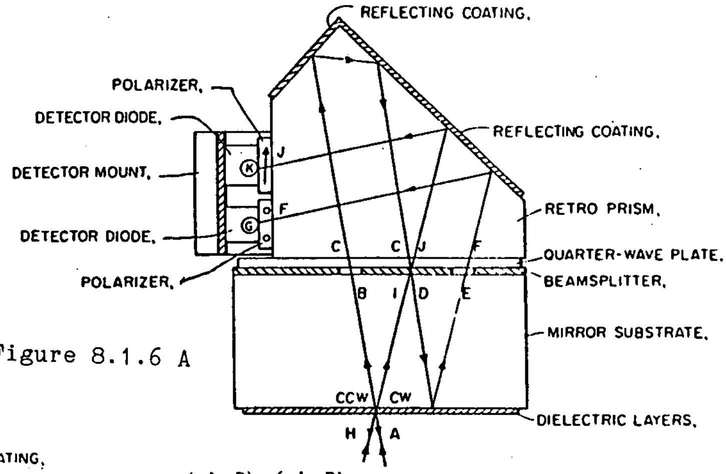
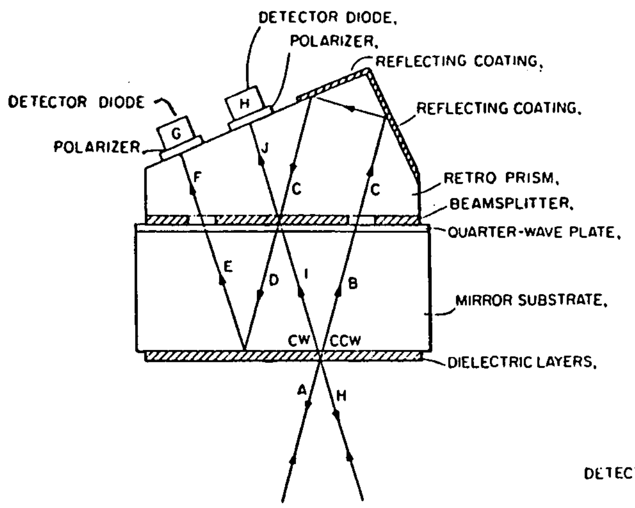
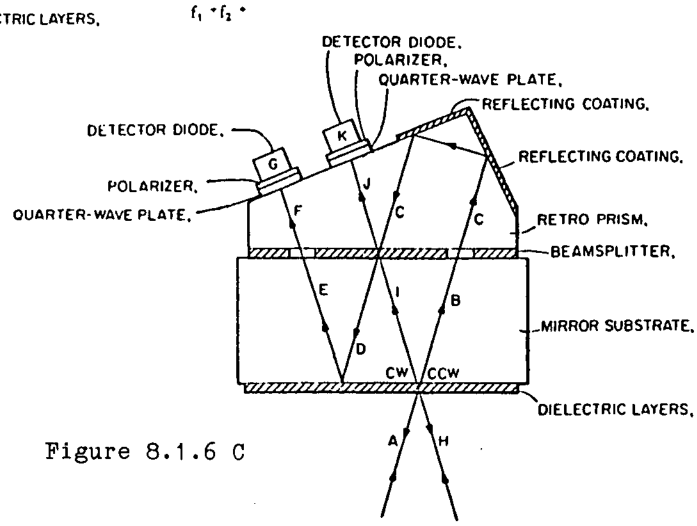


Figure 8.1.6 A



$(f_2Lf_3R)_{ccw}(f_1Lf_4R)_{cw}$
f_2Lf_3R
$f_2 - f_3 +$
$f_1Rf_2Rf_3Lf_4L$
$f_1Lf_2Lf_3Rf_4R$
$f_1 - f_2 - f_3 + f_4 +$
$f_3 + f_4 +$
$(f_1Rf_4L)_{cw}(f_2Rf_3L)_{ccw}$
f_1Rf_4L
$f_1 + f_2 + f_3 - f_4 -$
$f_1 + f_2 +$

Figure 8.1.6 B



A: $(f_2Lf_3R)_{ccw}(f_1Lf_4R)_{cw}$
B: f_2Lf_3R
C: f_1Lf_3R
D: $f_1Lf_4Rf_2Lf_3R$
E: $f_1Rf_4Lf_2Rf_3L$
F: $f_1Rf_4Lf_2Rf_3L$
G: f_3Hf_4H
H: $(f_1Rf_4L)_{cw}(f_2Rf_3L)_{ccw}$
I: f_1Rf_4L
J: $f_1Rf_4Lf_2Rf_3L$
K: f_1Vf_2V

Figure 8.1.6 C

8.2 MASER GYROSCOPE

The first masers emitted microwaves, later designs emitted in other parts of the electro-magnetic spectrum, and these were also called masers for a time, until laser came to be adopted as the general name for all such devices.

The etymological derivation and the original work were due to GORDON, J.P. ZEIGLER. H.J. and TOWNES. C.H. (1955). They explained that their device made use of a molecular beam in which molecules in the excited state of a microwave transition are selected. Interaction between these excited molecules and a microwave field produces additional radiation and hence amplification by stimulated emission. they called the apparatus utilizing this technique a 'maser' - which is an acronym for 'microwave amplification by stimulated emission of radiation'.

A proposal for a new type of solid state maser was put forward early by BLOEMBERGEN. N. (1956).

A microwave gyroscope said to provide advantages over the ring laser gyroscope is due to FELSENTHAL. H.D. Jr. US 3861220 (1975). The instrument has one wave guide loop with a modulated signal travelling in one direction only, use being made of the dispersion characteristic of the waveguide, which causes the propagation velocities of the carrier and the modulation to differ. This system has the advantage that phase locking at low rates of angular rotation is eliminated. The only system somewhat similar to the invention involves the ring laser. All known embodiments of the ring laser, including those with frequency biasing to avoid frequency locking, are subject to random fluctuations in the output frequency, due to several causes, including convection currents in the laser plasma and variations in the relative and total intensities of the two contra-rotating beams, i.e. winking. These fluctuations may have a non-zero average, resulting in a drift less than but analogous to the drift experienced with mechanical gyros. The winking results from the fact that both contra-rotating beams are generated by the lasing action of the same set of atoms, i.e. those having a given instantaneous thermal velocity component parallel to the light path, with all types of frequency biasing.

Several schemes have been used in the past, none of which have all the advantages of the invention. for example, prior schemes involving lasers have used: frequency biasing with Faraday rotators and birefringent material; frequency biasing by Zeeman effect frequency splitting; frequency biasing by mechanical vibration of a ring laser, i.e., sinusoidal plus random noise, with no net bias in either direction; frequency biasing by mechanically rotating transparent discs; frequency biasing by suppressed carrier double sideband frequency modulation of one beam but no modulation of the other beam, resulting in two frequencies in one direction and one frequency in the other direction; and, a three-frequency scheme, with no frequency biasing, where the frequencies are in three mutually perpendicular loops in one unit comprising a three-axis gyro.

In all prior cases, convection currents are present with resultant irreducible error.

None of the earlier schemes, due for example to WALLACE. A. US 3102953 (1963) DRESSLER. R. and HALBERSTAM. M. US 3218871 (1965) and SPELLER J.B. US 3395270 (1968), makes use of a dispersive propagating medium, and none treats the random fluctuations in the output frequency.

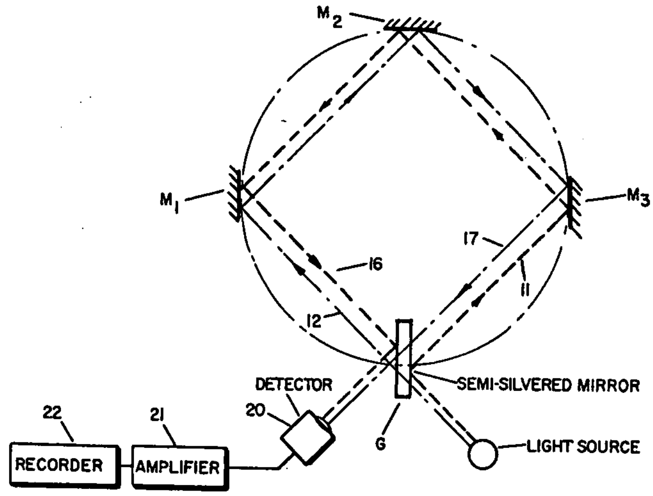


Figure 8.3.1 A

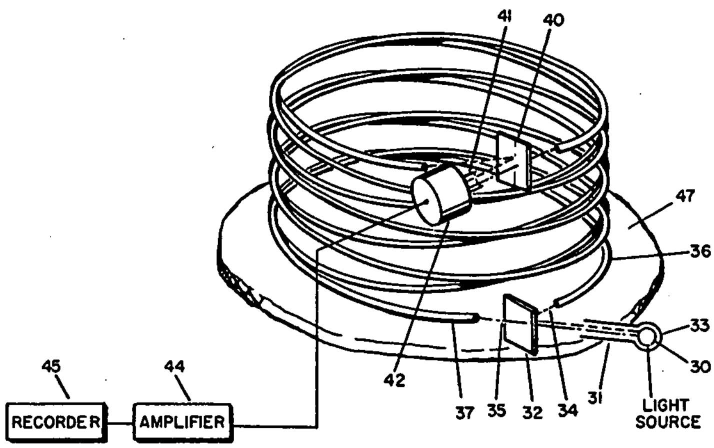


Figure 8.3.1 B

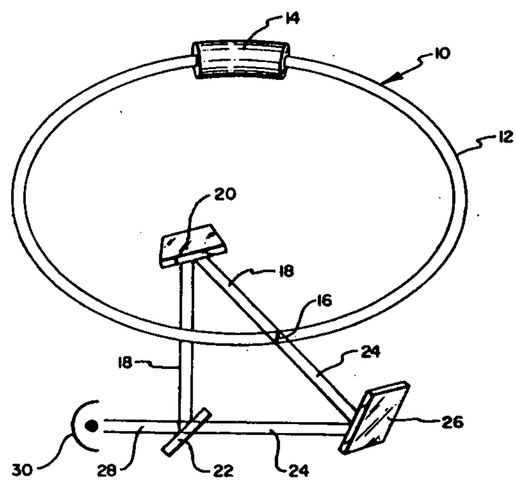


Figure 8.3.2

8.3 SAGNAC FIBRE-RING INTERFEROMETER GYROSCOPE

VALI and SHORTHILL (1976) point out that the sensitivity of a ring interferometer as a gyroscope can be increased considerably by making the counter rotating beams travel around an area many times, a technique not applicable in a ring laser. When the ring interferometer is rotating with an angular velocity ω the observed fringe shift is according to POST. E.J. (1967) and others

$$\Delta Z = \frac{4\,\omega NA}{\lambda_c}$$

where N is the number of round trips the counter rotating beams make around an area A, is the free space wavelength, and c is the free space velocity of light. Such an optical gyroscope would be undesirably large if the area A is made bigger than a few hundred centimetres squared, or if an air (or vacuum) path is used and N is made larger than one hundred.

An optical fibre waveguide would, according to VALI & SHORTHILL, keep the size quite small even for N at a value of 10^4 or larger. The enclosed area A is made circular, hence $A = \pi R^2$ where R is the radius of the circle. The fringe shift is

$$\Delta Z = \frac{4\omega N \pi R^2}{\lambda_c} = \frac{2\,\omega LR}{\lambda_c}$$

where L is the length of the fibre used. LEEB, SCHIFFER and SCHEITERER (1979) attribute the idea of a multi-turn coil in an optical fibre gyroscope to VALI and SHORTHILL (1976) but they disagree with them in regard to the fringe shift or scale factor depending upon the effective index of refraction and upon the dispersion of the fibre. Both groups of authors were totally unaware of a clear proposal by WALLACE. A. US 3102953 (1963) some thirteen years earlier to a multi-turn Sagnac interferometer using a coil of 'LUCITE'*. WALLACE gives the mathematical relation between the displacement of the fringes and the angular velocity as

$$\Delta Z = \frac{2\,\Omega S}{\lambda_c}$$

where S is the area of surface enclosed by the light path (See Fig 8.3.1A and B). It is made clear by WALLACE that an increase in the value of S decreases the smallest value of Ω (the angular velocity of rotation) that can be detected, and this is done by increasing the number of turns.

VALI, SHORTHILL, GOLDSTEIN & KROGSTAD US 4013365 (1977) of The University of Utah were concerned to improve on the ability of the laser gyroscope to measure path differences of less than 10^{-6} A and frequency changes of less than 0.1Hz; a precision that is superior to one part in 10^{15}; thereby to enable the device to read rotation rates of less than 0.1 degree per hour. The proposed means for removing a portion of each of the clockwise and counter clockwise laser radiation from the optical fibre waveguide; which means comprises an imperfect splice in the optical fibre waveguide. (See Fig 8.3.2). The imperfect splice (16) serves to detect a small portion of each laser radiation. Beam (18) is deflected by mirror (20) to a beam splitter (22) where it is combined with the counter-directionally travelling beam (24) which has been deflected by mirror) (26) into beam splitter (22). Beam splitter (22) combines each of beams (18) and (24) into a combined beam (28) where the beat frequency developed between the two beams is detected by a conventional detector (30). The observed beat frequency Δf and the angular rotation rate ω is

* LUCITE: a commercially produced acrylic resin or plastic consisting essentially of polymerized methyl methacrylate.

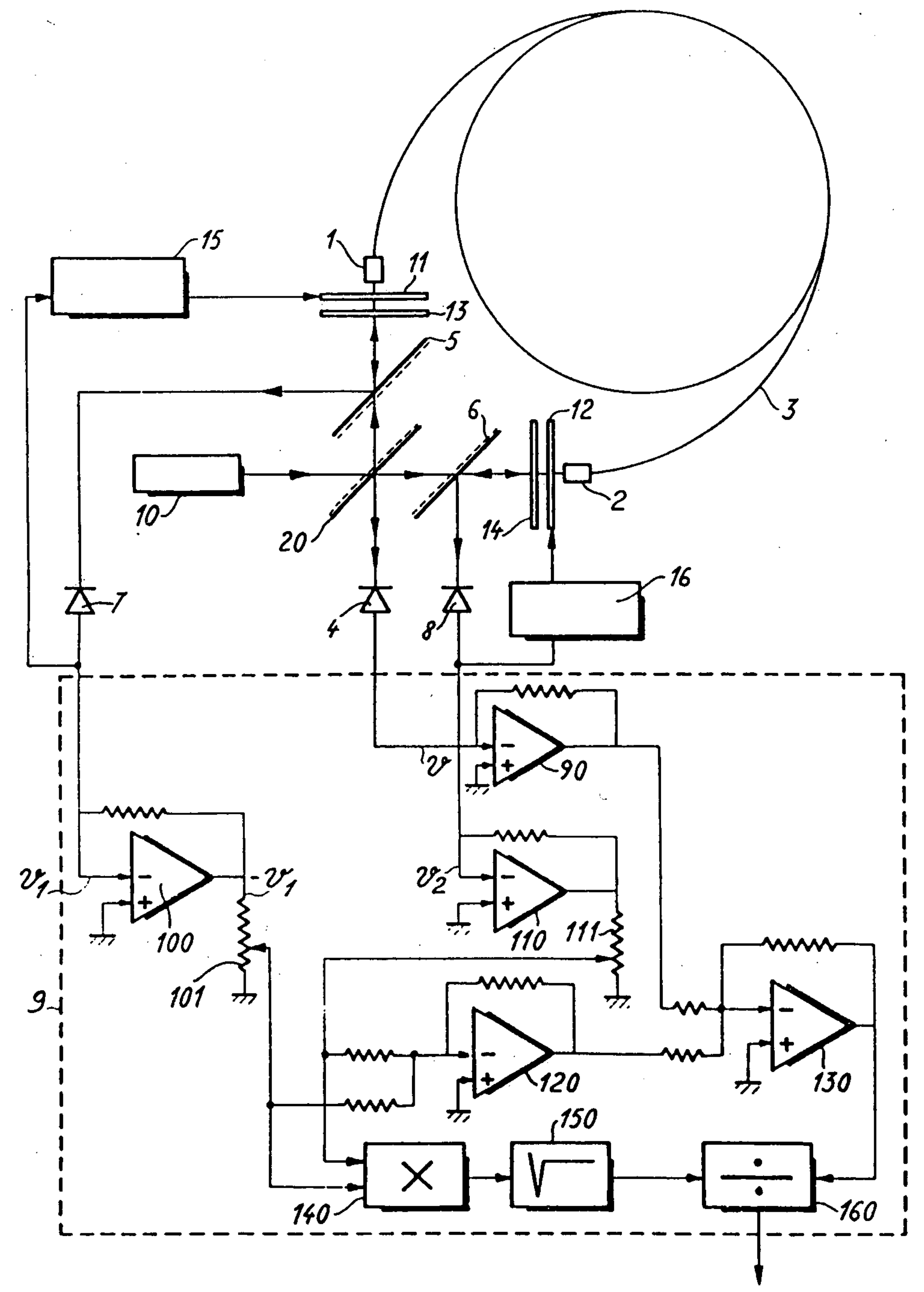

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
20
90
100
101
110
111
120
130
140
150
160

Figure 8.3.3

$$\Delta f = \frac{4A\omega}{\lambda L}$$

where λ is the laser wavelength and L is the cavity length.

REDMAN. C.M US 4133612 & US 4138196 (1979) proposes inter alia means for greater dimensional phase shifts by providing a counter rotating coherent optical signal at a first frequency in an endless closed-loop multiple turn fibre interferometer and means for generating first and second reference optical signals that are offset in frequency from the first frequency by a second frequency. The coherent optical signals are mixed with the first and second reference signals to provide first and second signals of substantially the same frequency and means for detecting the relative phase shift between the first and second signals.

THOMPSON, ANDERSON, YAO and YOUMANS (1978) (See also THOMPSON, ANDERSON AUGUST and YAO US 4208128 (1978) propose an interferometer that may conveniently use an incandescent source and include a balanced heterodyne phase detection circuit. First and second modulated light components are inflicted in counter rotation into the optical fibre wherein they experience phase changes due to the apparent change in optical length of the optical fibre upon rotation of the fibre about an axis of sensitivity. The said light components are, after removal from the fibre, combined and compared with the signals applied at the input. The phase difference therebetween is then established to measure the rate of rotation of the optical fibre about the axis of sensitivity.

LACOMBAT & LEFEVRE GB 2046434 (1983) provide an optical-fibre interferometric gyroscope comprising means for withdrawing part of the waves emerging from the ends of the optical fibre before recombination, and a signal-processing device arranged, on the basis of the detected signal and the characteristic signals of the two waves before recombination, to measure the phase-shift due to the rotational speed of the gyroscope, and consequently to measure the speed itself. The gyroscope is shown in Fig 8.3.3.

Light from a laser 10 is conveyed by a separator 20 towards the two ends 1 and 2 of a monomode optical fibre 3. The two waves from the two ends of the optical fibre, after travelling in the fibre in opposite directions, are recombined at the output by the same separator 20. A detector 4 receives light of intensity l corresponding to the interference between the two waves from the fibre, which have intensity l_1 and l_2 respectively. The intensity l depends on the intensities l_1 and l_2 and the phase shift ϕ between the two waves. The phase shift ϕ is the sum of a phase-shift ϕ_o due to asymmetrical reflection and transmission in the separator, and a phase-shift $\phi\Omega$ related to the speed of rotation Ω of the assembly.

$$\phi_\Omega = \alpha\,\Omega, \text{ where } \alpha = \frac{4\pi RL}{\lambda . c}$$

λ is the wavelength of the light used, L is the length of the coiled fibre, R is its radius and c is the velocity of light.

The detected intensity is:

$$l = l_1 + l_2 + 2\sqrt{l_1 l_2}\,\mathrm{Cos}\,(\phi\Omega + \phi_o)$$

The separator 20, which is a partially reflecting plate, can be chosen so that the fixed phase-shift ϕ_o, due to the difference in motion between the two waves caused by the asymmetry of the two paths in the plate, is equal to $\pi/2$. Under these conditions, the signal detected by detector 4 is substantially linear for low values of Ω. The speed of rotation Ω is thus expressed directly in dependence on the intensities l_1 and l_2 of the two waves from the two ends of the fibre, and the detected intensity l :

$$\Omega = \frac{l - (l_1 + l_2)}{2\alpha\sqrt{l_1 . l_2}}$$

The device according to the invention comprises means which, before the two waves recombine, detect a fraction of the two waves proportional to the intensities I_1 and I_2, and a processing device for directly obtaining the rotation speed from the detected intensities I and $k_1 I_1$ and $k_2 I_2$.

To this end, the gyrometer comprises a partially reflecting plate 5 disposed between the semi-transparent plate 20 and end 1 of the fibre and a partially reflecting plate 6 placed between plate 20 and end 2 of the fibre. the radiation reflected by plate 5 is detected by a detector 7. The radiation reflected by plate 6 is detected by a detector 8. The voltage v supplied by detector 4 and the voltages v_1 and v_2 supplied by detectors 7 and 8 respectively are characteristic of the intensity of light received by these detectors.

The gyrometer includes an analog signal-processing device 9. The processing device comprises an operational amplifier 90 connected as an inverting amplifier and receiving the output voltage from the detector 4. The device also comprises two operational amplifiers 100 and 110 connected as inverting amplifiers and receiving the output signals of detectors 7 and 8 respectively. The outputs of amplifiers 100 and 110 are connected to earth by potentiometers 101 and 111 respectively. An operational amplifier 120 connected as a summing amplifier receives output voltages from potentiometers 101 and 111 and supplies a voltage proportional to $I_1 + I_2$. A second operational amplifier 130 connected as a summing smplifier receives the output voltage of amplifier 120 and the output voltage of operational amplifier 90 proportional to -v. The output voltage of operational amplifier 130 is therefore equal to: k $(I - (I_1 + I_2))$ when the gain of amplifiers 100 and 110 and the adjustments of potentiometers 101 and 111 are such that, at $\Omega = 0$, the output signal of amplifier 130 is zero. (k is constant). A multiplier 140 also receives the output voltages of potentiometers 101 and 110, the output voltage of multiplier 140 being proportional to the product $I_1 I_2$. The multiplier output is connected to the input of a circuit 150 for extracting the square root $\sqrt{I_1 . I_2}$; the output signal of circuit 150 is applied to an input of a divider 160 whose other input receives the output signal of operational amplifier 130. The output signal of divider 160 is therefore proportional to

$$\frac{I - (I_1 + I_2)}{\sqrt{I_1 . I_2}} \quad \text{and therefore to Cos } (\Phi_\Omega + \Phi_o).$$

If Φ_o is equal to $\pi/2$ and if Ω is small, the signal is directly proportional to the rotation speed Ω. The processing device 9 can therefore be used for analog processing of the detected signals. Alternatively, the same operation can be performed by digital treatment of the signals after analog/digital conversion of the signals from detectors 4, 7 and 8. The digital processing can be carried out, for example, by a microprocessor. Since the pass band of the system is limited to a few Hertz only, the digital processing device need not be high-speed. A gyrometer according to the invention comprising the aforementioned processing device can therefore provide an accurate measurement of the rotational speed at which it is carried.

We have referred above to the valuable and detailed analysis of the Sagnac effect by POST. E.J. (1967) and we should recall that in the true Sagnac interferometer (See DITCHBURN. R.W. (1976) p 424) the whole apparatus including the source and camera is rotated.

VALI. SHORTHILL & BERG (1977) distinguish clearly between a rotating ring interferometer with co-rotating medium viz., the classical Sagnac interferometer; a rotating interferometer with stationary medium and a stationary interferometer with rotating medium, the last being the rotational version of the Fresnel-Fizeau* drag experiment. The fring shift is

$$\Delta Z = \frac{2\omega LR}{\lambda c} n^{2\alpha}$$

where n is the index of refraction of the fibre and α is the Fresnel drag coefficient. They show an interferometer Fig 8.3.4 in which the fibre coil is mounted on a rotating table and the coil end brought out to a stationary table. The coil had a radius of 25.5 cms and 53 turns giving some 85 metres of single mode fibre, this leaves about 5 cms of fibre free to bend as the coil is rotated through 180 degrees of arc.

To perfect a low noise fibre optic ring interferometric gyroscope the errors caused by the temperature variation of the index of refraction of the fibre and the dispersion term in the Fresnel drag coefficient of the fibre have to be measured and this is examined experimentally by VALI. BERG and SHORTHILL (1978) using the Sagnac interferometer. (SI) and the Fresnel Drag (FD) optical device shown diagramatically in Fig 8.3.5. HILL. K.O. and KAWASAKI. B.S. US 4107628 (1978) disclose a continuous wave Brillouin+ ring laser using a length of low loss optical fibre for generating a Brillouin shifted Stokes beam that is applied in contra-rotation with two distinct beam outputs. VALI and SHORTHILL US 4159178 (1979) disclose a stimulated Brillouin scattering (SBS) ring laser gyroscope with means for pumping a single mode optical fibre wave guide in a first and a second direction with the laser means to induce SBS radiation counter-directionally therein. Fibre optic rotation sensors (FORS) giving a rotation rate sensitivity of a few milli-arc seconds per second, are described by GOSS W.C. and GOLDSTEIN (1979) and GOLDSTEIN R and GOSS W.C. (1979)

Two European patent specifications to THOMSON. CSF EP 0007826 and EP 0007827 (1980) are of interest. In the first, see Fig 8.3.6 a laser source uses a beam splitter to form two pairs of contra-rotating waves. The beam splitter produces a phase difference of $\pi/2$ between the two waves as they emerge from the ends of the fibre. A switch is provided to direct the waves alternately to the ends of the fibre so as to measure the supplementary phase changes corresponding to the speed of rotation. A synchronous demodulator receives the detected signal after amplification and a signal synchronous with that applied to the switch. From these the demodulator produces a signal proportional to the speed of the rotation.

In the second, see Fig 8.3.7, a coherent light source provides a beam which is directed to a prism having two mirror surfaces which split the beam into two components that pass into an optical fibre (6) made into a coil that is able to rotate on its axis (x). When the beams emerge from the coil they are combined and passed to a detector. The difference in phase between the two beams is a measure of the rotational speed of the coil.

An improvement for a pulsed ring laser fibre gyro is disclosed by FLETCHER. P.C. and HILDEBRAND V.E. US 4258336 (1981) and assigned to the Secretary of the US Navy. Clockwise and counter clockwise travelling pulsed optical signals are generated by a pair of optical amplifiers and a beam-splitter/coupler feeds the signals to a detector. The clockwise and counter clockwise signals do not meet in either of the optical amplifiers, but do arrive in coincidence at the beam splitter and detector. Although low rotation rate lock-in limits the minimum detection rate in a continuous wave laser gyro, it does not so restrict the ability of the pulsed laser gyro to detect low rotation rates, because there is little, if any, coupling between the clockwise and counter clockwise modes of signal propagation.

* FRESNEL. A.J. (1788-1827)
FIZEAU A.H.L. (1819-1896) Drag experiments C.1851
(Comptes Rend. 33 (1851))p.349

\+ BRILLOUIN. L.M. (1854-1948). Brillouin scattering, not Raman scattering, but one in which photons exchange energy and momentum with the thermal vibrations.

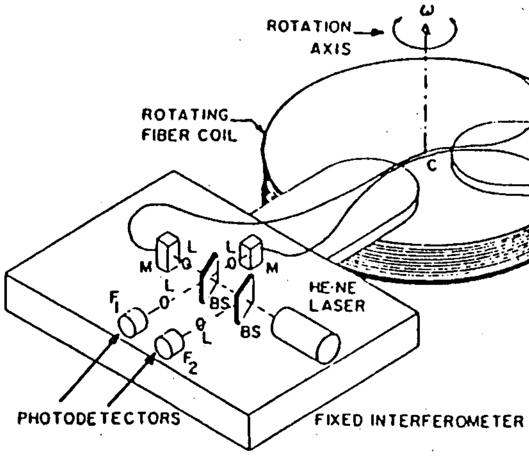


Figure 8.3.4

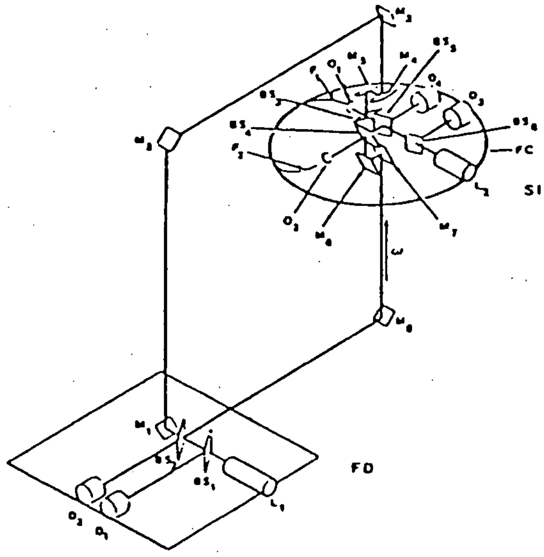


Figure 8.3.5

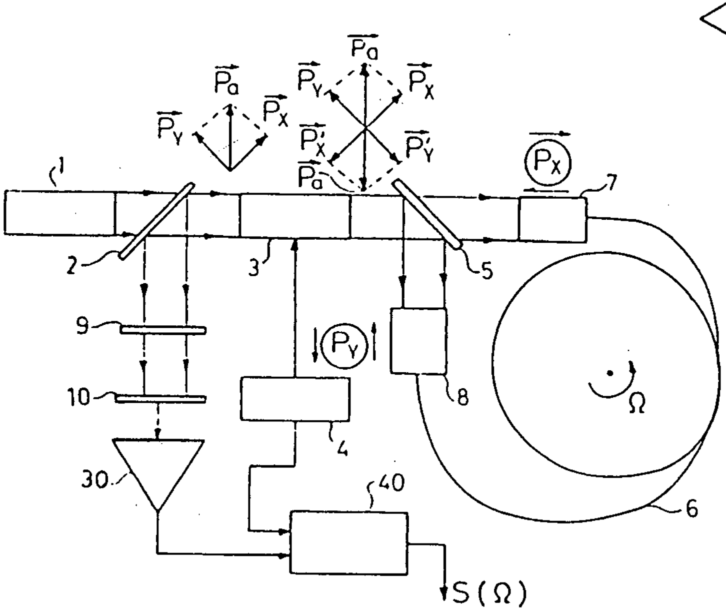


Figure 8.3.6

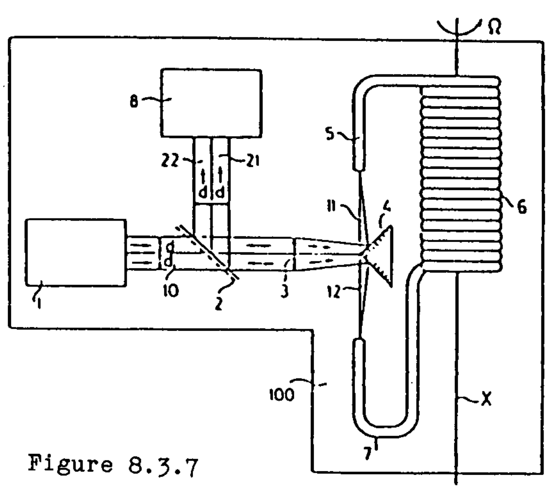


Figure 8.3.7

8.4 MACH-ZEHNDER HETERODYNE INTERFEROMETRIC GYROSCOPE

We are indebted to LAVAN, VAN DAMME, and CADWALLENDER. (1977) for a proposal that the well known Mach-Zehnder+ interferometer* may offer advantages over the usual laser ring gyroscope as a rotation sensing device. The device is shown in Fig 8.4.1B; polarized light of frequency ω emitted by a HeNe laser is partially reflected from the beam splitter B_1 to the mirror R_1 from which it passes to and through the beam splitter B_2 to the detector D. The fraction of the radiation which passes through B_1 to acoustic-optical modulators M_1 M_2 is reflected by the mirror R_2 and from B_2 to the detector D.

The modus operandi is as follows:

Laser light of frequency ω travelling in the clockwise direction passes first through the acousto-optical modulator M1, which is driven at the frequency ω_1 by the crystal oscillator XI, and then through M2, which is driven at ω_2 by X2. Because M1 is operated in the upper Bragg mode, while M2 operates in the lower Bragg mode, the result is that the laser light is shifted in frequency to $\omega + \overline{\omega}$, where the frequency offset $\overline{\omega}$ is equal to $\omega_1 - \omega_2$. In the system described ω_1 and ω_2 were equal to 40 MHz and 30.5MHz, respectively, with $\overline{\omega}$ equal to 0.5 MHz. This radiation is then photomixed on the detector D with the laser radiation of frequency ω propagating in the counter-clockwise direction. The time-varying current resulting from the superposition of the electric fields on the detector is

$$\overline{i} \propto \cos(\phi + \overline{\omega} t + \delta_R),$$

where the constant phase ϕ depends upon the geometrical path lengths travelled by the two beams and δR is the relativistic phase factor induced by the rotation. To determine δR the detector current is compared to a reference current derived by mixing the radio-frequency outputs of the crystal oscillators in a double balanced mixer and filtering the output to isolate the $\overline{\omega}$ component. Any frequency drifting in the crystal oscillators affects both the detector current and the reference current so it is eliminated when the phases of the currents are compared by the phase comparing subsystem PCSS.

The relativistic phase shift is given approximately by

$$\delta R = \frac{4\pi A \Omega}{C\lambda}$$

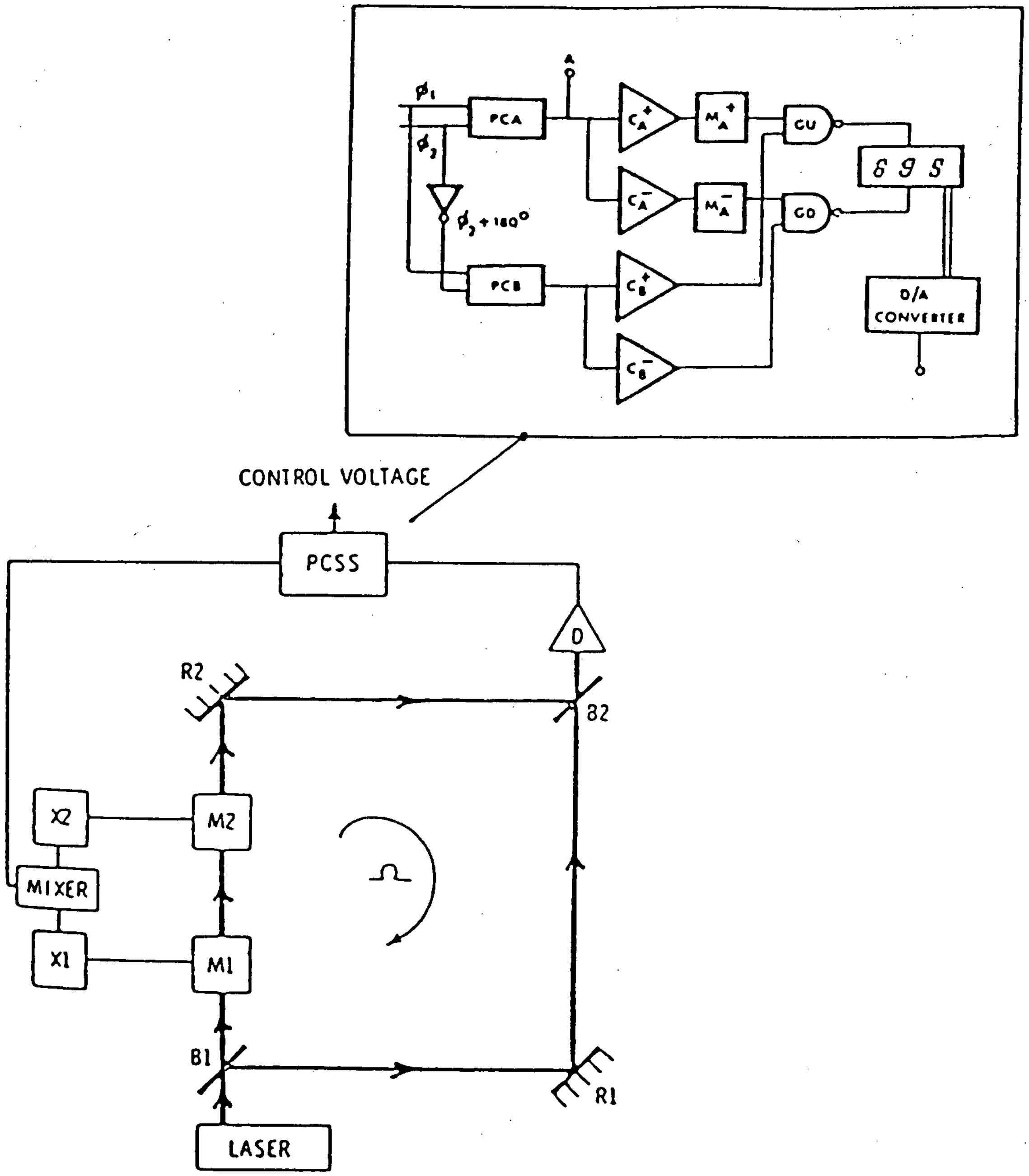
where A is the area enclosed by the interferometer, Ω is the rotation rate, and λ the wavelength of the light. Note that this phase shift is only half that of an equivalent Sagnac Interferometer, because the light beams travel only half the distance around the interferometer.

+ MACH. L. "Über ein Interferenzrefractometer"

Sitzungsberichte der Akademie der Wissenschaften in Wien. Mathematische-naturwissenschaftliche 101 (1892) 102 (1893) 107 (1898)

ZEHNDER. L.A. "Ein neuer Interferenzrefraktor" Zeits.f.Instrumentenkunde 11 (1891) p275.

* See BENNETT. F.D. & KAHL. G.D. A generalized vector theory of the Mach-Zehnder Interferometer. Journal of the Optical Society of America 43 (1953) P71-78.
POLSTER. H.D. Interferometry McGraw-Hill. Encyclopedia of Science and Technology 7 (1977) p217
DMITRIEV. A.K. etal. Twin wave Mach-Zender (sic) Laser Interferometer with Stabilisation of the position of the operating point. Instruments and Experimental Techniques 19 (1976) p1484.



ϕ_1
PCA
A
ϕ_2
$\phi_2+180°$
PCB
C_A^+
C_A^-
C_B^+
C_B^-
M_A^+
M_A^-
GU
GD
D/A
CONVERTER
CONTROL VOLTAGE
PCSS
D
R2
B2
X2
M2
MIXER
X1
M1
B1
R1
LASER

Figure 8.4.1

The phase comparing subsystem PCSS is shown in the upper box of Figure 8.4.1, and is given by LAVAN, CADWALLENDER and DEYOUNG (1975) it comprises two phase comparators PCA.PCB and four voltage comparators $C_A^+.C_A^-\ C_B^+C_B^-$ electrically connected to multivibrators and gates MA±GU.GD.

RASHLEIGH. S.C. US 4495411 (1985) proposes a Sagnac rotation sensing interferometer that uses a Mach-Zehnder interferometer to provide incident light beams that counter-propagate through an optical-fiber loop. The Sagnac interferometer operates at maximum sensitivity for zero rotation rates when the Mach-Zehnder is adjusted so that the intensities of the incident light beams are equal. By periodically varying the position of a mirror in the Mach-Zehnder the interferometer is switched into and out of quadrature so that the amplitude of the interferometer output signal is modulated at frequency f_o. Phase sensitive detection at $2f_o$ or multiples thereof reduces the background noise level several orders of magnitude below the level for dc operation.

For a useful report on current work see below.+

8.5 NUCLEAR GYROSCOPE (MAGNETIC INDUCTION GYROSCOPE)

Spin is the sine qua non of the gyroscopic condition, and the phenomena now known under the generic name of paramagnetic resonance* rely on the spin of the electron and of the atomic nucleus. The former is termed electron spin resonance (ESR) and the latter, nuclear magnetic resonance (NMR). Strictly speaking paramagnetic resonance refers to the magnetic resonance of permanent magnetic dipole moments and it encompasses not only the magnetic resonance of electrons but also nuclear magnetic resonance.

ABRAHAM. M. (1903) was the first to consider, according to THOMAS. L.H. (1927) an electron with an axis. Many have since considered spinning electrons, ring electrons and the like, see for example. COMPTON. A.K. (1921) who suggested a quantitized spin for the electron, but it remained for UHLENBECK. G.E. and GOUDSMIT. S. (1925/1926) to show how this idea explained the anomalous Zeeman effect and took on something of a physical reality. The first experimental observations were due to ZAVIOSKY. E. (1945/1946) of Kazan using several concentrated salts of the iron group.

The first observations of nuclear magnetic resonance (NMR) were made by BLOCH. F, HANSEN. W.W. and PACKARD. M. (1946) using the protons of water and independently in the same year by PURCELL. E.M., TORREY. H.C. and POUND. R.V. (1946) using the protons of paraffin; a detailed discussion of NMR is due to ANDREW. E.R. (1955). In 1962 it was reported by CULVER. W.H. (1962) staff physicist at the Institute for Defence Analyses, late of Rand Corporation the author of a seminal report CULVER (1960) (repeated in part in CULVER (1962)) that - no one has yet demonstrated a useful nuclear gyroscope. I will return to CULVER's ideas below but first I would like to refer to the works of LEETE. B.D. US 2720625 (1955) and HANSEN. A. Jr. US 2841760 (1958) both commenced in late 1952; who are the first, as far as I can discover, to propose workable nuclear gyroscopes and at the same time to put before the public in open letters exemplary expositions of the modus operandi of these complex instruments. Both workers were assignors to General Electric Company of New York and their disclosures are substantially identical being prepared by their attorney Richard. E. Hosley.

\+ LASER GYROS AND FIBRE OPTIC GYROS
One day Symposium - Proceedings. 25th February 1987.
Royal Aeronautical Society p1.1 - 8.13.

* See PAKE. G.E. Paramagnetic Resonance W.A. Benjamin Inc. N.Y. (1962) 205 pages.

LEETE & HANSEN show that it is known that the nuclei of many atoms have an angular momentum or spin, and likewise have a magnetic moment. When such nuceli are placed in a magnetic field, their magnetic moments tend to precess about the field direction at a rate known as the Larmor frequency,* the value of which is given by the relation $2\pi V = \gamma H$, where V is the Larmor, or precession frequency, H is the magnetic field intensity, and γ is a quantity known as the gyromagnetic ratio which is proportional to the quotient of the magnetic moment of the nucleus divided by its angular momentum. For any one given kind of atomic nucleus, the gyromagnetic ratio is a constant, so that the Larmor frequency is directly proportional to the magnetic field intensity. The nuclei of twenty-six or more kinds of atoms and isotopes are known to have magnetic moments which precess in this manner. For simplicity in the following discussion, the proton, or hydrogen nucleus, will be considered as a typical example, but the invention is not limited to the use of protons, since other nuclei may be used without materially altering the principles involved. Chemical bonds appear to have no appreciable effect on the Larmor frequency. Consequently, the protons used in magnetic resonance apparatus may be hydrogen nuclei in any convenient chemical combination for example, ordinary water. However, in practice small quantities of other substances, such as manganous sulphate or other paramagnetic salts, are often dissolved in the water for well-known reasons having to do with the "relaxation time," which need not be discussed to explain the present invention. A 1/250 molar solution of manganous sulphate in one cc. of distilled water has been employed with good results as a proton sample in magnetic resonance equipment.

The gyromagnetic ratio γ of the proton is approximately 2.67×10^4 per oersted-second. Thus, the Larmor frequency of proton precession in a magnetic field of H oersteds is

$$\frac{2.67}{2\pi} \times 10^4 x H \text{ cycles per second}$$

For example, if H is 10,000 oersteds, the Larmor frequency is approximately 42.5 megacycles per second.

Assume that a large number of protons are placed in a homogeneous, unidirectional magnetic field, so that the magnetic moments precess about the field direction at the Larmore frequency. Considering the geometric projections of the proton magnetic moments on the field direction, it will be found that two orientations of the protons exist: some of the protons have their magnetic moments aligned with the field, which is called the parallel orientation, while others have their magnetic moments aligned against the field, which is called the anti-parallel orientation. The anti-parallel orientation represents a higher energy level than the parallel orientation, since work must be done to turn the magnetic moments against the field. However, at temperatures normally encountered, the energy difference between the two orientations is very small compared to the energy of thermal agitation. Because of the thermal effects, frequent transitions of individual nuclei from one orientation to the other occur, but if the protons are in thermal equilibrium, it is known that the probability as a function of time for a transition from the higher energy level to the lower energy level is slightly greater than the probability for the reverse transition, so that, under such conditions, on the average a slightly larger number of protons will be found in the lower-energy parallel orientation than in the higher-energy anti-parallel orientation. For example, of 2,000,000 protons in thermal equilibrium at room temperature, 1,000,007 may have the parallel orientation, while the remainder have the anti-parallel orientation. Assume that a second magnetic field is introduced at right angles to the first field, and that the second field alternates at the Larmor frequency of the protons precessing in the first field. Now, those protons having the low-energy parallel orientation can absorb energy from the alternating field, which increases the probability of transitions from the parallel orientation to the higher-energy anti-parallel orientation and thus tends to equalize the numbers

* See notes on Larmor frequency after this discussion.

of protons in the two energy levels. If the alternating field is sufficiently strong, the proton populations in the two energy levels soon become substantially equal, and no more energy is absorbed. But if the alternating field is a bit weaker, the absorption of energy by the protons from the alternating field may be balanced by their tendency to return to thermal equilibrium, so that there can be substantially continuous absorption of energy from the alternating field. Therefore, there is an optimum strength of the alternating field, which can be determined by experimental adjustment, at which maximum energy is absorbed by the protons. Although quantum considerations prohibit more than two orientations of protons in a magnetic field, other nuclei may have as many as 10 possible orientations. This does not affect the basic principles involved, since transitions may still take place between adjacent energy levels represented by different orientations. This absorption of energy by precessing nuclei, which generally occurs only when the frequency of the alternating field is substantially the same as the Larmor frequency of the nuclei, is called nuclear magnetic resonance. Similar phenomena, known as electronic magnetic resonance, can occur in substances having uncoupled electrons. In general, there are two classes of such substances, one class being strongly paramagnetic salts, and the other class being ferromagnetic metals and alloys. For electronic magnetic resonance in paramagnetic salts, sometimes called paramagnetic resonance, the same relations apply as in proton resonance, except that the gyromagnetic ratio of the electron is used in place of the gyromagnetic ratio of the proton. Since the electron has a gyromagnetic ratio which is about 700 times as large as that of the proton, the Larmor frequency for paramagnetic resonance is about 700 times as that for proton resonance in the same magnetic field.

In ferromagnetic metals and alloys, the magnetic induction B inside the metal is not substantially equal to the magnetic field intensity H. In this electronic resonance case, sometimes called ferromagnetic resonance, the Larmor frequency v is given by the relation

$$2\pi v = \delta\sqrt{BH}$$

where δ is the gyromagnetic ratio of the electron, H is the magnetic field intensity, and B represents the magnetic induction which is equal to the product of the magnetic field intensity and the permeability of the metal. The invention may utilize either nuclear magnetic resonance or electronic magnetic resonance. The generic term "magnetic resonance" includes both. Since the basic principles are the same, only nuclear resonance need to be discussed in detail.

The magnetic resonance phenomenon can be detected by various means, several of which are well known. For example, assume that the alternating field is supplied by a suitably energized coil placed around the proton sample, which is a common arrangement in nuclear resonance apparatus. Energy absorption from the field by the protons at resonance causes a measurable decrease in the apparent "Q" of the coil, where Q is the well known symbol for the ratio of energy stored per cycle to energy dissipated per cycle. This is known as the absorption effect. There is also a small, but measurable, change in the apparent inductance of the coil. This is known as the dispersion effect. Furthermore, when a second coil is placed near the proton sample with its axis orthogonal to the respective directions of the two applied magnetic fields, at resonance the precessing proton magnetic moments induce an alternating voltage in the second coil. This is known as the induction effect. While any of these three effects may be used to detect the existence of magnetic resonance conditions, in practice the absorption and induction effects are most frequently used.

As set forth in the co-pending applications of ALBERT HANSEN, Jr., and entitled "Method for Measuring Angular Motion," it has also been found that the nuclear or electronic precession frequency, as given by the magnetic resonance relations, must be referred to irrotational space coordinates; that is, the Larmor frequency v computed from the magnetic resonance relations as hereinbefore stated must be taken with reference to a coordinate system which does not rotate in space. Assuming that the magnetic resonance apparatus, and in particular the means providing the alternating magnetic field, rotates in space about the axis of the other field with an angular velocity , the frequency f of the alternating field which satisfies the resonance condition, as it appears to an observer rotating with the apparatus, differs from the Larmor frequency according to the relation

$$v = f + \frac{\omega}{2\pi}$$

Since v can be computed when the magnetic intensity H and the gyromagnetic ratio δ are known, and since f can be measured, this relation can be used to measure values of the angular velocity ω, and, if desired, the velocity can be integrated to determine angular displacement. For example, assume that f, δ and H are all kept precisely constant. Then the magnetic resonance absorption of energy will vary as a function of ω, and these variations can be measured to determine values of the absolute angular motion of the apparatus. However, the problem of keeping f and H constant with sufficient precision presents a substantial practical difficulty.

A principal object of LEETE'S invention is to provide improved apparatus for measuring angular motion, based upon magnetic resonance phenomena, in which the precision of measurement is not materially decreased by small variations in the values of f and H. Another object is to provide improved apparatus for indicating changes in heading of airplanes, ships or other craft. Other objects and advantages will appear from the description below.

Briefly stated LEETE provides two magnetic resonance systems which are similarly affected by changes in the values of f and H, and are differently affected by changes in the value of ω. In particular, he provides equal but oppositely directed magnetic fields in two magnetic resonance samples. Since the two fields are oppositely directed, the respective directions of precession in the two samples are opposed, so that the angular velocity ω is positive with respect to one sample and negative with respect to the other. That is, with respect to one sample.

$$v = f + \frac{\omega}{2\pi}$$

while with respect to the other sample

$$v = f - \frac{\omega}{2\pi}$$

Since the two fields are equal, changes in the value of H affect both samples in the same way. A common frequency source is used for both samples, so that changes in the value of f also affect both samples in the same way. However, changes in the value of affect the two samples differently.

The invention will be better understood from the following description taken in connection with the accompanying drawings, in which:
Fig 8.5.1 is a schematic representation of apparatus embodying principles of the invention, and Fig 8.5.2 is a schematic representation of an alternative magnetic structure useful in such apparatus.

Referring now to Fig 8.5.1, a magnetic structure comprises two similar parts, 1 and 2. These parts have respective centre legs comprising identical pairs of small permanent magnets, 3,4,5 and 6, arranged with substantially equal gaps between magnets 3 and 4 and between magnets 5 adn 6, as shown. These magnets provide north poles at 7 and 8 and south poles at 9 and 10, so that equal, but oppositely directed, magnetic fields are provided across the two gaps. Although such factors as temperature variations may cause small changes in the strength of the magnets, or small changes in the dimensions of the gaps, the two fields remain substantially equal since the two magnetic circuits are identical.

Magnetic resonance samples 11 and 12 are located in the respective gaps as shown. If proton resonance is to be used, the magnetic resonance samples may be water contained in small glass tubes sealed at each end, or other suitable containers. Coils 13 and 14 are positioned about samples 11 and 12 respectively, with the coil axes perpendicular to the magnetic fields across the gaps. To assist the detection of magnetic resonance conditions, the magnetic intensities within the gaps are modulated at an audio frequency by suitable means, such as modulating windings 15 and 16 energized, for example, by an audio-frequency oscillator 17.

Coil 13 is connected in a bridge circuit 18 energized by a radio-frequency oscillator 19, so that radio-frequency current is supplied to coil 13, whereby an alternating magnetic field is applied to sample 11 perpendicular to the field across the gap between magnets 3 and 4. The magnetic intensity within the gap and the frequency of oscillator 19 are chosen such that substantial magnetic resonance conditions exist in sample 11.

Bridge circuit 18 may be of the well-known bridged-T type as shown, but it will be appreciated that numerous other bridge circuits may be used with good results. The bridged-T circuit has zero transfer admittance when it is adjusted to balance. Assume, for example, that the bridge is balanced when there is no magnetic resonance absorption of energy by sample 11. At magnetic resonance, sample 11 absorbs energy, thereby increasing the apparent resistance of coil 13, and unbalancing the bridge. Consequently, as the magnetic intensity in sample 11 is modulated through the magnetic resonance value, a radio frequency signal appears across the output terminals 20 of bridge 18, which is amplitude modulated at an audio frequency. This signal is demodulated by suitable means, such as rectifier-demodulator 21, and amplified by an audio-frequency amplifier 22 to provide an audio-frequency error signal having an amplitude which is related to deviations of the average magnetic intensity in sample 11 from the magnetic resonance value, and having a phase which depends upon whether the average magnetic intensity in sample 11 is above or below the magnetic resonance value. In practice, is is not essential that the bridge circuit be precisely balanced - in fact, some unbalance may be desirable. Similarly, coil 14 is connected in a bridge circuit 23 which is also energized by radio-frequency oscillator 19. The output signal from bridge circuit 23 is demodulated by rectifier-demodulator 24 and amplified by audio-afrequency amplifier 25 to provide a second audio-frequency error signal having an amplitude which is related to the difference between the average magnetic intensity in sample 12 and the magnetic resonance value, and having a phase which depends upon whether the average magnetic intensity in sample 12 is above or below the magnetic resonance value.

The gyromagnetic ratios γ for the two samples being equal, and the magnetic intensities also being equal, the Larmor frequency for sample 11 is precisely the same as the Larmor frequency for sample 12. Furthermore, coils 13 and 14 are energized at exactly the same frequency by the common radio-frequency oscillator 19. Therefore, when there is no rotation of the apparatus, identical magnetic resonance conditions exist in the two samples, and the two error signals supplied by amplifiers 22 and 25, respectively, must be identical. However, when the apparatus rotates in space the magnetic resonance conditions in the two samples are not identical, as is evident from the relations hereinbefore given, and the two error signals are not identical. Preferably, the magnetic resonance apparatus is enclosed in a housing 26, supported upon a shaft 27 which is rotatable about bearings 28 and 29. Thus, housing 26 is rotatable about an axis parallel to the direction of the magnetic fields across the two gaps of the magnetic structure. Housing 26 may be of ferromagnetic material to provide shielding of the magnetic structure from external magnetic fields. The error signals from amplifiers 22 and 25 are applied to opposed control winding 30 and 31, respectively, of a two-phase induction servomotor 32. The field winding 33 of servomotor 32 is energized by connections to audio-frequency oscillator 17, so that motor 32 operates at a speed and in a direction corresponding to differences between the two error signals. Motor 32 is connected to a chaft 34 which rotates a pinion 35 in engagement with a

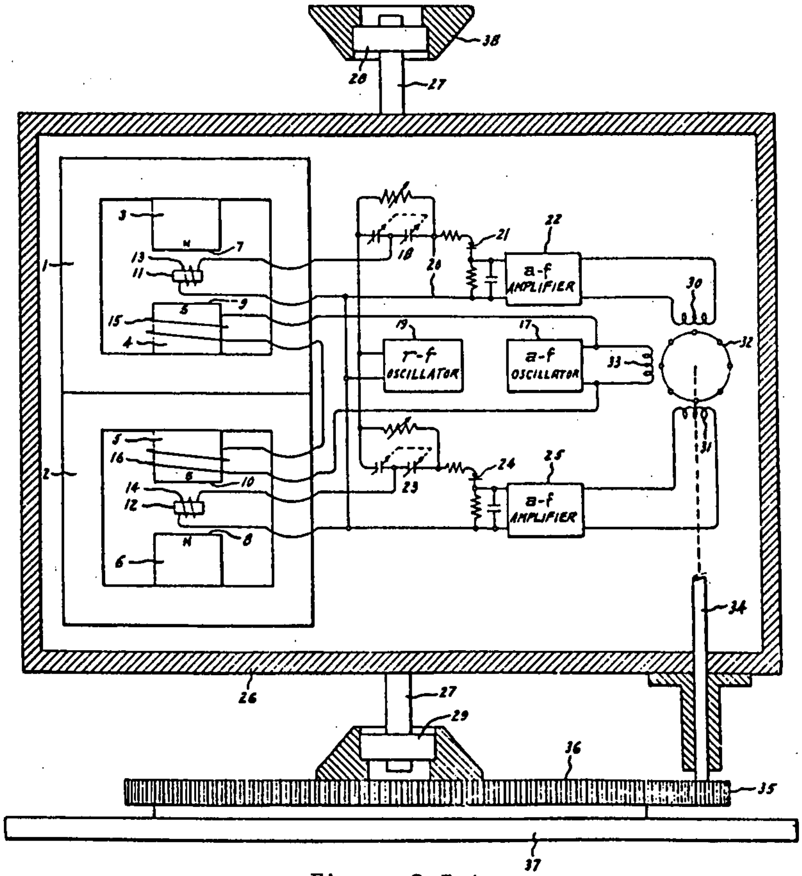


Figure 8.5.1

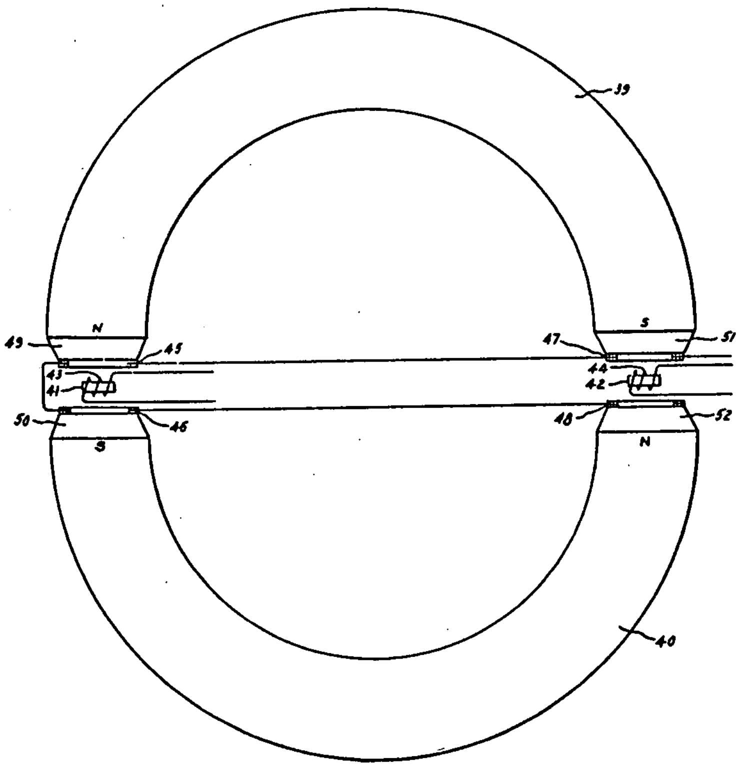


Figure 8.5.2

stationary gear 36 mounted upon a suitable base plate 37. Any small rotation of the apparatus produces a difference between the two error signals, as hereinbefore explained, which operates motor 32 and thus, by rotation of pinion 35, rotates housing 26 in a direction to oppose such rotation, and thus to keep the housing aligned in a fixed direction.

The apparatus described may be installed in an airplane, ship, or other craft, with shaft 27 vertical, and base plate 37 and the upper bearing bracket 38 suitably attached to the craft. When the craft changes heading, the resulting rotation of the apparatus operates motor 32 to rotate housing 26 relative to the craft, and keep the housing 26 substantially aligned in its original direction. Thus by observing the alignment of housing 26 relative to the craft, changes in heading of the craft can be determined. If desired, electric power for the oscillators and amplifiers can be supplied through slip rings, not shown, attached to shaft 27.

Fig 8.5.2 shows an alternative magnetic structure which may be used in an improved apparatus. The magnetic structure comprises two substantially semi-circular permanent magnets 39 and 40, arranged as shown, with substantially equal gaps between the respective ends of the two magnets. Since the two gaps are in series in the same magnetic circuit, the magnetic flux across the two gaps must be the same, except for differences in "fringing" fluxes around the respective gaps. These differences in fringing fluxes are minimized by the symmetry of the construction. Consequently, the magnetic fields of equal strength are provided in the two gaps, and these fields are oppositely directed since the two gaps are on opposite sides of the circuit. Magnetic resonance samples 41 and 42 are located within the respective gaps as shown, and radio frequency coils 43 and 44 are provided about the samples in the usual manner. The modulating windings preferably are in the form of small annular coils 45, 46, 47 and 48 placecd at the tips of the magnet poles. The portions 49, 50, 51 and 52 at each end of each permanent magnet are preferably soft iron pole pieces, to provide more homogeneous fields within the gaps. Other parts of the apparatus may be identical to that shown in Fig 8.5.1.

Note on LARMOR and LARMOR frequency

LARMOR. J 1857-1942 Irish theoretical physicist, responsible in 1897 for the introduction of the precession which orbiting charges undergo when subjected to a magnetic field.

It is shown by LAUKIEN. G. & GESCHWIND. S. in Flügge's Handbüch der Physik 38/1 (1958) p. 38/128 that the precession frequency (LARMOR frequency) is independent of the opening angle of the precession cone.

The same result can also be directly derived in two different ways for single nuclei. If we designate as the angle between the axis of the nuclear moments a and μ and the field Ho a torque to the value of $/\mu/.Ho.\sin\alpha$ tends to rotate the nuclei in the direction of the field. In order to achieve this state of motion a gyromoment of value $/a/\omega.\sin$ must act on the nuclear gyro. Equating

$$\omega = \frac{/\mu/}{/a/} H_o = \gamma H_o = \omega_o$$

Again the possible neighbouring orientations about equidistant energy quanta $\mu Ho/I$ are distinguished quantum theoretically according to the energy levels for the symmetric top

$$W_r = hBJ(J+1) + h\ (A-B)K^2.$$

Here $B = h/8\pi^2 I_b$ and $A = h/8\pi^2 I_a$ where I_a and I_b are the moments of inertia parallel and perpendicular respectively to the symmetry axis. $\frac{h}{2\pi}\sqrt{J(J+1)}$ is the total angular momentum of the molecule and $K\frac{h}{2\pi}$ its projection on the symmetry axis. If the molecule possesses a permanent electric dipole moment it can absorb a quantum of microwave energy hv and make a transition to a higher rotational state governed by the selection rule $\Delta J=+1$, $\Delta K=0$. The pure rotational absorption frequency is then given by $v=2BJ$ where J is the angular momentum quantum number for the upper state of the transition.

If this energy quantum is set at $hv_o = h\omega_o$ then we obtain $\omega_o = \gamma H_o$.

Some years later OVERHAUSER. A.W. (1953) discussed a new method of polarizing nuclei, applicable only to metals. He was able to show that if the electron spin resonance of the conduction electrons is saturated, the nuclei will be polarized to the same degree that they would be if their gyromagnetic were that of the electron spin. This action results from the para-magnetic relaxation processes that occur by means of the hyperfine structure interaction between electron and nuclear spins. A shift of the electron spin resonance due to the same interaction will occur for large amounts of polarization and should provide a direct indication of the degree of polarization; now known as the OVERHAUSER effect. This is appealed to by FRASER. J.T. US 3103620 (1963) who explains that in this phenomenon, during the pick-up by induction of signals from nuclei after excitation at their Larmor frequency, if assocated electrons also be excited at their electron Larmor frequency the nuclear signal is greatly increased in amplitude. The nucleus and the electron may be those of the same kind of atom or of two different kinds of atoms. As example, the nucleus of the common isotope of hydrogen may be employed. The hydrogen may be in any form; in chemical combination as in water or in a solid, of as the free elemental gas. The electron may be the unpaired electron found in the manganous ion, which is preferably in the form of a water solution of a salt such as manganous sulphate. Any other molecule having an unpaired electron exhibiting the phenomenon of electron resonance may be employed. The physical nature and material of the bottle, bottles or other containers containing the subatomic particle substances is unimportant so long as the material is nonmagnetic, does not react with the substance, and does not exhibit magnetic resonance in the region of interest. The aggregate protons and the aggregate electrons should be in such form as to be immersed in and equally acted upon by a constant-direction magnetic field which is to be described, and the materials containing them must be intimately mixed when the Overhauser effect is employed.

The nucleus of an atom of common hydrogen, H^1 , consisting of a single proton, is considered to be in rapid spinning rotation and therefore has mechanical properties like those of a rapidly rotating gyroscope. Like the gyroscope, its axis of rotation will move in a circle, or precess, if a torque be applied to it. Such a moment is supplied if the proton be subjected to a magnetic field, and the rate of precession, ω_p, is

$$\omega_p = \gamma_p H \qquad (1)$$

in which γ_p is a constant of nature termed the gyromagnetic ratio or the magneto gyric ratio, and is known with accuracy. H is the strength of the magnetic field. The rate of precession, ω_p, is termed the Larmor frequency. Similarly, all electrons are in a spinning rotation and undergo precessional rotation when subjected to the force of an external magnetic field. Under certain conditions the precessional rate of electrons in certain environments can be observed and measured, just as that of nuclei can be measured. The Larmor frequency, ω_e, of the electron is

$$\omega_e = \gamma_e H \qquad (2)$$

in which γ_e is a constant of nature which is known with accuracy and H is the magnetic field strength. Fraser has determined that the free precessional rotations of the nucleus have the property of rigidity in space. If therefore, the instrument containing the particles, say proton particles, itself has a rotation relative to the inertial frame in a plane perpendicular to the axis of precession, the instrumental rotation, ω , is added to or subtracted from the Larmor rate. That is to say, the apparent Larmor frequency is varied by an amount which is dependent on the rate of instrument rotation. Equation 1 is modified in Equation 3 to take account of this and the resulting apparent Larmor rate, ω_p', is that which would be observed by an observer rotating with the instrument.

$$\omega_p' = \gamma_p H + \omega \qquad (3)$$

Thus, rotation of the instrument relative to inertial space can be detected and measured but, of course, translatory motion is not detected. When the particle is a proton, Equation 3 applies. In the case of free precession of any other particle, an equation similar to Equation 3 applies. In the case of slaved precession, that is, when large-angle precession is effected by the continuous application of an alternating magnetic field of substantially Larmor frequency the particle still has the property of gyroscope rigidity in space. However, an equation of the simple form of (3) is not adequate. In the case in which the particle is an electron the situation is quite complicated but is generally described by

$$\omega_e' = \gamma_e H + F\ (\omega_e,\ \omega\ ,\ P) \qquad (4)$$

in which ω_e' is the Larmor frequency which an observer rotating with the instrument would see. The term ($\gamma_e H$) is inserted separately to show the explicit dependence of ω_e' on it in the first degree. P stands for all other parameters among which are H, and T, the relaxation time of the electron. For the purpose of this description, when the electron's precessional frequency is not employed directly in the computation of ω, but merely in a convenient method of controlling the continuous magnetic field, Equation 4 may be approximated as

$$\omega_e' \cong \gamma_e H \qquad (5)$$

and this approximation may be employed as correct enough in this use.

One way in which a precessing particle may be coerced by an external force is by application of a constant-direction magnet field. If an aggregate of particles be subjected to a steady magnetic field, after a time depending on the nature of the particles, they all will have been coerced to positions in which their axes of precession are all either parallel or antiparallel to the magnetic field, there being a preponderance in one direction causing a net macroscopic magnetic

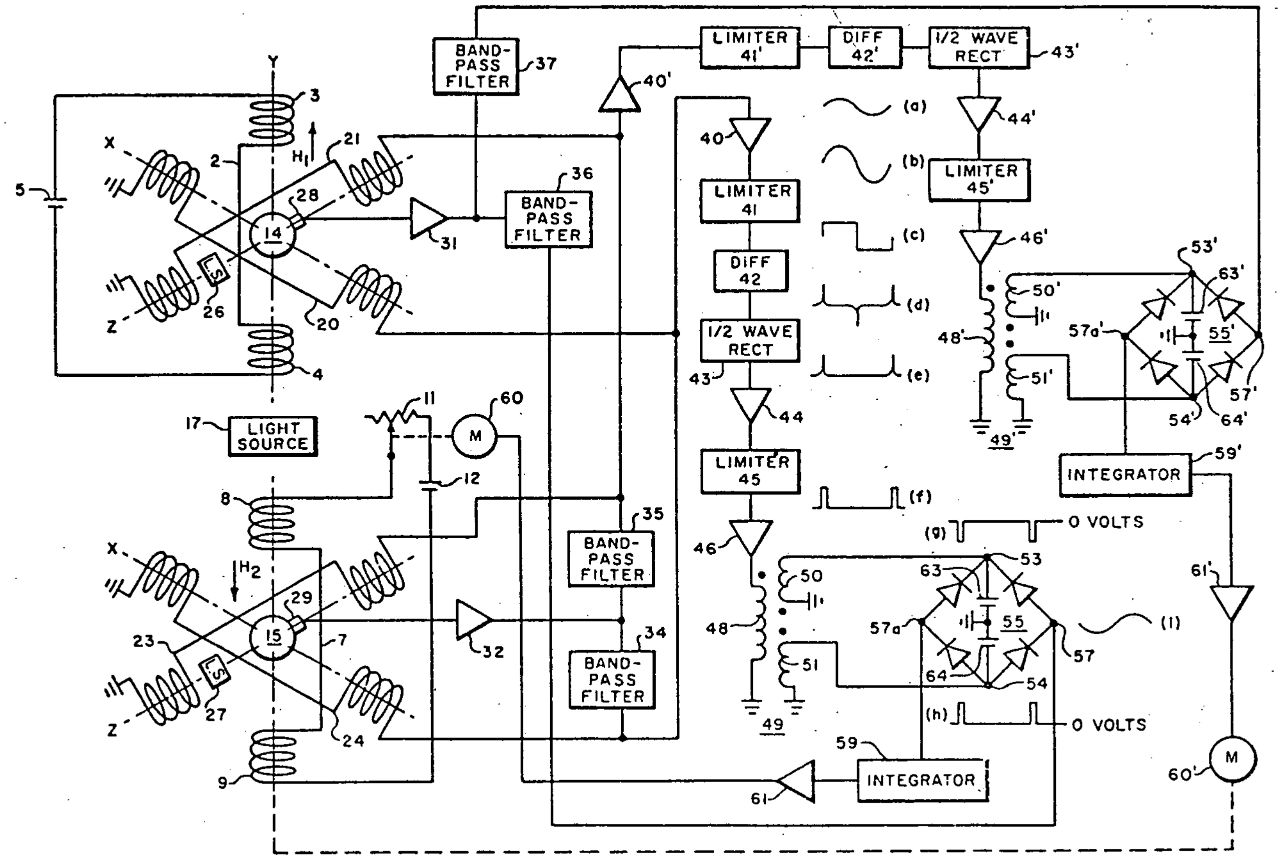

BAND-PASS FILTER
LIMITER 41'
DIFF 42'
1/2 WAVE RECT
LIMITER 41
DIFF 42
1/2 WAVE RECT
LIMITER 45'
LIMITER 45
LIGHT SOURCE
BAND-PASS FILTER
BAND-PASS FILTER
BAND-PASS FILTER
INTEGRATOR
INTEGRATOR
0 VOLTS
0 VOLTS
M
M
L.S
L.S
X
Y
Z
X
Z
H1
H2

Figure 8.5.3

moment. The above equations contain the tacit assumption that this has been done, that the direction of the field H is that of the macroscopic indication of the direction of the common axis of precession, and that the plane of instrumental rotation, ω, is perpendicular to the field H.

Such a gradual alignment or realignment of the aggregate precessional axis of a number of freely precessing particles to the direction of a constant field is termed relaxation. The relaxation times of interest in this invention may be small or large depending on many factors. The particles also can be coerced by an oscillating magnetic field and, if the frequency of oscillation be correct the oscillating field will change the angle of precession. If the direction of an incident constant-direction magnetic field be changed, the particle precessional axes will gradually realign themselves to the new direction. While they are doing so, the precessional motion can be detected and the detected signal will have an amplitude varying with axial direction and with time. The amplitude of this detected signal is a measure of the amount of rate of axial displacement and can be employed together with the before-described frequency determination in accordance with Equation 3, for the complete determination in three dimensions of the direction of pointing of the field direction relative to an inertial frame.

FRASER. J.T. US 3103621 (1963) introduces an optically pumped magnetic resonance gyroscope and direction sensor. It has already been shown that the phenomenon of magnetic resonance is a potent device suitable for replacing the mechanical gyroscope that suffers from its inherent drift limitation.

But before replacement could be made a number of problems required solution. the most important of these is stabilization of the strength of the field (H) since any reading of (ω) which one might obtain is dependent thereon. If the value of (H) is not stabilized or accurately known, the apparent Larmor frequency, designated $^{\nu}LA$, which equals $\gamma_{eff}H\pm\omega$ in the case of the electron and $\gamma H\pm\omega$ for the nucleus may be influenced by either a variation in the field strength (H) of any angular displacement of frequency (ω). A second perplexing problem is the measurement of (ω) since the difference between $^{\nu}L$ and $^{\nu}LA$ which is equal to (ω) when (H) is held constant is small compared to the magnitude $^{\nu}L$.

In Fig 8.5.3 there is shown an optically pumped magnetic resonance gyroscope for stabilizing a structure about a single axis.

A coil 2 which is preferably of the Helmholtz type has upper and lower portions 3 and 4, respectively, connected in series across a battery 5 to provide a substantially uniform field H_1 in the area between the two coils. The connection of battery 5 and the arrangement of windings 3 and 4 are such that the field H_1 is directed upwardly and parallel to the axis marked Y. A second coil 7 is similar to coil 2 has upper and lower portions 8 and 9, respectively, connected in series with a potentiometer 11 across a battery 12 to provide a substantially uniform field H_2 in the area betwee coils 8 and 9. The connection of battery 12 and windings 8 and 9 are such that the field H_2 is directed downwardly and parallel to the Y axis. Potentiometer 11 is provided to automatically adjust the current in coils 8 and 9 so that the fields H_1 and H_2 will be of equal strength. The operation of potentiometer 11 will be discussed in full detail at a later time.

As shown in the drawing fields, H_1 and H_2 are colinear. That is they are in a line parallel to and equally spaced about the Y axis. This is not a requirement and is only one way of arranging the fields. The only limitations on the location of the fields is that they be parallel to the Y axis and angularly fixed with respect to the Y axis. Stated differently, the coils can be moved in any direction and at any time as long as they do not move around or about the Y axis except as will be described later. They may be moved up or down toward or away from the axis and in opposite directions without affecting the operation. If, however, they should be moved or rotated about the Y axis the readings supplied by the device would be altered by an amount depending on the rate of such rotation. This will become obvious as the description progresses. Notwithstanding what has been said the coils may be in any angular position before the device is operated and the limitation as to angular movement applies only once the device is in operation.

In all of the discussion so far, both the earth's magnetic field and any stray fields which might be present have been ignored since the device is enclosed within a magnetic shields, not shown, to eliminate their effect.

Two identical containers 14 and 15, constructed of a transparent non-magnetic material, are located in fields H_1 and H_2 respectively. The contents of containers 14 and 15 are identical and each contains two different substances which exhibit magnetic resonance when properly stimulated in the manner which will be described in detail later. The two substances may be selected from amongst those substances containing nuclei and paramagnetic materials which exhibit magnetic resonance. Two paramagnetic materials which exhibit electron magnetic resonance and are quite suitable for use in this apparatus are rubidium 87 and sodium 23 both in their vapour form. If desired a buffer gas such as argon may be mixed with the rubidium and the sodium to increase the efficiency and accuracy of the device. The buffer gas extends the relaxation time of the electrons by decreasing the collision rate between the rubidium atoms and between the sodium atoms which results in a decrease in the rate of coherence loss between the phase of the precessing electrons. A source of monochromatic circularly polarized sodium light 17 is positioned such that containers 14 and 15 are irradiated with the energy emitted by source 17. When the rubidium and sodium samples in the containers are subjected to the fields H_1 and H_2 the electrons associated with each assume or are constrained to two orientations or levels, often called "Zeeman levels," of different energy values. The sodium electrons absorb the energy radiated by the source 17 and those electrons in the lower energy state are transferred to the higher energy level thus increasing the population of the higher energy level at the expense of the lower energy level to provide a net macroscopic moment.

In the embodiment chosen for illustration source 17 emits only sodium light and the rubidium electrons are transferred to a higher energy level by spin exchange collisions with the sodium electrons. If desired, source 17 could be arranged to emit photon energy of both rubidium and sodium wave length and the transitions for each of the electrons would be independent of each other. The technique described for implementing the transitions of the electrons from their lower energy state to the higher energy state is called "optical pumping" and has been used for many purposes with single samples. Optical pumping is highly desirable since substantially a 100% transition can be effected with a very small unidirectional field strength.

When the sodium electrons in the container are subjected to an alternating magnetic field at right angles to the unidirectional field provided by coils 2 and 7 and whose frequency is equal to or substantially the same as the Larmor frequency which was previously defined they will precess about the unidirectional magnetic fields produced by the coils at the Larmor frequency. This also applies to the rubidium electrons which have been aligned by the spin exchange collisions with the sodium electrons. However, in the case of the rubidium electrons the frequency of the alternating fields and the precession frequency of the rubidium electrons will differ since the effective gyromagnetic ratio of the rubidium electron differs from that of the sodium electron. Actually sodium and rubidium were chosen in the first place because their Larmor frequencies differed by a substantial amount. The reason for this choice will become apparent as the description continues.

Two alternating fields at right angles to the unidirectional field H_1 are produced by a pair of coils 20 and 21 each of which is energized by a current of the correct frequency in a manner which will be described later. Similarly the two alternating magnetic fields at right angles to the unidirectional field H_2 are produced by a pair of coils 23 and 24. The angular position of coils 20, 21, 23 and 24 is immaterial and the only restriction on their position is that the fields produced by each coil be normal to the unidirectional fields H_1 and H_2 as the case may be.

The frequency of precession of the sodium electrons about the unidirectional magnetic fields H_1 and H_2 when the platform on which the containers 14 and 15 rest is stabilized about the Y axis with respect to inertial space may be computed from the formula:

$$\text{Frequency of precession of sodium electrons} = \gamma_{eff}\, \text{NaH}$$

where γ_{eff}Na is the gyromagnetic ratio of the sodium electron and H is the strength of the field H_1 or H_2 depending on which of the containers we are considering. Thus, if both H_1 and H_2 are identical the frequency of precession of the sodium electrons in containers 14 will be identical to the frequency of precession of the sodium electrons in container 15. If, however, H_1 and H_2 differ, the precessional frequencies of the sodium electrons in containers 14 and 15 will differ by an amount which corresonds to the difference in the field strength. Likewise the frequency of precession of the rubidium electrons about fields H_1 and H_2 under the conditions previously set forth may be determined by the formula:

$$\text{Frequency of precession of rubidium electrons} = \gamma_{eff}\text{RbH}$$

where the symbols denote the same qualities as stated above except with respect to the rubidium electrons. Here also the precessional frequency of the rubidium electrons in containers 14 and 15 will differ if the fields H_1 and H_2 differ in strength. This difference also corresponds to the difference in field strength. While the magnetic moments of the sodium and the rubidium electrons in container 14 precess at the same frequency as their respective counterparts in container 15, they are, however, precessing in the opposite directions due to the 180° phase reversal of field H_2 with respect to field H_1 and therefore we may say the precession of the magnetic moments of the electrons in one container are of opposite phase to their counterparts in the other container.

If we now impart an angular velocity to containers 14 and 15 which causes said containers to rotate at a frequency (ω) about the Y axis the observed frequency of precession will be reduced in one container and increased in the other by the value of (ω) since the angular velocity, depending on its direction, augments the precession of the moments in one container while it detracts from those in the other container. This may be expressed algebraically by the following equations:

$$f\,(\text{precession sodium electron}) = \gamma_{eff}\,\text{NaH} \pm \omega$$
$$f\,(\text{precession rubidium electrons}) = \gamma_{eff}\,\text{RbH} \pm \omega$$

Thus, if the angular velocity imparted about the Y axis is such as to add to the observed precessional frequencies of the magnetic moments of he sodium and rubidium electrons in container 14 those observed in container 15 will be reduced by a similar amount, and the difference in the precessional frequencies of similar electrons in the two containers will equal twice the frequency of the angular velocity about the Y axis.

We now have a means for determining the angular rotation about the y axis with respect to inertial space if we can be sure that the difference in the observed precessional frequency is due solely to an angular rotation. This can be accomplished by insuring that both fields, H_1 and H_2 are always equal since under this condition any observed difference in frequency can only be due to an angular rotation.

It should be obvious from what has been said that if a difference in the precessional frequency between two similar electron magnetic moments is observed and that difference is the result of a change in the value of one of the fields which results in one precessional frequency increasing or decreasing, depending on whether the field increased or decreased, then the difference in frequency between the observed frequency of the magnetic moments of the other similar electrons will undergo a proportional change. Thus if we continually measure the difference in the precessional frequency of the sodium electron moments in containers 14 and 15 we may derive an error signal which can be used to adjust one of the fields so as to null the difference in frequency between the sodium electron magnetic moment precessions and the rubidium electron magnetic moment precessions and at the same time maintain both fields (H_1 and H_2) at the same field strength. If, however, the difference in frequency is due to an angular displacement about the Y axis which was caused by a rotation whose frequency is (ω) the mere adjustment of the strength of one of the fields can never eliminate the difference between the precessional frequencies of the magnetic moments of the rubidium electrons in the two containers, since an angular displacement which was caused by the rotation about the Y axis does not effect the difference frequencies of the two similar precessing moments in the containers proportionally. This is so because the frequency of the rotation causing the displacement is linearly added to and subtracted from the precessional frequency of the magnetic moments of the electrons and therefore no proportionality factor exists. Thus, by properly adjusting time constants we may first utilize the difference between the precessional frequencies of the sodium electron magnetic moments to adjust the fields to equilibrium and if unsuccessful after a predetermined time we have established that the difference was caused by an angular displacement and second utilize the difference between the precessional frequencies of the rubidium electron magnetic moments to generate after the predetermined time delay, an error signal for opposing the angular displacement of the containers about the Y axis.

By so doing we can provide stabilization, with respect to inertial space, of the containers, about the Y axis. The time delay is adjusted to permit sufficient time for a field correction to take place. How this is implemented will be explained as the description of Fig 8.5.3 continues.

The precessional frequencies of the magnetic moments are detected by directing a pumping beam similar to that supplied by source 17 at right angles to the magnetic field. In this instance the intensity of the pumping beam as it traverses the container is modulated by the precessing magnetic moments of both the sodium and rubidium electrons by the well known absorption process. Thus both precessional frequencies are impressed on the light which passes through the containers in the form of an amplitude modulation. Two light sources 26 and 27 are located adjacent to containers 14 and 15, respectively, and arranged so that the light energy is directed toward the container with which each is associated. A pair of photocells 28 and 29 are located adjacent to containers 14 and 15, respectively, opposite light sources 26 and 27, respectively.
To facilitate the processing of the photocell outputs we can secure the desired 180° relationship of the output signal by placing the light sources 26 and 27 so that each source is displaced the same angular distance from coils 20 and 24, respectively, and the same distance from coils 21 and 23, respectively. In order to do this it may be necessary to rotate the coils about the Y axis so that the distances will be as specified but such movement does not otherwise affect the output. The modulated light passing through container 14 is detected by a photocell 28 and an electric signal which is its analogue is supplied to an amplifier 31 where it is amplified and undergoes a 180° phase shift.

The modulated light passing through the container 15 is detected by a photocell 29 and an electric signal which is its analogue is supplied to an amplifier 32 where it is amplified and undergoes a 180° phase shift. A pair of band-pass filters 34 and 35 are connected to the output of amplifier 32. Filtering 34 is centered at the Larmor frequency of the sodium electron and passes only the frequency component of the photocell output contributed by the precession of the sodium electron magnetic moment. Filter 35 is centred at the Larmor frequency of the rubidium electron and passes only the frequency component of the photocell output contributed by the precession of the rubidium electron magnetic moment. The output of band-pass filter 34 is connected to coils 24 and 20 to supply the alternating magnetic field which produces forced precession. It should be noted that this field leads the sodium electron magnetic moment precession by about 90° since the 180° phase shift introduced by amplifier 32 in combination with the 90° lag of the current in coils 24 and 20 result in about a 90° lead of the alternating field. The output of band-pass filter 35 is connected to coils 23 and 21 to provide an alternating field for forcing the precession of the magnetic moments of the rubidium electrons.

Another pair of band-pass filters 36 and 37 similar to filters 34 and 35, respectively, are connected to the output of amplifier 31. Filter 36 is centred at the Larmor frequency of the sodium electron and passes only that frequency component of the output of photocell 28 which is contributed by the precession of the sodium electron magnetic moment which is precessing in container 14. Filter 36 is centred at the Larmor frequency of the rubidium electron and passes only that frequency component of the ouput of photocell 28 which is contributed by the precession of the rubidium electron magnetic moment which is precessing in container 14.

It has been previously pointed out that the difference in the precessional frequency of the magnetic moments of the similar electrons in containers 14 and 15 which might be due to either a difference in field strength or an angular velocity of frequency (ω) about the Y axis is slight compared to the Larmor frequency. Therefore, some form of comparison other than frequency which will give the frequency difference is necessary. Here this takes the form of a phase comparison. When two alternating signals of very high frequency differ in frequency by a small amount the difference may be measured as a phase shift which is direct proportional to the difference in frequency.

The output of band-pass filter 34, curve (a) is amplified by an amplifier 40 whose output, curve (b), is applied to a limiting circuit 41 which provides a square wave output, curve (c). A differentiating circuit 42 differentiates the leading and trailing edges of the limiter output to provide pulses, curve (d), at the zero voltage or axis crossings of curve (a). A half-wave rectifier 43 clips the negative pulses to provide a single pulse once each cycle occuring at one zero voltage level or axis crossing and illustrated by curve (e). The output of rectifier 43 is amplified by an amplifier 44 and then limited by a second limiter 45 to provide a pulse, shown in curve (f), which has a uniform width and height and which has a very sharp rise and fall. An amplifier 46 amplifies the pulse output of limiter 45 and applies the amplified pulse to a primary winding 48 of a transformer 49. Transformer 49 has a pair of secondary windings 50 and 51 which are oppositely wound so that the pulse outputs of the two windings are of opposite polarity. The pulses are applied to terminals 53 and 54, respectively, of a bridge circuit 55. The output of band-pass filter 36 is applied to another terminal 57 and the bridge is so arranged that the instantaneous voltage of the output of filter 36 is passed through the bridge to an integrator 59 each time pulses are applied to terminals 53 and 54. Thus, if the outputs of filters 34 and 36 are of the same frequency and 180° out of phase with each other, which is the case when fields H_1 and H_2 are equal and containers 14 and 15 are stabilized with respect to inertial space about the Y axis, the voltage at terminal 57 will be zero when the pulses are applied to terminals 53 and 54. If however, fields H_1 and H_2 differ in strength or the containers experience an angular velocity of frequency (ω) about the Y axis, the frequencies will differ and the voltage present at terminal 57 when the pulses are applied to terminals 53 and 54 will be at some value greater than or less than zero depending on the relative change in fields or the relative direction of the angular velocity about the Y axis and integrator 59 will integrate this voltage and apply it to a motor 60 through a servo amplifier 61 to adjust the voltage across winding 7 and the strength of field H_2. If the difference in frequency is due to a field difference the integrator output applied to motor 60 via amplifier 61 will adjust potentiometer 11 to adjust field H_2 to that it equals field H_1 and the integrator output will go to zero after a minimum period of hunting with a properly designed servo system.

Bridge 55 comprises two pairs of oppositely connected diodes connected between terminals 57 and 57A and a pair of biasing batteries 63 and 64 connected between terminals 53 and 54. The common junction of batteries 63 and 64 is connected to ground and thus the terminals 53 and 54 are positive and negative, respectively, so that the diodes connected between terminals 53 and 57, and 54 and 57 are back biased except when pulses from windings 50 and 51 are applied to terminals 53 and 54, respectively. The voltages of batteries 53 and 54 must be selected so that they individually exceed the maximum value of the voltage applied at terminal 57 and low enough so that the same diodes will be forward biased whenever the pulses from windings 50 and 51 are applied to terminals 53 and 54, respectively. The output of band-pass filters 35 and 37 are applied to similar phase comparison circuits wherein identical numbers bearing a prime designate similar components. In this instance, however, servo motor 60' is mechanically coupled to the structure bearing containers 14 and 15 to rotate that structure about the y axis so as to oppose any angular motion about the y axis and maintain stability of the support structure about the y axis with respect to inertial space. As was previously mentioned the time constant of the servo system which includes motor 60 is less than the time constant of the servo system which includes motor 60'. This permits a field correction prior to any platform correction and since no amount of field correction will null both frequency differences if they are the result of an angular velocity of frequency (ω) about the Y axis, servo motor 60' will eventually take over and restore the support structure to null the system. This arrangement eliminates a certain amount of oscillation in the system and contributes greatly to smoothness of operation.

With the bridge circuit described for measuring the phase shift it is possible to detect without ambiguity phase shifts of as much as ±90°. This is more than adequate to provide effective control since it corresponds at 90° phase shift to an angular velocity about the Y axis having a frequency of about 50 r.p.s. The system will ordinarily operate on the substantially linear portion which permits accurate and substantially linear correspondence between the phase shift and the voltage applied to integrators 59 and 59' during normal operation of the system.

Bridges 55 and 55' are arranged so that the polarity of the pulses supplied to integrators 59 and 59' is dependent on the direction of the phase shift which is in turn dependent on either the direction of the angular velocity of support structure about the Y axis with respect to inertial space or on which of fields H_1 or H_2 is the greater. Thus, the polarity of the output of integrator 59 determines whether or not the field will be increased or decreased and the polarity of the output of integrator 59' determines whether or not the support structure will be turned clockwise or counterclockwise about the Y axis. In both cases, however, if the frequency of the voltage applied to terminals 57 and 57' increases, the bridge output will be negative to secure one type of correction and if the frequency decreases the polarity of the bridge output will be positive to secure the opposite type of correction. In the case of bridge 55, a positive output will cause a decrease in the field H_2 by rotating motor 60 in such a way to as to increase the resistance of potentiometer 11 to reduce the current through winding 7. Conversely, if the output of bridge 55 is negative, motor 60 will be rotated oppositely to decrease the resistance of potentiometer 11 and increase the current through winding 7 to thus increase the strength of field H_2 and null the bridge output. In the case of bridge 55', a positive output indicates an apparent decrease in the frequency of the rubidium electron magnetic moment precessing in container 14 with respect to those in container 15 which is due to a clockwise rotation of container 14 about the Y axis. Therefore, motor 60' is arranged to rotate the support structure counter-clockwise to oppose the rotation whenever the bridge output is positive. On the other hand, a negative output indicates an apparent increase in the frequency of the rubidium electron magnetic moment precessing in container 14 with respect to those in container 15 which is due to counterclockwise rotation of container 14 about the Y axis. Therefore, motor 60' is arranged to rotate the support structure clockwise to oppose the rotation whenever the bridge output is negative.

GREENWOOD. I.A. US 3103623/4 (1963) provides a nuclear gyroscope using one or at least two unidirectional magnetic fields cooperate with an aggregate containing two selected kinds of atomc nuclei. He gives examples of pairs of nuclei that may advantageously be used, such as the nuclei of fluorine of mass number 19 and of hydrogen. These have the nuclear constants 4007 and 4257 cycles per second per gauss. Hydrogen and phosphorus nuclei in an aqueous solution of orthophosphoric acid are another suitable pair of nuclei having constants 4257 and 1723 cycles per second per gauss. A third suitable pair consists of the nuclei of deuterium and of hydrogen exhibiting the constants 653.6 and 4257 respectively. One special form of the gyroscope is discussed in general terms by SIMPSON, FRASER and GREENWOOD (1963), the resonant nuclei being mercury 199 and mercury 201 contained in a fused quartz absorption cell situated in a homogeneous d.c. magnetic field produced by Helmholtz coils.

A not dissimilar gyroscope is the subject of a disclosure by GREENWOOD. I.A. US 4104577 (1978) again using the two odd isotopes of mercury ie 199Hg 201 H_g, but with an absorption cell having surface elements the normals of which simultaneously form angles of about 55 degrees of arc with lines parallel to the alignment axis that is itself parallel to a unidirectional magnetic field.

Another nuclear magnetic resonance gyroscope is due to GROVER. B.C., KANAGSBERG. E., MARK. J.G. and MEYER. R.L. US 4157495 (1979) in which the NMR cell contains an alkali metal vapour, such as rubidium, together with two isotopes of one or more noble gases, such as Krypton-83 and Xenon-129. A buffer gas such as helium may also be contained in the cell. The NMR cell is illuminated by a beam of circularly polarized light that originates from a source such as a rubidium lamp and which passes through the cell at an angle with respect to the steady magnetic field. Absorption of some of this light causes the atomic magnetic moments of the ribidium atoms to be partially aligned in the direction of the steady magnetic field. This alignment is partially transferred to the nuclear magnetic moments of the noble gases and these moments are caused to precess about the direction of the steady magnetic field, which in turn creates magnetic fields that rotate at the respective Larmor precession frequencies of the two noble gases.

These rotating fields modulate the precessional motions of the rubidium magnetic moments which in turn produces corresponding modulations of the transmitted light, thereby making it possible optically to detect the Larmor precession frequencies of the two noble gases. The authors refer to seven related US patent specifications: 3103623, 3103624, 3396329, 3404332, 3500176, 3513381, 3729674.

ALEKSEEV. B.F. BELONOGOV. A.M. and GRAMMAKOV. A.G. et al Russian 428-268 (1970) discuss means for measuring the rotation of gyroscopic instruments using nuclear magnetic resonance techniques and a paramagnetic working element.
The invention concerns gyroscopic devices used in navigation systems. The proposed nuclear hydro-tachometric device, based on the effect of nuclear magnetic resonance is an improved version of a conventional one in that the effect of the outer magnetic field variations on the working of the device is reduced, and the accuracy measurement is increased. These objectives are achieved by using a paramagnetic element such as solution of sodium in ammonia, in which the working nuclei are polarised by means of saturating the electron transitions with a frequency different from the resonance frequency for a given intensity of the magnetic field. The chosen magnetic field intensity corresponds to the quasi-flat section of the produced absorption signal of the nuclear magnetic resonance. The value and sense of the power absorbed by the nuclei and expressed as rotation, are coupled to the value and sense of the rotational velocity of the object under investigation.

Susan. P. POTTS and J.C. GALLOPHAVE assigned to the Secretary of State for Defence GB 2007847 (1979) a cryogenic nuclear gyroscope in which a cylinder of niobium is cooled within a helium cryostat so as to be superconducting and to provide a substantially homogenous magnetic field, within it is a helium-3 sample having nuclei possessing a net magnetic moment. Coils provide a polarization of the sample and a superconducting quantuminterference device (SQUID) using a Josephson- like barrier is coupled to the sample to detect changes in the precession of the nuclear moments of the sample caused by rotation of the gyroscope about an axis parallel to the direction of the homogeneous magnetic field.

KELLOGG and MILLMAN (1946) published the nuclear moments of some twenty six nuclei as shown in the table below.

Nuclear moments measured by the molecular beam magnetic resonance method

Nucleus	Observed g value	Spin i	Moment	Diamagnetic Correction %
H^1	5.5791 ± 0.0016	1/2	+2.7896	0
H^2	0.8565 ± 0.0004	1	+0.8565	0
Li^6	0.8213 ± 0.0005	1	+0.8213	0.01
Li^7	2.1688 ± 0.0010	3/2	+3.2532	0.01
Be^9	0.784 ± 0.003	3/2*	-1.176	0.02
B^{10}	0.598 ± 0.003	1*	+0.598	0.03
B^{11}	1.791 ± 0.005	3/2*	+2.686	0.03
C^{13}	1.402 ± 0.004	1/2*	+0.701	0.03
N^{14}	0.403 ± 0.002	1	+0.403	0.04
N^{15}	0.560 ± 0.006	1/2	0.280	0.04
F^{19}	5.250 ± 0.005	1/2	+2.625	0.06
Na^{23}	1.4765 ± 0.0015	3/2	+2.215	0.08
Al^{27}	1.452 ± 0.004	5/2	+3.630	0.10
Cl^{35}	0.547 ± 0.002	5/2*	+1.368	0.14
Cl^{37}	0.454 ± 0.002	5/2*	+1.136	0.14
K^{39}	0.260 ± 0.001	3/2	+0.391	0.16
K^{40}		4	-1.290	0.16
K^{41}	0.143 ± 0.001	3/2	+0.215	0.16
Kr^{83}	0.2148	9/2*	-0.967	0.38
Rb^{85}	0.536 ± 0.003	5/2	+1.340	0.39
Rb^{87}	1.822 ± 0.006	3/2	+2.733	0.39
In^{113}	1.22 ± 0.01	9/2	+5.49	0.58
In^{115}	1.22 ± 0.01	9/2	+5.50	0.58
Cs^{133}	0.731 ± 0.002	7/2	+2.558	0.67
Ba^{135}	0.554 ± 0.002	3/2	+0.831	0.68
Ba^{137}	0.619 ± 0.002	3/2	+0.929	0.68

* Nuclear spins uncertain

A Table of the characteristics of various types of gyroscope

Name of Gyroscope	Type of Suspension	Basic Characteristics	
		Kinetic Moment	Drift Value
Electro-mechanical	Rotorace bearings	Usually bronze	0.1 - 0.4 degrees/hr
Float	Combination mechanical and hydro-static	Rotor, bronze, beryllium, etc.	0.01 - 0.03 degrees/hr
With electro-static suspension	Electro-static	Hollow sphere, beryllium	10^{-4} - 10^{-5} degrees/hr
Cryogenic	Electro-magnetic	Hollow sphere, titanium and nobium	Drift deter-mined by sensitivity threshold of final am-plification devices
Hydro-dynamic	Bearings	Liquid of high specific gravity	0.01 degrees / hr
Magneto-hydraulic	Mechanical	Liquid, usually mercury	Drift inversely proportional to dimensions
Corpusular	No suspension	Particles of helium and other inert gases	Drift 10^{-4} - 10^{-8} degrees/hr
Gyrotron	No rotating parts	Elastic inertial masses	
Laser	No rotating parts	None	No drift; sensitivity up to 0.01 degrees/hr
Electro-vacuum	Suspension under weight-lessness	Spherical rotor	0.01"/year
Gyroscope operation of which is based on eddy prin-ciple	No suspension	Liquid or gas	
Helitron Josephson ring	No suspension No rotating parts	Plasma Electron flux	10^{-4} - 10^{-5}
Nonreasonant ring, using radiowaves	No suspension No rotating parts	None	0.01-0.001 degrees/hr
Nuclear	None		Sensitivity better than 0.01"/year

9. GYROSCOPIC GEARS AND TRANSMISSIONS

The idea to use the peculiar forces and characteristics of the gyroscope to minister to the needs of a variable gear transmission was reserved to RUSSELL GB 12361/1905. His proposal, Fig 9.0, that was to prove seminal to engineers over some three decades, was to make use of the principle that when a longitudinal member turns rapidly end-over-end and also serves as an axis for a rotary flywheel, then the end-over-end motion of the longitudinal member opposes the rotation of the flywheel upon its axis. For example, if a gyroscopic top be spinning upon a stationary axis and that axis then be turned end-over-end, or if the axis be quickly inverted, the top is almost instantly stopped, no matter in which of two directions it may be rotating.

The gear wheels considered as a unit behave a good deal as if they were connected by a thick viscous material permitting of but little relative movement. Similar devices have continued to provide instructive mechanisms and some of these are presented in the works of GOODER GB 221725 (1924) and PALMER US 1966357 (1934).

9.1 TYPES OF DEVICES

SEILLIERE GB 22418/1911 had proposed a gyrotransformer using four gyroscopic flywheels and HUNT GB 107251 (1916) used an inclined axle to the main flywheel which reappears in the device of CANTIER GB 1421309 (1976). Meantime French engineers had produced transmissions not dissimilar to the original ideas of RUSSELL as can be seen in the mechanisms of de MARTINO FRENCH 774746 (1934) Fig 9.1.1 and KAST FRENCH 918516 (1945).

LINDSAY GB 137205 (1920) also proposed a system similar to that of RUSSELL but with a simple shaft for the opposed flywheels.

MAUVILLIER GB 204062 (1924) and BIDOIRE GB 256230 (1926) separately both proposed a differential or balance gear rendered automatic by gyrating members. BIDOIRE was able to quantify the gyroscopic couple as $C = \frac{2\pi^2 MR^2 N^2}{n}$.

His short mathematical dissertation is as follows:- Let I represent the moment of inertia of a circular member of a mass of M kilogrammes and a radius of R metres. The value of I is given by the relation $I = \frac{MR^2}{2}$ that of ω, for a number N of revolutions per second is given by $\omega = 2\pi N$. If $\underline{n}$ is the ratio of reduction due to the presence of wheel and pinion the speed ω^1 is connected to the speed ω by the relation $\frac{\omega}{\omega^1} = \underline{n}$, whence we deduce $\omega^1 = \frac{2\pi N}{n}$.

The value of the gyroscopic couple is therefore given by the relation for C expressed above. The variation of the couple C is therefore linear relative to the mass M. By giving to the quantities R, N and $\underline{n}$ considered as parameters, fixed values and by causing the quantity M to vary, we obtain a network of straight lines and by giving to R.N and $\underline{n}$ in succession different values we superimpose on the first network a number of networks of straight lines and we obtain an abacus giving the values of the gyroscopic couple for fixed values of the quantities M. R. N and $\underline{n}$.

BIDOIRE uses two similar units, each unit has a primary gyro and housed in this is a secondary gyro. With each rotation of the primary gyro the reaction torque of the secondary gyro changes in direction, consequently the torques transmitted to the two driven axles vary all the time between maximal and minimal values, and the drive produces pulsating torques.

TURNER GB 238423 (1924) had already proposed a similar device combined with epicyclic gears and WOOLER GB 450360 (1936) advanced the idea of increasing and decreasing, with the output demanded by the gear transmission, the weight of the gyroscope *per se* by the use of a fluid fed to hollow rotors.

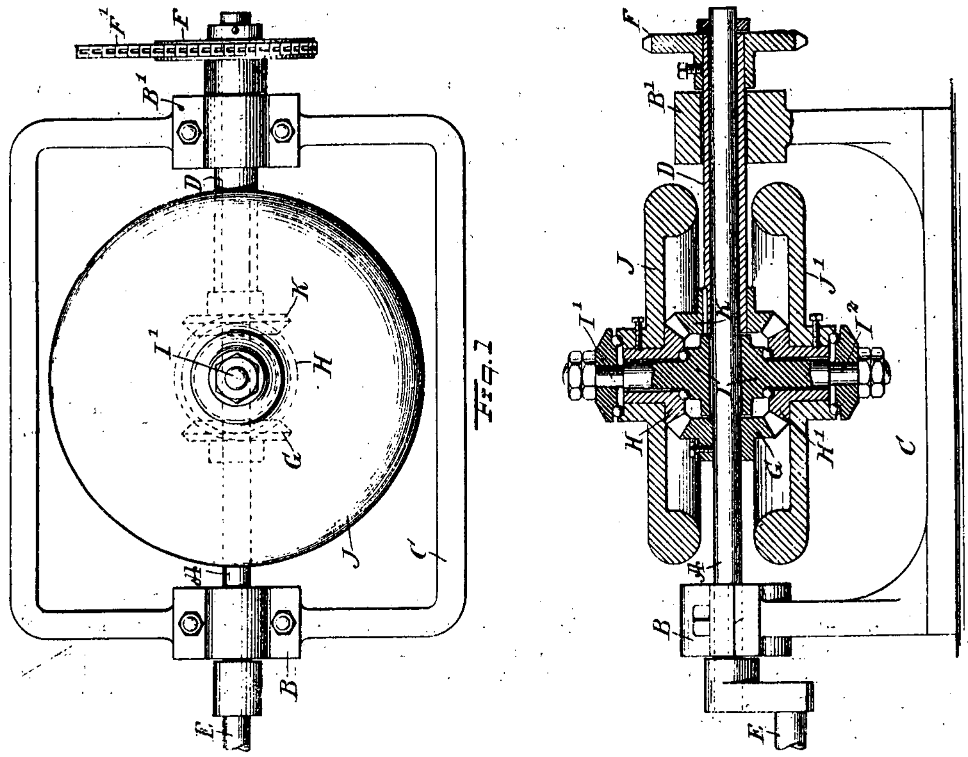


Figure 9.0

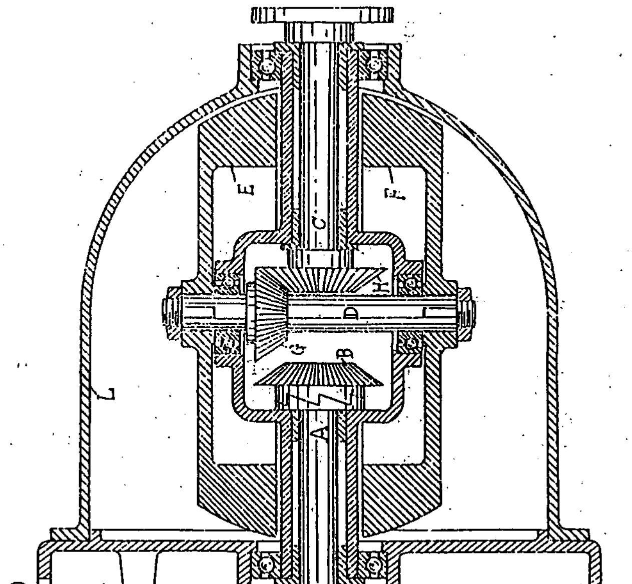


Figure 9.1.1

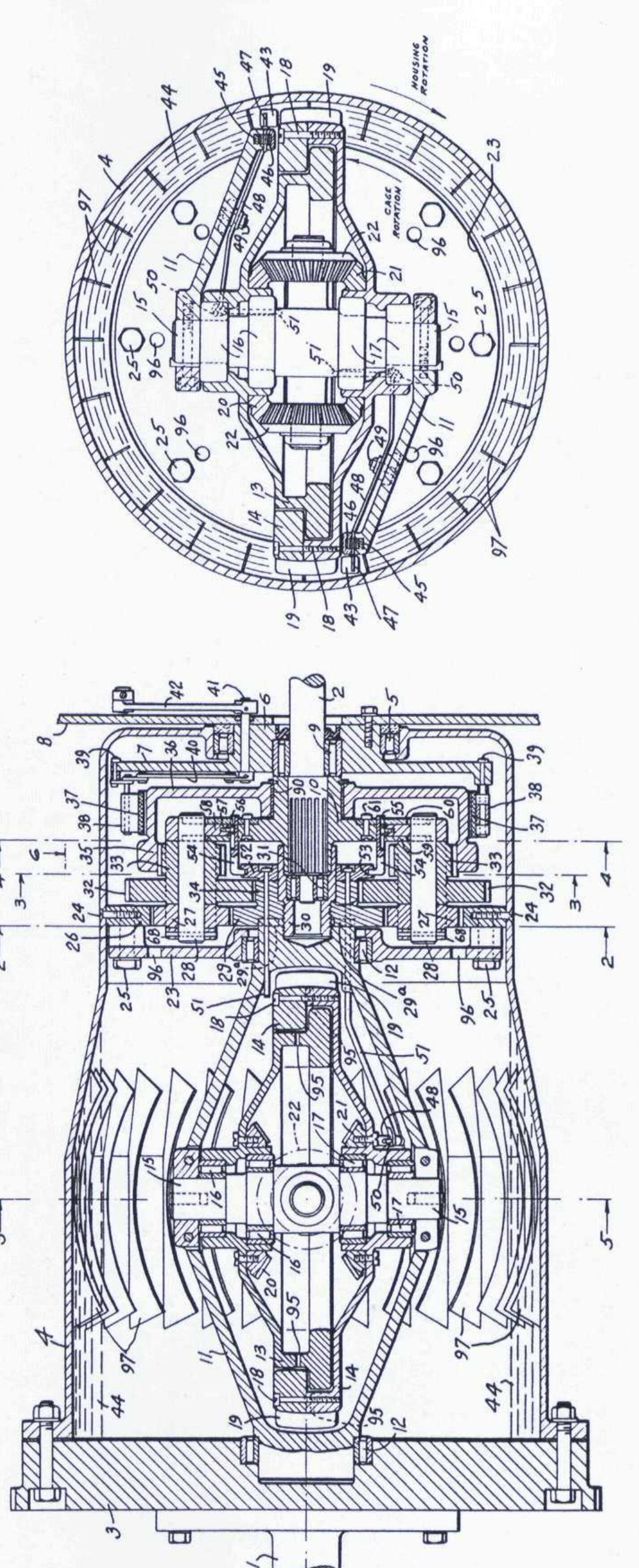


Figure 9.1.2

LENOX GB 510518 (1939) proposed coupling a power shaft through differential gearing to a rotary cage and flywheel, the gyroscopic effects produced by rotation of the flywheel about an axis normal to the cage operating to vary the relative speeds of rotation of the differentially driven shafts. A more sophisticated variable gyroscopic gear was proposed by STEIN et al US 2296654 (1942), yet in OSGOOD US 2571159 (1951) we find a reversion to a very simple structure. VOIGT GB 695671 (1953) proposed the use of a gear drive having built-in gyroscopic masses arranged above and below the axes of the drive shafts, the masses being circular and telescopically intercalated at the rim so that the rim of each mass is rotatably mounted on the hub of the other mass. FINCH US 2744422 (1956) allows a pair of nested gyroscopic rotors to be attached to a cage directly on the flywheel of the driving shaft. The outer gyroscope carries vanes for cooperating with a hydraulic fluid. The device is shown in Fig 9.1.2 from which it can be understood that rotation of the housing 4 will cause an increasing rotation of the driving fluid 44 and if the control vanes 43 are even at a slight angle to the vanes 19 on the outer gyroscopic rotor, a tendency to rotate the rotors will be apparent. This rotation will obviously be in a plane at right angles to the plane of the section of the housing 4 and about the axis 15. The beveled gears 20 and 22 will move both the rotors, resulting in a proper balancing effect. At the same time, the housing 4 is rotating about an axis defined by the axes of the shafts 1 and 2 and the internal gear 26 is correspondingly rotated. The planetary gears 27 will be accordingly rotated about the sun gear 29. At the same time, the gyroscopic rotors are rotating about their supporting shaft 15 and any tendency of the sun gear 29 to turn the supporting cage 11 of the gyroscopic rotors will be resisted by the gyroscopic action, wherein the plane of rotation of the same tends to remain fixed. The sun gear will accordingly be rotated at an increasingly higher speed about its own axis and will impart increasing rotation to the driven shaft 2. Rotation of the housing 4 also tends to rotate the planetary gears 27, because they are in mesh with the internal gear 26 fastened to the housing. Part of this driving motion is transmitted to the driven shaft 2 by way of the sun gear 29 and this torque is in the same direction as the rotation tendency imparted through the cage 11 by the gyroscopic action. As the shaft 2, connected to the load, gains momentum, due to the gyroscopic and fluid action on the vanes, the torque required to propel shaft 2 decreases to an amount where the gyroscopic action approaches the moment of greatest efficiency, the planetary gears and the sun gear arrive at such a speed relationship to each other, that when this condition exists, the housing 4 rotates the driven shaft 2 at the same speed. This creates a direct drive between the shafts 1 and 2. The power ratio between the shafts varies automatically as the slippage, due to the gyroscopic action, and when the torque required to drive shaft 2 increases sufficiently to cause gyroscopic action, the power ratio will increase as the torque required increases. An over-drive relationship between the shaft 2 and the driving shaft is also often accomplished by this mechanism.

Two not wholly dissimilar gear transmissions were advanced by VOIGT US 3153353 (1964) and by RASS US 3495479 (1970) both having to some extent been exploited earlier by SCHAFER GERMAN 210296 (1909) and 227068 (1910).

An improved power transmission due to PLOGER US 2877667 (1959) highlighted the main problem in such transmissions, namely that a flywheel changes the direction of its gyroscopic torque every time it revolves through two right angles, thus no unidirectional torque can readily be obtained. We shall deal below more fully with this matter, but first let us look briefly at the reversible and continuously variable torque converter of PLOGER since it resembles some of the gear transmissions described above, yet leads beyond them to torque converters *per se*. What PLOGER introduces via a lever and shaft system is a means such that each time during operation of the drive the distance between each gyro and the centre of mass of the supporting housing is varied continuously. This is necessary in order to ensure that the total torque applied about the axis of rotation of the frame is always in the same direction. The resulting disadvantage is that the cyclic changing of the moments of inertia of the gyros by shifting masses results in costly complications. Moreover a pulsating torque is obtained, because each individual resulting torque depends in the first place on the changing angular position of the gyro spin axis and secondly, on the moment of inertia of the gyro which changes in response to a particular function. This pulsation necessarily prevents the drive from functioning quietly and promotes undesirable abrasion in the moving parts.

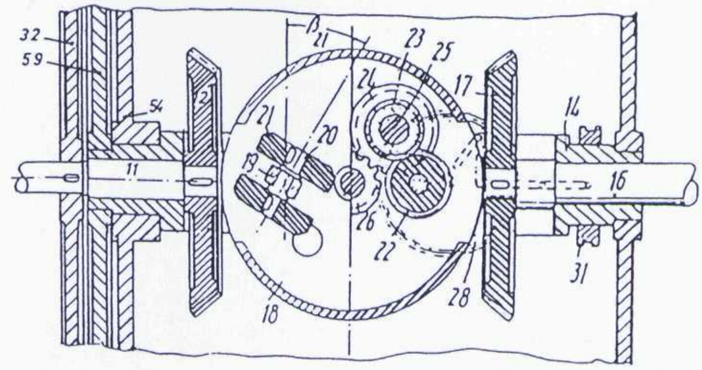


Figure 9.1.3

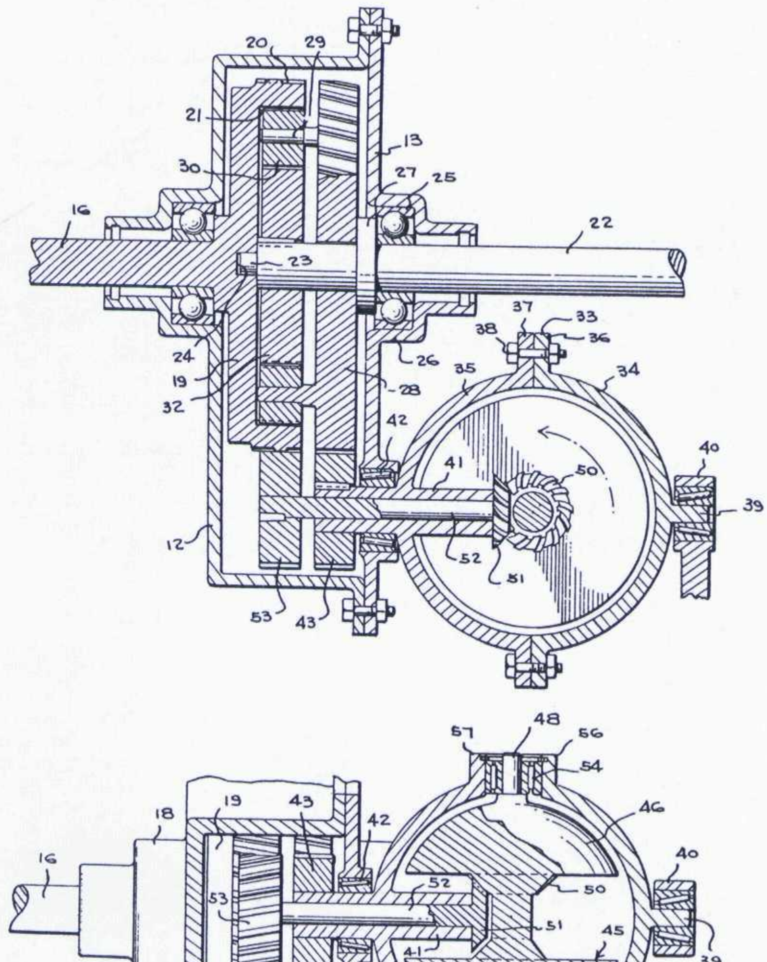


Figure 9.1.4

GUMLICH US 3851545 (1974) saw the disadvantages of the transmission of PLOGER and also that of BIDOIRE. He proposed a constant torque transmission by ensuring that the gyro axes remained parallel to themselves, that is to say the angle between the gyro spin axis and the precession axis remain unchanged during operation.

GUMLICH achieved this desideratum by using a plurality of coupled gear wheels (Nos. 22. 23. 24 and 26 see Fig 9.1.3) of which the radii are related according to the expression

$$r22 = \frac{[r23\ (r24 + r26)]}{r26}$$

He was, however, partly anticipated by BRIGGS US 2088834 (1937) who also saw clearly the need for structural parallelism of the gyro axes and stated it succinctly in the following terms.

The present type of transmission is based upon, and makes use of, the variable resistance to the change of direction of the axes of a gyrostatic wheel or rotor which is mounted in a revoluble carrier supported in a revolving frame. If one or more gyrostatic wheels, or rotors, are so mounted in such a revolving frame or driver with their individual axes of rotation arranged in a plane parallel to, or included in, a plane that is transverse to the axis of revolution of the frame or driver and be mounted in a support or carrier that is free to revolve about an axis at right angles to the axis of the gyrostatic rotor wheel but parallel with the axis of revolution of the frame, so that the gyrostatic wheel or rotor is capable of maintaining complete parallelism of its axis throughout its entire path of revolution, along with its carrier, the gyroscopic element may be coupled constantly with a driven shaft, preferably arranged in coaxial alignment with the axis of revolution of the gyroscopic driver. This constant coupling between each gyroscopic element comprising a rotor and its revoluble carrier member and the load shaft that is to be driven thereby, may be utilized to cause rotation of the driven shaft against resistance or load at different rates of speed according to the strength of the opposition that the rotor opposes to any change in the direction of its own axis. The greater the speed and the mass of the rotor, the greater will be the torque exerted upon the driven shaft, while at the same time with this arrangement, increase in load while the device is running, by affording a greater resistance to the tendency of the rotor to maintain parallelism of axis, tends to partially overcome the resistance of the rotor to any change in direction of axis, thereby lessening the speed of rotation of the load shaft to compensate for heavier loads.

The use of a rotatable spherical mass with a special lissajou slot to permit some degree of precession was exploited by RICHARDS GB 416032 (1934) and by KEYSER US 2960889 (1960), the latter using a plurality of spheres as gyroscopic mass, each sphere having a circumferential groove and a cooperating pivot pin to form a universal connection for rotating the gyroscopic mass about an axis tiltable with respect to the axis of rotation of a yoke. A cam rider is held in an annular track and rotation of the gyroscopic mass forces the cam rider against the annular track to produce a component of force on the gyroscopic yoke that tends to resist rotation of a torque reaction cage so that torque is transmitted from the drive shaft to the driven shaft.

In the transmission of PRICHARD US 2811050 (1957) we see the same basic idea as that of RUSSELL exploited in a manner allowing a massive gyro rotor to combine with an internally and externally toothed bell gear and associated planetary gears. It will be readily apparent from Fig 9.1.4 that if there is a difference in speed between the input and output shafts, the planetary gears 30 will tend to travel around the sun gear 32 and will thus apply a torque to the idler 28 which will be transmitted through the gear 43 to the gyro housing 33. However, the massive gyro rotor 45 rotates at a speed in accordance with the speed of rotation of the input shaft 16 and creates a resistance to the tendency of the gyro housing to rotate in accordance with the input shaft speed. The resistance of the rotor housing 33 to rotate freely is transmitted back through the gear 43 and the idler gear 28 to the planetary gear shafts 29 and acts to restrain the movement of the planetary gears 30 around the internal gear teeth 21 of bell gear 19 and thus allows the bell gear 19 to transfer torque to the driven gear 32 at a ratio in accordance with the amount of resistance to rotation imposed on the idler gear 28 by the gyro housing 33.

It will thus be seen that the load, transferred from the input shaft 22 to the gear 32, is transmitted back through the gyro housing 33, developing a tendency for the gyro housing 33 to rotate, which tendency is opposed by the rotating gyro member 45, the degree of opposition depending upon the speed of rotation of the input shaft 16. This is reflected as a restraint on the freedom of gear 28 to rotate and hence as a restraint on the freedom of the planetary gears 30 to move around the sun gear 32. The restraint on the free movement of the planetary gears allows torque to be transmitted from the bell gear 19 to the sun gear 32 to sustain the load.

A departure is made from this well known construction in the mechanisms of WHITLOW US 2310724 (1943), REMINGTON US 1760850 (1930) and CASCAJARES GB 744645 (1956) who chose to use the force necessary to upset the gyroscopic balance in a flywheel or rotor disc to produce a check to the free rotation of a member to produce an inertial override. REMINGTON, provides in combination with an epicyclic train having a movable ratio-controlling device, a gyroscope driven from the source of power of the train and operatively connected to the ratio-controlling device to be deflected relative to its own axis and thus yieldingly opposing the motion of the ratio-controlling device. CASCAJARES, uses a frustoconical flywheel as a gyroscopic rotor that tends, with increase of speed, to reach a stabilized position and in so doing force a ring to resist the free rolling of a planetary gear and give a direct drive thereby to the driven shaft. WHITLOW, uses two tilted rotors, the gyroscopic balance of which is upset to provide braking forces on a rotable casing that is coupled to the shafts by complex gearing.

Whether all of these constructions are intrinsically sound is open to doubt since none has proved to be commercially viable for any length of time. One of the major difficulties is to allow in any mechanism the gyroscopic forces of precession to exhibit themselves in a full and useful manner. It is to this refinement that STERN and LARSEN GB 455963 (1936) directed themselves. They point out that in known planet wheel gearing of the type in which the planet wheel carrier is revolvable about coaxial shafts of a driving and a driven sun wheel, one or more gyroscopes being arranged on the carrier so as to be driven at a speed of rotation dependent upon the speed of revolution of the carrier, the axes of rotation of the gyroscopes being arranged at an angle to the sun wheel shafts and in which the sun wheels and planet wheels are constituted by bevel gear wheels; the gyroscopes are rigidly connected directly to the planet wheels, the axes of rotation of which are radially arranged and stationary with respect to the planet wheel carrier. (They may have in mind the construction of GOODER GB 221725 (1924) but see also EASTON GB 595866 (1947)). Through the autorotation of the gyroscopes together with the planet wheels, a damping force is produced on the planet wheel carrier for the purpose of adapting the ratio of the gear to the load on the driven shaft, which force counteracts the movement of the planet wheel carrier, and which will be the greater the faster the gyroscopes rotate about their own axes. As a matter of fact, however, this effect cannot be obtained in the desired degree. The gyroscopes in their capacity as inertia masses can only exert small forces on the planet wheel carrier or on the other parts of the gearing according to the inertia of the mass counteracting the acceleration or retardation of the said masses. Thus an automatic adaptation of the gearing to the load on the driven shaft in every instance will not be possible. Besides, the gyroscopes in addition to their rotatability about their own axes and about the axis of the planet wheel carrier must have a third degree of freedom, in order that the gyroscope may precess. This third degree of freedom, however, cannot be realized on the basis of the known planet wheel gearing. Besides, when the planet wheels are used to drive the gyroscopes directly, it will be difficult to attain the necessary high rotational speeds of the gyroscopes. They continue their analysis of the difficulties and drawbacks experienced in the mechanisms of the earlier art explaining that the utilization of the gyro effect has been proposed also for planet wheel gearings of the type where the planet wheel carrier is rigidly connected with the driving shaft, and where the planet wheels cooperate with a single sun wheel arranged on the driven shaft. In one construction of this type of gearing, the driving shaft, for instance, is provided with radial arms connected each at their ends to a separate gyroscope by a ball and socket-like joint, so that each gyroscope will be swingable about its own axis as well as in all directions, within certain limits, relatively to the appertaining supporting arms. Rigidly connected to each gyroscope is a bevel gear meshing with a bevel sun wheel on the driven shaft. The teeth of the bevel gears must be worked in such a manner as to permit the limited swingable movements of the gyroscopes without causing mutual jamming. This arrangement aims at a direct impulse-like operation of the central bevel wheel of the driven shaft through the movements of precession of the gyroscopes caused by continuous alterations of the direction of the axes of the gyroscopes at the rotation of the driving shaft. This effect, however, is not obtainable in a sufficient degree on account of the direct locking of the gyroscopes against lateral tilting cuased by the friction of the teeth; on the contrary, the gyroscopes will tend to adjust themselves into a constant position of the axes relatively to the radial arms of the driving shaft, so that no appreciable transmission of power can take place from the planet wheels to the sum wheel on the driven shaft.

In a further construction of the same type of gearing, a single conical planet wheel is arranged in the planet wheel carrier connected with the driving shaft, which planet wheel serves to operate a spherical gyroscope arranged centrally within the planet wheel carrier and suspended by a universal joint therein. In this gearing, power transmission from the driving onto the driven shaft can take place only when the rotation of the planet wheel with respect to the planet wheel carrier is braked. This braking effect cannot set in, however, as by reason of its universal suspension the spherical gyroscope cannot exert any inertia effects on the planet wheel carrying the same. The gyro effect at any rate will be perceptible only in so far as the rotary movement of the planet wheel carrier and thus of the driving shaft is braked up. Also the improvements which have likewise been proposed to the effect that the central gyroscope be given a positive own rotation in dependence on a second planet wheel, about an axis which is stationary relatively to the first planet wheel, will make no difference in this respect. Here, the gyroscope performs three different rotary movements at the same time, but still it cannot bring about the braking effect required for the power transmission, inasmuch as it has no third degree of freedom relatively to the planet gears. Disregarding this fact, the construction of this gearing is a very complicated one.

Considering the type of gearing described in the first paragraph of the specification, see Fig 9.1.5, each gyroscope according to the invention in addition to its rotatability about its own axis and the axis of rotation of the planet gear carrier is adapted to be turned within certain limits with respect to the planet gear carrier about an axis extending at an angle to the two first-mentioned axes, the gyroscopes being then driven from one of the sun wheel shafts by means of an intermediate gearing separate from the planet wheel carrier. In this way only will a sufficient inertia effect of the gyroscope be obtained, which solely occurs at the planet wheel carrier rotatable per se, and which may in its entirety be utilized in supporting the power transmission. The most advantageous effect will be obtained, when the additional axis of rotation of the gyroscope is arranged tangentially to the circular path described by the centre of gravity of the gyroscope. With respect to its characteristics the novel gearing is comparable to the ship gyroscope according to Schlick. In the novel gearing, the influence of the gyroscope on the planet wheel carrier corresponds to the damping effect of the gyroscope on the rolling ship.

9.2 APPROACH TO UNI-DIRECTIONAL TORQUE

We have noted that PLOGER was concerned with an approach to a uni-directional torque; but earlier references to the problem and different solutions are offered by ANDERSON & HOWE GB 414693 (1934), VAN ASPEREN GB 422577 (1935), WALTON GB 437950 (1935), STALKER US 2223745 (1940), BIASI GB 622337 (1949) and TAYLOR US 2639631 (1953).

ANDERSON & HOWE in an early torque converter, to be described more fully below, noted that during one half of a revolution the movement of the couple varied from a maximum in one direction of rotation to a maximum in the opposite direction and that it was necessary to incorporate a ratchet and pawl device to resist the tendency of the shaft to rotate in one direction yet allowing it rotational freedom in the other direction. They propose the use of elliptical gears to give maximum angular velocity in a predetermined position a modus operandi not to be more fully exploited for two decades, until it was taken up

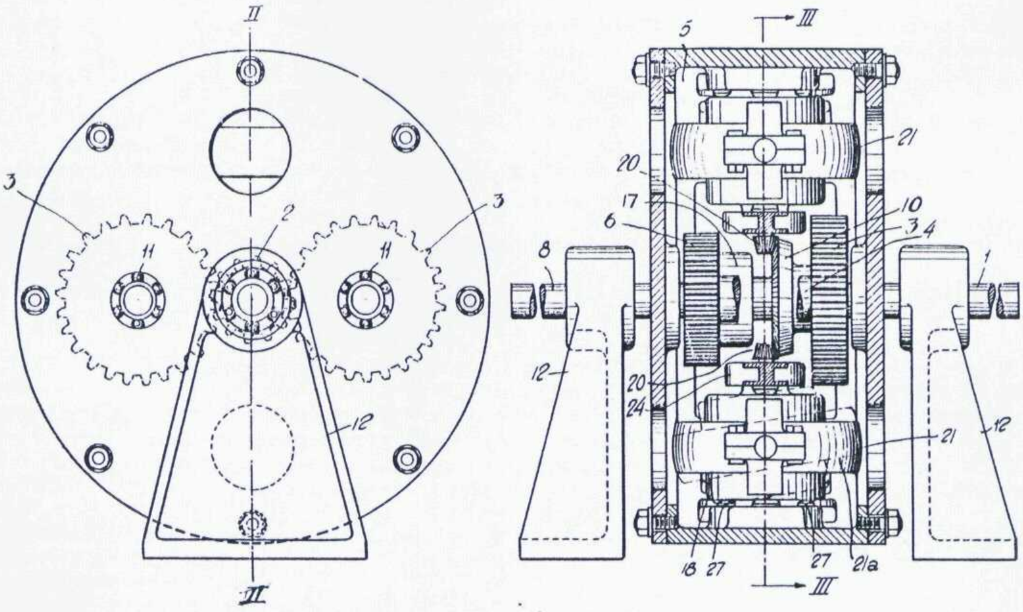


Figure 9.1.5

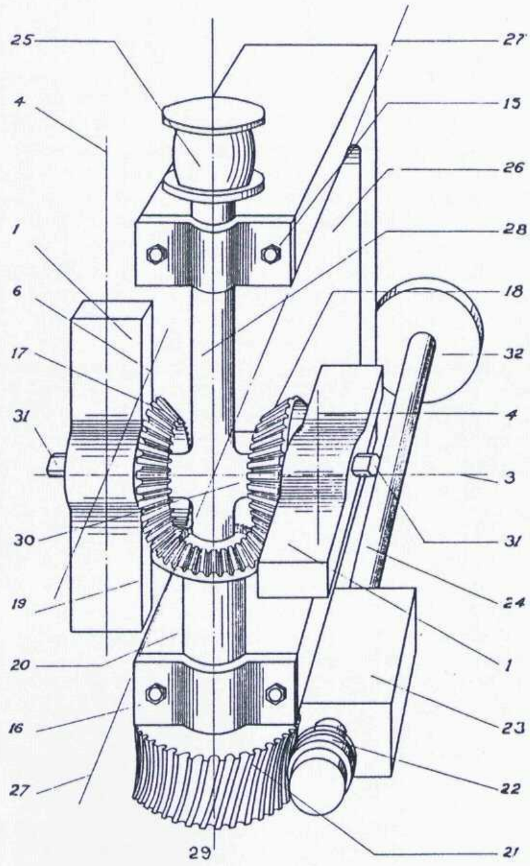


Figure 9.2.1

by TAYLOR (1953). VAN ASPEREN, grasps the nettle firmly, however, and directs himself to a gyroscopic structure for the production of an unidirectional pulsating couple or torque for use in gyroscopic machines with no reference to gear transmissions as such. He gives the magnitude of the gyroscopic couple as proportional to the cosine squared of the angle of spin (A) for a concentrated gyroscopic body which he defines as a mass, the majority of the particles of which are situated in or close to a plane containing the axis of spin. Thus when A is zero degrees of arc the value of the torque is at a maximum and when A is a right angle it is at a minimum. For A to be zero the plane of symmetry of the rotor (the gyroscopic body) is parallel to the axis of turn and at a minimum when they are disposed at a right angle to one another.

VAN ASPEREN's preferred construction for his gyroscopic structure is shown in Fig 9.2.1.

A double constrained gyroscope has two concentrated gyroscopic bodies that revolve about a common rotatable turnshaft the direction of the axis of which is the direction of the axis of turn of the gyroscopic bodies while spinning about a common axis of spin 3. The two concentrated gyroscopic bodies 1 are turnably mounted on two trunnions 30 on opposite sides of a common rotatable turning shaft 28. The axis of the trunnions is the axis of spin of the gyroscopic bodies 1 and is at right angles to the axis 29 of the shaft 28. The bodies are prevented from flying from the shaft, under the influence of centrifugal force, by thrust bearings, shown in the drawing as nuts 31. The two bodies 1 have rigidly attached to each bevel gear 17 and 18 respectively, these two bodies and gears are respectively alike, and placed at equal distances from the axis 29. They form therefore, a balanced whole about the axis 29. One end of the turning shaft 28 is surrounded by a hollow sleeve 20 in which it finds a bearing. The sleeve 20 has rigidly attached thereto at its upper end, a bevel gear 19 which is provided with a central hole for the passage of the shaft 28; at the lower end of the sleeve 20 is a worm gear 21 rigidly attached thereto, engaged in which is a self locking worm 22. The upper end of the shaft 28 is provided with means for rotating the shaft, here represented by a pulley 25. The bevel gears 17 and 18 are the spinning gears; they engage the bevel gear 19 which is the stationary gear. The shaft and the sleeve with their mountings are journalled in the bearings 15 and 16 of the frame 26 which forms a part of a gyroscopic machine. The self locking worm 22 is journalled in a bearing 23 connected to the frame 26, and is provided with a stem 24 and a hand wheel 32. The gear ratio between the spinning and the stationary gear is made two to one. This preferred construction provides a simple means for balancing the different stresses which act within this mechanism when in operation. It also reduces the weight and space requirements when more than one gyroscope is applied in a gyroscopic machine. In order to follow the operation of this construction only one of the gyroscopes will be considered, for example the one on the left side of the figure. The operation of the other one being identical. When the shaft 28 is rotated by means of the pulley 25, the gyroscopic body 1 revolves with its trunnion 30 about the axis 29 and in doing so, it rotates at the same velocity about its axis of turn 4. At the same time the spinning gear 17, being meshed with the stationary gear 19, is rotated by it and because of the chosen ratio 2 to 1, the body 1 will spin about the axis 3 with half the angular velocity of turn. The stationary gear 19 does not move, being held by the self locking worm 22. When during rotation the body passes through the vertical position, it will be subjected to a maximum couple about its gyroscopic axis 6. In this instance the top part of the body will be subjected to a force to the left, away from the shaft 28, and the lower part to a force in the opposite direction, or a smaller force in the same direction, thus resulting in a couple about the axis 6. During the rotation this couple will diminish until it becomes practically zero after half a revolution of shaft 28, which causes an angle of turn through 180 degrees, and an angle of spin through 90 degrees. It is then in the position in which the other body has been drawn. After a full revolution the angle of turn has been 360 degrees, and that of spin 180 degrees. Because of the symmetry of the body it has been returned to its original position and the couple is again at maximum, which occur in a fixed position in relation to the frame 26. If the shaft revolves with a constant speed the angular velocities of turn and spin are also constant, and this maximum will occur regularly after equal time intervals. These intervals are the period of the pulsating gyroscopic couple and are equal to the time required for one complete revolution of the shaft. The same is true for any other value of the couple provided that the same

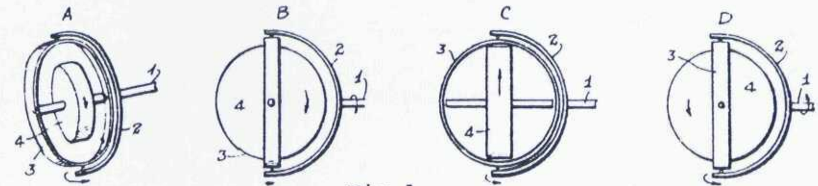


Fig.1

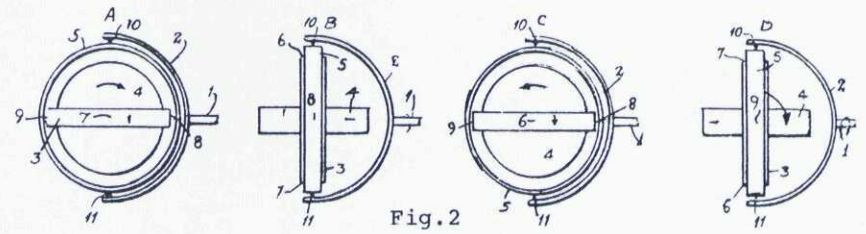


Fig.2

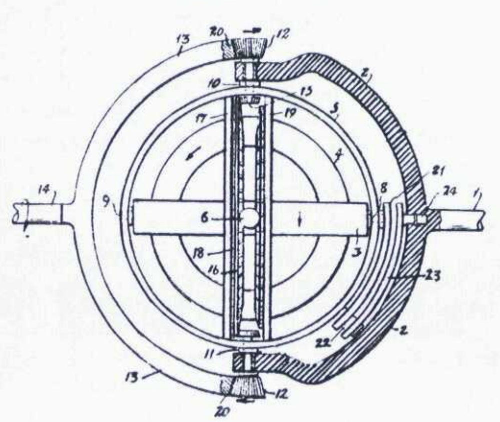


Figure 9.2.2

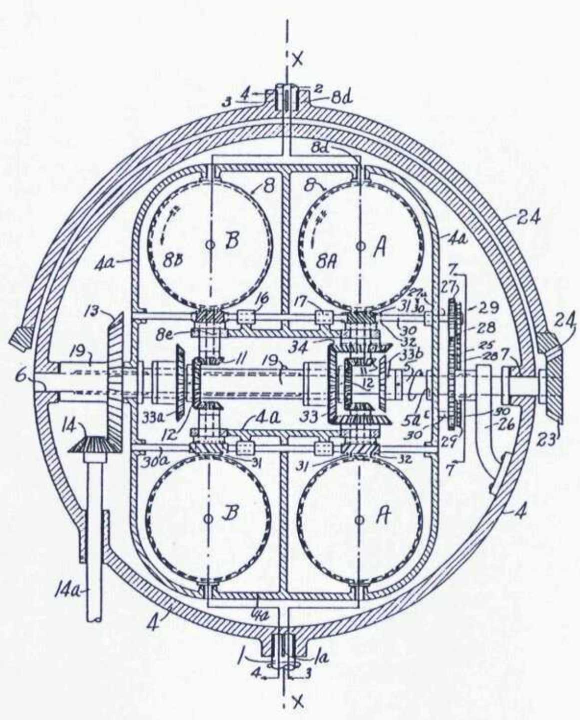


Figure 9.2.3

consecutive values are taken both when increasing or decreasing. The magnitude of the gyroscopic couple has therefore the same periods in time as its gyroscopic axis in space. As the magnitude of the couple is proportional to the square of the angular velocity of turn and as the above equality in periods is independent of the constant angular velocity of turn, this equality will exist independent of the absolute magnitude of the constant velocities of turn and spin. Although these determine the length of the period they do not, of themselves, cause a phase shift in space of the period. If, by means of the hand wheel 32, the worm 22, the worm gear 21, and sleeve 30 an angular displacement is given to the stationary gear 19 then a transient acceleration or deceleration will be given to the spinning gear 17, the angular velocity of turn remaining the same. If the hand wheel 32 is turned a definite angle clockwise, then the teeth of the stationary gear will move to the left, that is opposed to the sense of rotation of the shaft 28. This will cause a transient acceleration in the angular velocity of spin, which will cause a permanent displacement of the gyroscopic couple. It also follows that a change in the speed of rotation of shaft 28, and therefore of the angular velocities of turn and spin do not effect the ratio 2 to 1 of the spinning gear and the stationary gear, and therefore do not affect the periodicity in space of the gyroscopic axis either before, during or after the change but only the length of the period and the magnitude of the gyroscopic couple. When applied for the transmission of energy from the driving element to a driven element, the frame 26 rotates about the axis 27 and is coupled to the driven element.

WALTON shows how an alternating torque may be converted into a unidirectional torque simply by inserting into the gyroscopic system an extra gimbal ring. The construction is made clear by a comparison of the alternating torque generation of a rotor mounted as shown in Figure 1 with a rotor mounted as shown in Figure 2 (See Fig 9.2.2). WALTON fits a precession ring that has attached to it a second ring that is in the plane of the flywheel. In this ring on one side of the precession ring and making an angle of 45 degrees of arc to the plane of the precession ring is a bearing in which runs the driving pin of a crank arm that enables the ring to make the gimbal swing backwards and forwards through two right angles. In Figs. 1 a driven shaft 1 has rigidly attached to it a fork or U-piece 2 in which is pivoted a ring 3 free to rotate about an axis at right angles to the axis of the shaft 1. A flywheel 4 has its spindle free to rotate in bearings in the ring 3, the axis of the spindle being at right angles to the axis on which the ring 3 turns in the fork 2. The axes of 1, 3 and 4 cross each other substantially at a common point in the centre of the wheel 4. It will be assumed that the flywheel 4 is driven in the direction shown by the arrow on it, and that the ring 3 is rotated about its axis in the direction of the arrow at the bottom of the drawings, by suitable means (not shown). The rotation of the ring 3 about its axis will be a precessional motion of the flywheel 4 which will bring into play a gyroscopic force tending to cause the spindle of the flywheel 4 to rotate end over end in the plane of the ring 3, this movement being prevented by the bearings of 4 in 3, the bearings of 3 in 2 and the bearings of 1. The gyroscopic force exerted depends on the speed of precession and the speed, the mass, and the radius of gyration of the flywheel 4. In Fig. 1, the four drawings A, B, C and D show consecutive positions of the ring 3. In A the spindle of the flywheel 4 is parallel to the shaft 1; in B the spindle of 4 is at right angles to 1, (i.e. 90° precessional movement from the A position); in C the spindle is 180° from the A position; and in D it is 270° from the A position. In A no gyroscopic force is exerted on the shaft 1 as a torque tending to rotate it on its axis. As the ring 3 moves from position A towards position B, a gradually increasing torque is applied to the shaft 1, tending to rotate it in the direction of the arrow encircling the shaft 1 in B. This torque is, for uniform speeds of 3 and 4, at a maximum in position B after which it gradually decreases to zero again in position C. Beyond position C torque is again exerted on the shaft 1, attaining a maximum at position D and decreasing to zero again at position A; but the torque between positions C and A is in the direction of the arrow encircling the shaft 1 in D. As the torque on the shaft 1 in position B is opposite to that in position D, the torque is such as to cause the shaft 1 to oscillate about its axis. As the gyroscopic force is constant for constant speeds of the ring and the flywheel 4, the torque in the plane of the ring 3 is also constant; but owing to the rotation of the ring 3, the gyroscopic force tending to rotate the shaft 1 is applied at a changing radius; consequently the torque on the shaft 1 is sinusoidally varied.

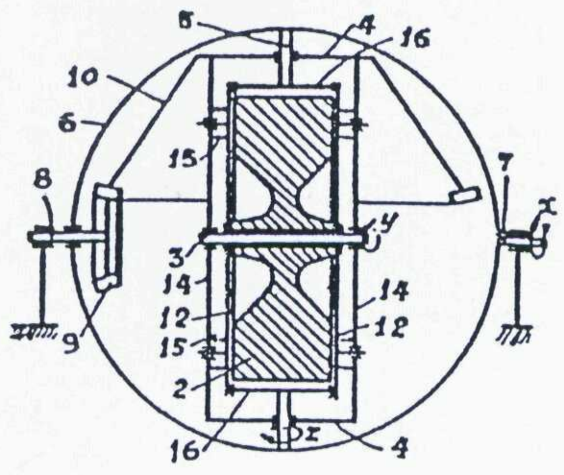


Figure 9.2.4

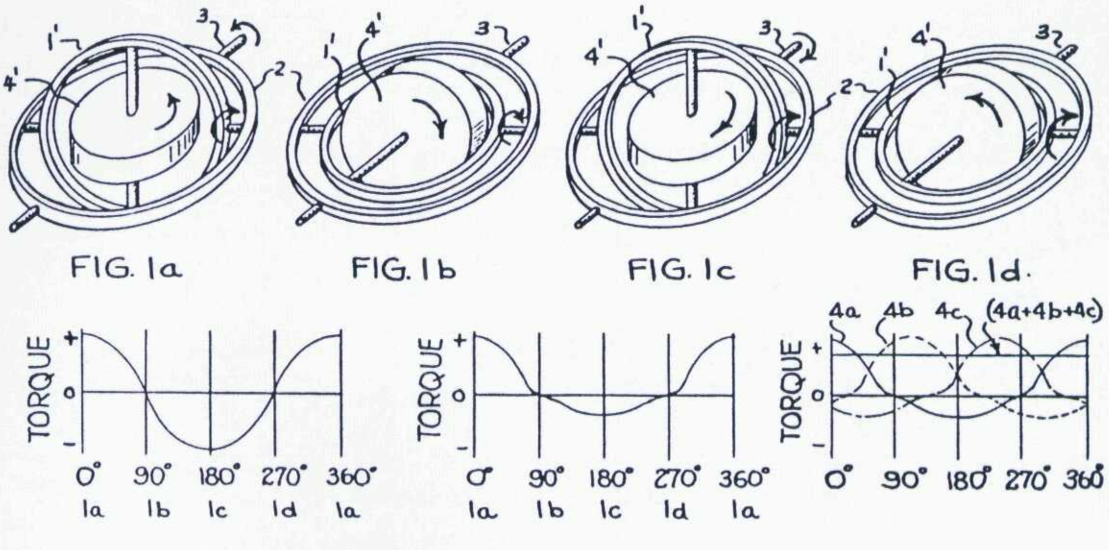


Figure 9.2.5

The arrangement of Figs. 1 does not meet the usual requirements in power transmission mechanisms for what is required is a unidirectional torque on the driven shaft. Some means are therefore required to "rectify" the oscillatory torque, and Figs. 2 show how this can be accomplished.

A driven shaft 1 has a fork 2 in which a gimbal ring 5 is mounted by pivots so that the ring 5 is free to swing on an axis at right angles to that of the shaft 1. In the ring 5 is mounted the precession ring 3, which is free to rotate on pivots about an axis at right angles to that of the gimbal ring, and in the precession ring 3 is mounted the flywheel 4, its spindle having bearings in 3 and rotating on an axis at right angles to that of 3. The axes of the parts 1, 5, 3 and 4 meet each other at a common point in the centre of the flywheel 4. The ends of the axes of the parts 5, 3 and 4 are denoted by 10 and 11, 8 and 9, 6 and 7 respectively, so that the form of the gyrations may be followed through drawings A, B, C and D of Figs. 2. It will be assumed that the flywheel 4 and the precession ring 3 are maintained in rotation about their respective axes by suitable means (not shown), the positions A, B, C and D showing 0°, 90°, 180° and 270° of the precessional rotation of the ring 3. In A the flywheel 4 is rotating clockwise and the pivot 7 moves downwards, a gyroscopic force being produced which tends to rotate the gimbal ring 5 about its axis in a direction such that the pivot 7 moves to the left. These motions continue towards the position B. In the aspect of B the precession ring 3 is rotating anticlockwise and the flywheel 4 towards the right, a gyroscopic force being produced which tends to rotate the shaft 1 about its axis in a direction shown by the encircling arrow. From position B motions continue to the position C where the flywheel 4 is rotating anticlockwise and the pivot 6 is moving downwards, so that the gyroscopic force tends to move this pivot to the right, i.e. the swing of the gimbal ring 5 about its axis is reversed relative to the swing in position A. The motions shown in C continue to the position D where the precession ring 3 rotates clockwise and the flywheel 4 rotates to the left, i.e. both these motions are reversed relative to those in position B; consequently the gyroscopic force in position D is in the same direction as in position B, tending to rotate the driven shaft 1 about its axis in the same direction, as shown by the arrow encircling this shaft. Thus between positions B and D the gyroscope has turned completely over by swinging about the axis of the gimbal ring 5, being in the middle of the swing in position C. Similarly between positions D and A the gyroscope turns completely over by swinging about the axis of the gimbal ring 5 in the opposite direction. From this it will be apparent that the swinging about the axis of the ring 5 is oscillatory, the angular velocity being zero in positions B and D and maximum in positions A and C. The torque on the shaft 1 is maximum in the B and D positions and minimum in the A and C positions, and in positions of precession intermediate to the positions shown in Fig. 2. The gyroscopic force has a component tending to rotate the shaft 1 as in positions B and D and another component tending to swing the gyroscope about the axis of the gimbal ring 5 in one or the other of the directions as shown.

Fig 9.2.2 shows how the swinging of the gimbal ring 5 may be controlled by a simple link motion 21 and 23.

STALKER makes an almost identical point to that of WALTON in his torque converter of 1940, he notes that if the gyroscopes rotate about the precessional axis (the axis of the central shaft) there is after a turn of two right angles a precessional torque that is reversed in direction; but if after the said turn the gyroscope is also inverted, the direction of the torque is in the same direction as at the beginning of the turn. This inversion according to STALKER should be made when the axis of spin of the gyroscope is parallel to the axis x.x of Fig 9.2.3, that is when the gyroscopes are at the top and bottom, the control axis being vertical.

BIASI, also sees that direction of the rotation of the gyroscope flywheel or rotor must be reversed with its revolution and he expresses this point succinctly in the formal language of his first claim that reads (see Fig 9.2.4).

> An infinitely variable gear characterised by the feature that it comprises supporting means (4) and counter supports (6) for at least one gyroscopic flywheel (2) connected the former with the driven shaft (8) and the latter with the driving shaft (7), and means

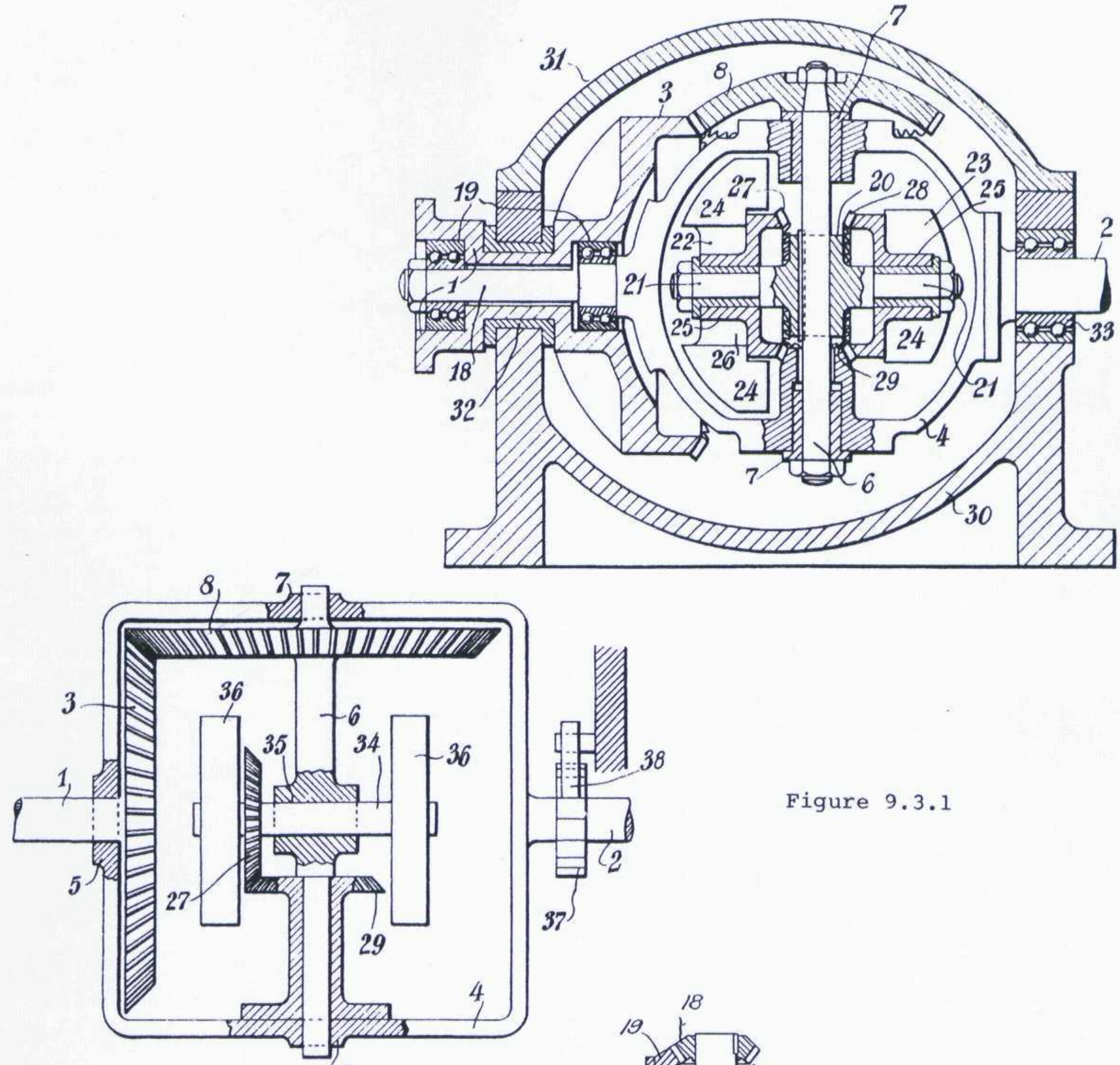


Figure 9.3.1

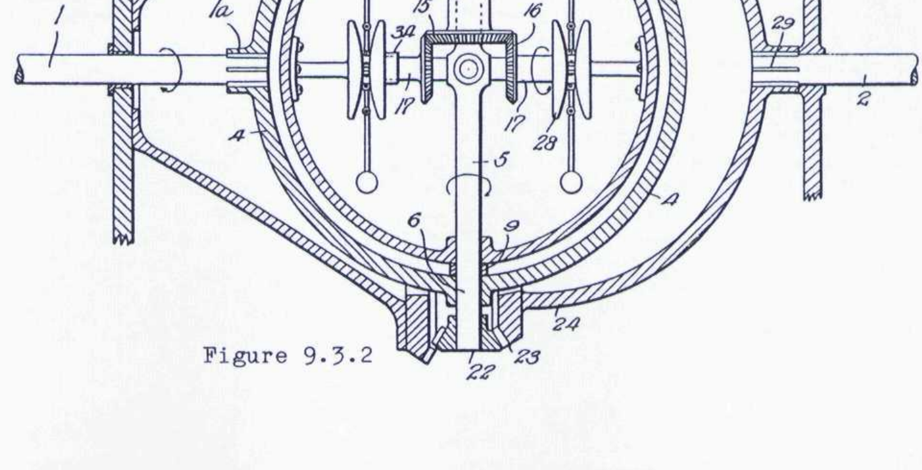


Figure 9.3.2

capable of reversing the rotary motion of the flywheel, so that on rotating the driving shaft the same will cause the supporting frame and counterframe of the flywheel to rotate, thereby obtaining on the driving and the driven shaft, torques and speeds entirely independent from each other.

TAYLOR, in a mechanism for power transmission, proposes another way toward success, he notes the cyclic change of torque (2_a of Fig 9.2.5) and he seeks to control it by shifting the phase of the cyclic variation of flywheel spin speed with respect to the period of rotation of the flywheel about the precession axis, that is to say their revolution in orbit. He achieves this by varying the spin speed in the following manner:-

Let it be maximum at the position shown in Figure 1a, gradually decreasing as the ring 1' rotates, until the spin speed is minimum at the position shown in Figure 1c; then gradually increasing as the ring 1' keeps rotating until the spin speed is once more a maximum at the position shown in Figure 1a. At the position in Figure 1a, the torque developed along the power axis will be maximum. As ring 1' turns and the flywheel passes through the position in Figure 1b, the direction of the torque developed about the power axis will change. However, the spin speed of the flywheel will be low during this portion of the cycle about the precession axis, hence the magnitude of the torque which is developed in the direction opposite to that in Figure 1a will be less. Generally stated, during that portion of the precession cycle when the torque is positive, the spin speed will be high and during the portion when the torque is negative, the spin speed will be low, thus the positive torque will be greater in magnitude than the negative torque. The variation of the gyroscopic torque about the power axis for the different positions about the precession axis will then take a form such as that shown in Figure 2b.

The spirit of his invention lies in this nonlinear cyclical variation of flywheel spin speed during the processional rotation to obtain a variable torque about the power axis which is greater in one direction. A number of flywheels given this motion and acting together will produce a smooth, unidirectional torque about the power axis.

9.3 TORQUE CONVERTERS

One of the earliest torque converters is due to JANSSEN US 1736789 (1929) who provides a power transmission mechanism adapted automatically to effect a smooth delivery of power from a power source to a point of use at a continuously varying mechanical advantage, so that a prime mover having high torque at higher speeds is caused to operate at maximum effectiveness throughout an entire acceleration range. To do this he arranges a conoidally rotatable rotor with peripheral cavities for mercury on one of two coaxially aligned shafts epicyclically coupled so that any relative rotational motion between the said shafts causes a gyroscopic couple to augment the transmission.

Another early torque converter still to carry the title of an improved variable speed gear and closely related to the end-over-end flywheel type of gyroscopic gear discussed above is that of ANDERSON & HOWE GB 414693 (1934). It is simple in construction (Fig 9.3.1) and it is able to convert a torque of given magnitude at any given angular velocity to a torque of greater magnitude at a correspondingly smaller angular velocity. The maximum moment of the couple provided by each 'flywheel' is obtained when each 'flywheel' lies along the transverse shaft (6) and the minimum moment is obtained when it lies across said transverse shaft.

ANDERSON & HOWE use what VAN ASPEREN calls concentrated gyroscopic bodies in the pleasing form of dumbell-like rotors with their axes orthogonally disposed. The patent document is notable since its authors see the need for a unidirectional torque and they propose a ratchet and pawl mechanism to remove the torque in one direction and the use of an elliptical gear cooperating with an undulatory mating surface to give maximum angular velocity at a predetermined position.

STALKER US 2223743 (1940), US 2223745 (1940) US 2389826 (1945) is responsible for three torque converters all using gyroscopic forces. In the first (Fig 9.3.2) four segmental gyroscopes are made to deliver an approach to an

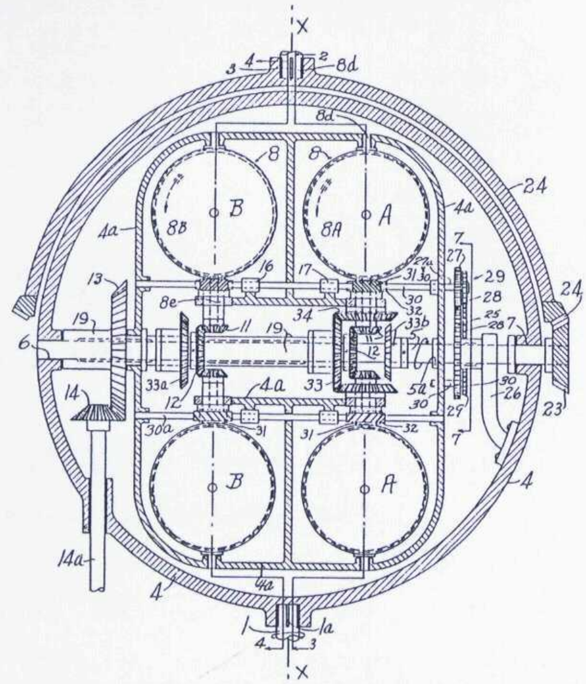


Figure 9.3.3

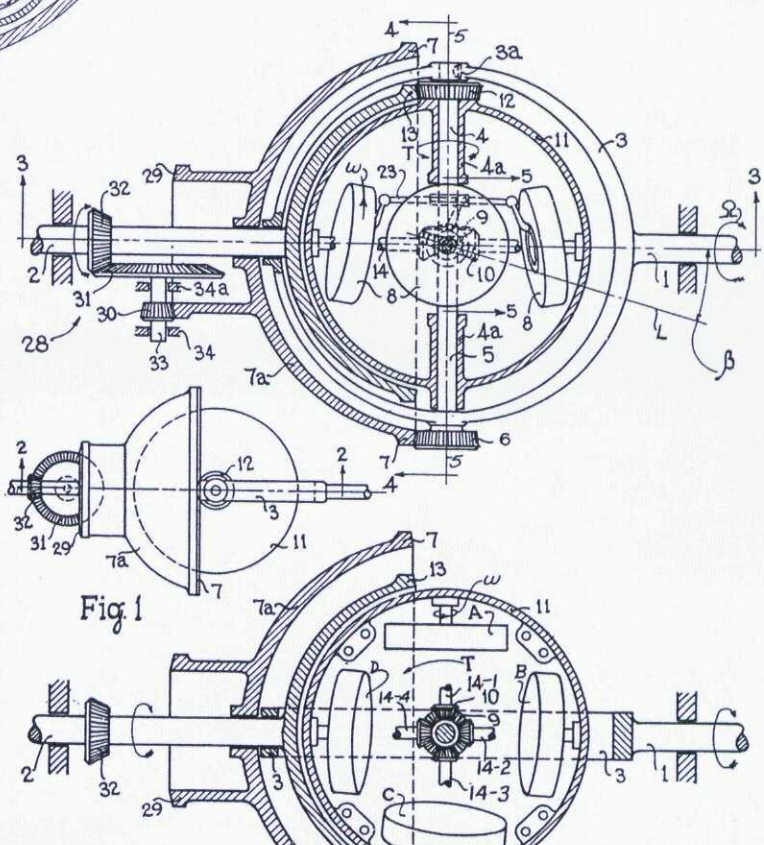
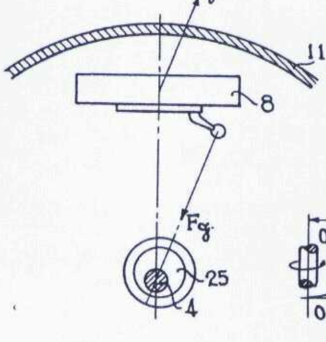


Fig. 1



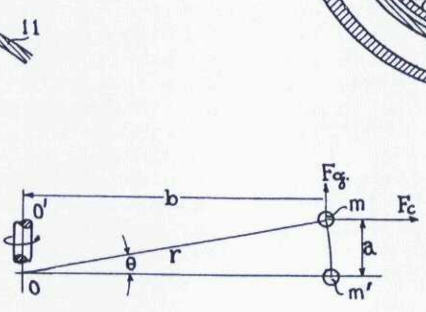


Figure 9.3.4

unidirectional torque by causing them to exert a substantial driving force only when the gyroscopes are on one side of the precessional axis. This is accomplished by the use of specially shaped guides that by a unique juxtapositioning allow the gyroscopes to be free to take up an unrestrained position when at the bottom and a restrained position developing a driving force at the top of any single rotation.

In the second, (Fig 9.3.3) twelve gyroscopes are deployed and the reversal of torque experienced inevitably every two right angles of the precessional revolution is corrected by the inversion of the gyroscopes when the spin axis is parallel to the axis of the coaxially arranged driving and driven shafts. The inversion is effected by the use of a train of gears including two eccentric gears.

In the third, four gyroscopes are orthogonally placed (see Fig 9.3.4) and they are provided with a tilting mechanism to ensure that there is a reduction in the gyroscopic torque such that the torque of rotors D.A.B. (in the lower figure of the set) predominates over the torque of B.C.D. In his U.S. patent specification STALKER explains what to his mind occurs under the tilting action.

Consider the mass m which is rotatable about the axis OO'. Let it represent any element or particle of mass of the gyroscope in position C. The position m' shows the mass acted upon by the centrifugal force only. The position m shows the mass acted upon by the centrifugal force F_c and the gyroscope force F_g. This force is found by dividing the precessional torque of the mass m by the radius r. (The particle m rotating about OO' is a gyroscope and subject to a precessional torque if the axis OO' is tilted.) The mass takes up a position of balance under the action of these two forces when the moments balance. That is

$$F_c a = F_g b \tag{1}$$

or

$$F_g r \cos\theta = F_c r \sin\theta \tag{2}$$

$$F_g = F_c \tan\theta \tag{3}$$

Since m was any particle these equations hold for all particles of the gyroscope in position C and hence for the whole gyroscope. When this condition prevails the gyroscope in the position C will exert no adverse torque. It's the function of the tilting to bring this condition about. From Equation 3 it follows that

$$\tan\theta = \frac{Fg}{Fc} = \frac{mr\Omega\omega}{mr\omega^2} \tag{4}$$

$$\tan\theta = \frac{\Omega}{\omega} \tag{5}$$

where Ω is the angular velocity about the torque input axis (axis of shaft 1) and ω is the spin about the spin axis (axis of shaft 14).

It is shown by Equation 5 that the angle θ depends on the ratio of angular velocities. However the "throw" of the eccentric 25 is fixed and constrains the gyroscope to a definite and unalterable angle of tilt θ. Hence the ratio of Ω to ω should be held constant so that the gyroscope in position C will not be acting against the gyroscope in position A.

<u>HELBERG</u> US 2693723 (1954) has proposed the use of a fluid gyroscopic weight for automatically regulating the torque transmitted to the driven shaft. A spherical container sensitive to relative motion between coaxially arranged input and output shafts contains a liquid that is free to precess, and from this is generated a reaction on a gear train that increases the torque on the output shaft. HELBERG's proposal is in some respects anticipated by the earlier work of <u>HARDING</u> GB 141139 (1920) and <u>BIASI</u> GB 622337 (1949).

<u>CASCAJARES</u> GB 744645 (1956) proposes the use of the regulating and controlling force included in the precessional force generated by an acclerated flywheel to press onto the shaft of a differential gear and thereby vary its ratio in accordance with the load on the driven shaft.

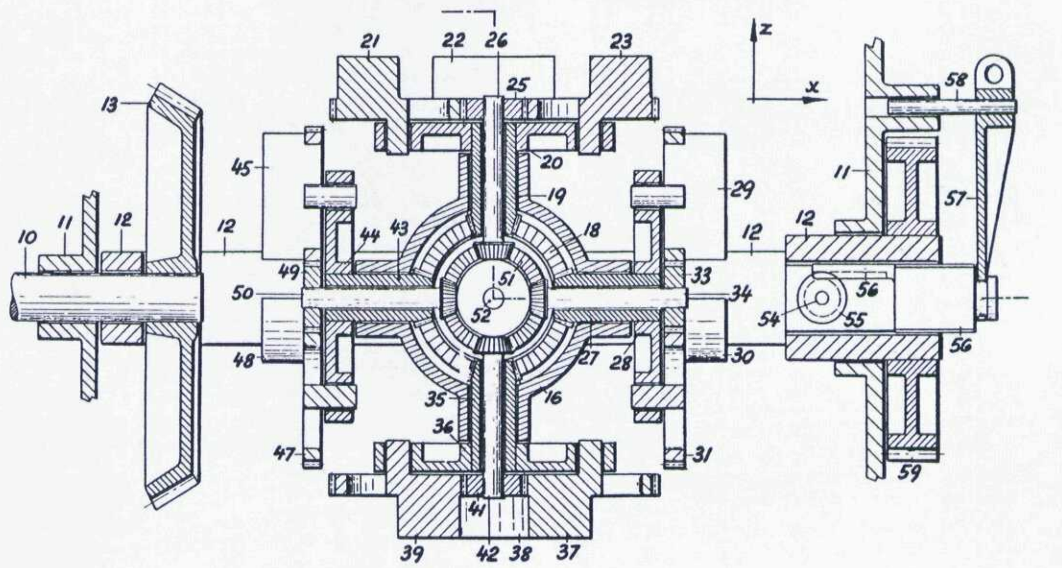


Figure 9.3.5

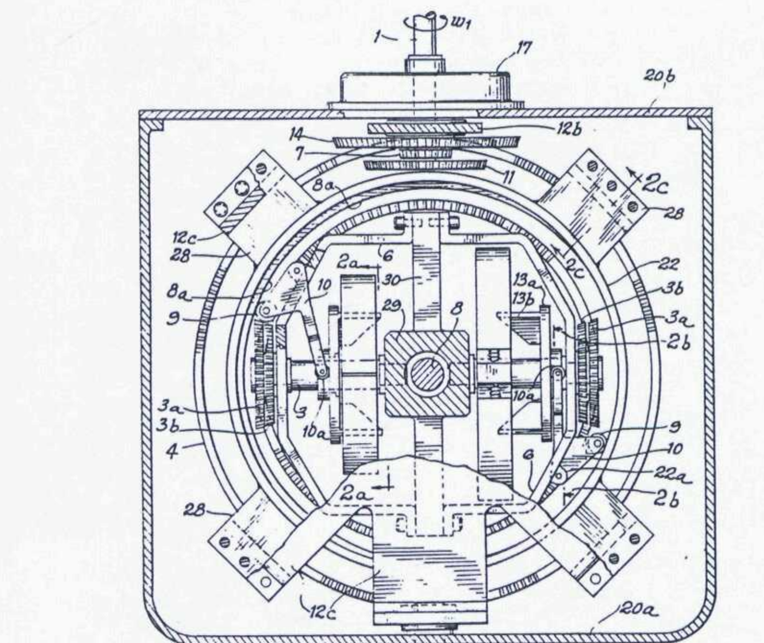


Figure 9.3.6

PLOGER US 2877667 (1959), seeks to solve the main problem in gyroscopic power transmission devices, that is the change of direction of a flywheels gyroscopic torque due to precession when its spin axis is revolved through two right angles, thus vitiating the generation of uni-directional torque. PLOGER does not wholly succeed in producing this desideratum but he approaches it by the use of a spinning device provided with eccentrics that has a periodically variable moment of inertia with respect to the spin axis (see Figure 9.3.5).

The period or periodicity in the motion of the eccentric elements is achieved by proper gear ratios. The gear ratio of the beveled gear integral with shaft 18 to the hollow shafts and beveled pinions 19, 27, 35, 43, the gear ratio of the beveled gear 51 to the beveled pinions and shafts 26, 34, 42, 50 and the gear ratio of the central gears 25, 33, 41, 49 to the gear rings of the eccentric elements are such, that a periodical motion of the eccentric elements relative to their corresponding cross-like supports of the following kind takes place: Each time when the spinning devices make a full turn (360 degrees) around the precession axis, all eccentric elements make a full turn, too, relative to their corresponding cross-like supports. Thus, a periodicity develops in the positions of the mass centroids of the eccentric elements with respect to their corresponding spin axes. With other words: the moments of inertia of the spinning devices with respect to their spin axes vary periodically between a maximum and a minimum value. The period is 2π in terms of angle of rotation of the spinning devices around the precession axis. The starting point of the period can be shifted continuously by moving the lever 57. The eccentric elements in the different spinning devices are so arranged that the spinning devcies passing through a determinate angular position in the plane of rotation around the precession axis have the same moment of inertia. In the positions of spinning devices shown in the figures the moment of inertia of the spinning device belonging to the cross-like support 20 is a maximu, and that belonging to the cross-like support 36 is a minimum. The moments of inertia of the remaining two spinning devices belonging to the cross-like support 36 is a minimum. The moments of inertia of the remaining two spinning devices belonging to the cross-like supports 28 and 44 are equal and have an intermediate value between the extreme values.

The modus operandi of the device is as follows:-
When the driving shaft 10 is rotated and the driven gear 59 restrained by a torque load, the spinning devices rotate, by means of the described gear trains, around their spin axes and, at the same time, are tilted around the precession axis. Thereby, according to well-known rules of mechanics, gyroscopic forces develop in the mass elements of the spinning devices. These forces yield torque components around the power axis. They are transmitted by the cylindrical and conical cage 16 to the frame 12 and, therefore, to the driven gear 59. The torque components are proportional to the moments of inertia of the spinning devices. The torque components of the spinning devices belonging to the cross-like supports 20 and 36, in the positions shown in the figures, are acting opposite to each other, but the difference between their moments of inertia yields a net torque around the power axis.

PRESTON US 3394619 (1968) US 3439561 (1969) US 3540308 (1970) in three useful and detailed documents proposes torque converters that are each said to provide a stepless variable-speed power transmission device. In all three devices the ratio of the input and output shaft speeds depends (a) on the external torque load applied to the output shaft and (b) on the speed of the power driven input shaft. The transmission of power from the input to the output shaft in the first device is by means of a pair of spinning rotors the kinetic energy of which undergoes cyclical changes involving both the rotational speed and the mass inertia of the rotors. During one phase of the working cycle energy is transmitted by a gear train from the input shaft to the axle of the rotors and during another phase energy is transmitted from the rotors by gyroscopic forces to the output shaft. The essential component of the device, the rotor, each comprise flyweights the rotary inertia of which is controlled by the angle of precession. It consists of weights slidingly arranged on spindles and these are preferably made of heavy metal such as a tungsten alloy.

Mathematically the device may be shown to exhibit the following performance equations:-

$$M_1 = c_1 (\omega_1 \omega_2 - c_2 \omega_2^2)$$

$$M_2 = c_1 (\omega_1^2 - c_2 \omega_1 \omega_2)$$

in which M_1 is the input torque, M_2 is the output torque, ω_1 is the speed of the input shaft and ω_2 is the speed of the output shaft. The constants c_1 and c_2 depend on several built-in parameters and can be defined as follows:-

$$C_1 = \frac{I'}{2\pi} \left(\frac{\tan\alpha}{\tan\beta}\right)^2 \times (\tan\alpha - \tan\delta)$$

$$C_2 = \frac{\tan\delta + \tan\gamma}{\tan\alpha}$$

in which I', as previously defined, is the maximum deviation of the axial rotor inertia from its median value,

α is half of the central angle of pinion 11,
β is half of the central angle of pinions 3a and 3b,
γ is half of the central angle of pinion 14,
δ is half of the central angle of pinion 7. (See Fig 9.3.6)

The concept of a segmented flyweight rotor to give an increase in the diameter of the rotor at specific positions in its revolution is to be found in the gyrotransformer of SEILLIERE FRENCH 887896 (1943).

In a second device power is transmitted from the input to the output shaft through a single spinning rotor having diametrically opposed, eccentrically revolving weights, the kinetic energy of which is therefore cyclically variable as the resulting rotational speed and mass inertia of the rotor also varies cyclically, as in the first device. During one phase of the working cycle energy is transmitted by a gear train from the input shaft to the rotor and during another phase part of the energy is recirculated to the input shaft and part of it is transmitted to the output shaft.
The rotor shaft undergoes a precessional motion but the device is in some respects closer to the centrifugal converter of CICIN US 3154971 (1964).

In a third device power is transmitted from the input to the output shaft through a spinning rotor the inertia of which is constant and the axis of which is forced to undergo a cyclic precessional motion. During subsequent alternate phases of this cyclic motion power is transmitted from the input shaft to the rotor and then from the rotor to the output shaft. The conversion of the reversing torque, that is to say its rectification to use an electrical analogy, is effected through a planetary gear set incorporating two one way clutches interposed between the input and output shafts. In some respects this device moves closer to the older gyroscopic gear transmissions of ANDERSON & HOWE and that of TARCIA & MENGHI ITALIAN 460372 (1950).

KEMPER US 3955432 (1976) discloses a balanced torque transmission device of considerable complexity incorporating a rotor block that enjoys a conical movement and is said to exhibit gyroscopic torque that moves the parts of a two-part shell away from one another or toward one another depending upon the ratio of certain cooperating radii. The device is thought to bear some resemblance to the old mutator of WEISS US 1728383 (1929).

We have referred above to the work on unidirectional torque by WALTON and it is in the nature of a pleasure and a surprise to see his work appear again with certain modifications in GB 1292613 (1972) where he discloses a gyroscopic torque converter integrated with inter alia a following gearing of a fixed ratio. The invertion of the rotor is effected, as in 1935, by the use of a crank and a link and a fully rectified torque is obtained.

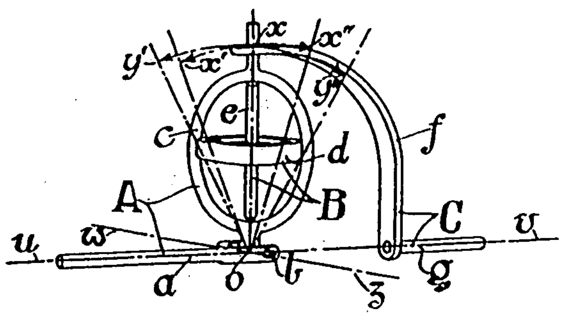


Figure 9.4.1

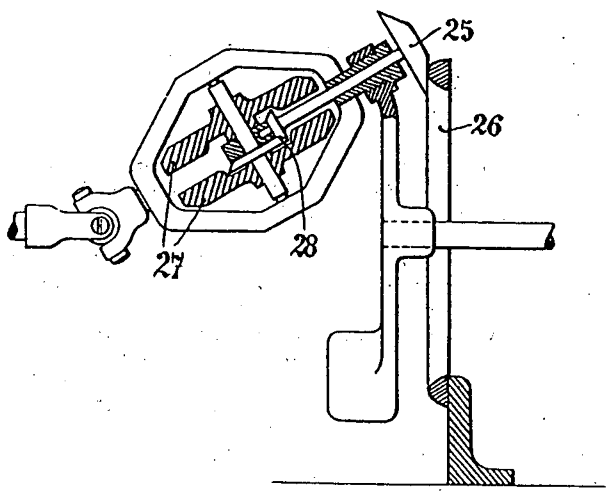


Figure 9.4.2

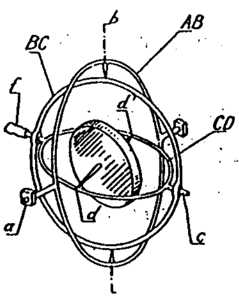


Figure 9.4.3A

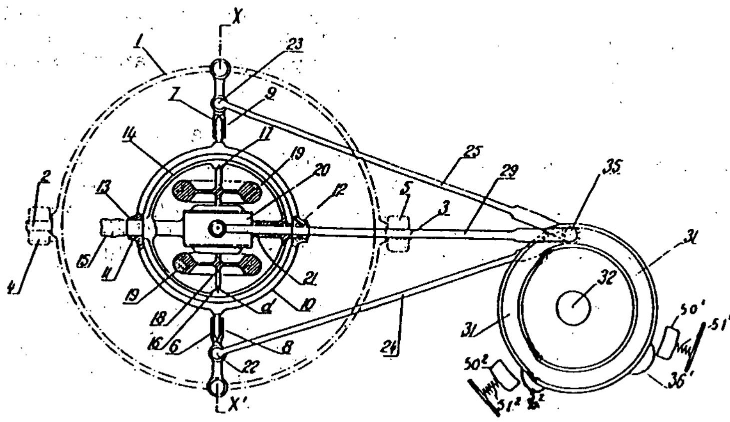


Figure 9.4.3B

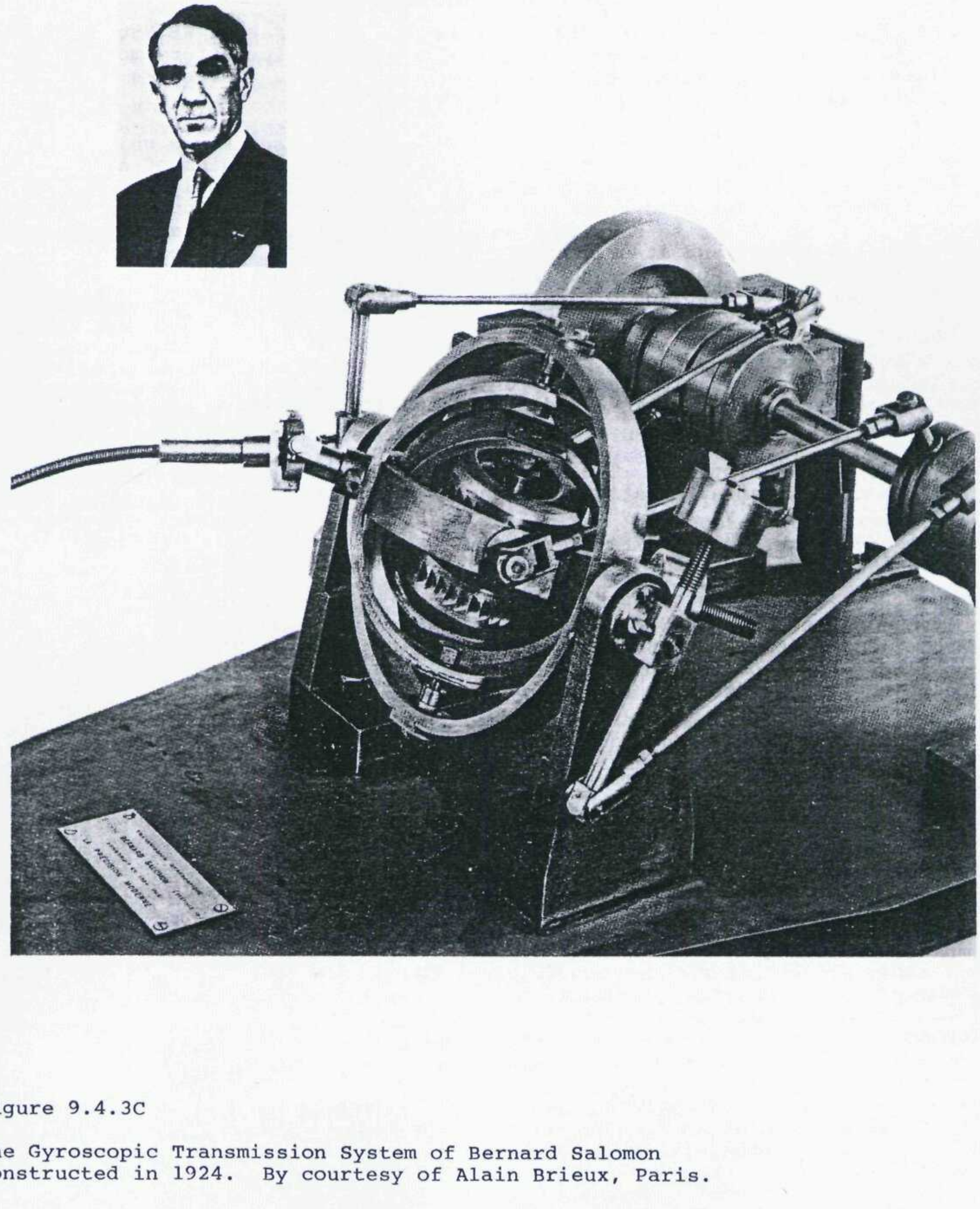


Figure 9.4.3C

The Gyroscopic Transmission System of Bernard Salomon constructed in 1924. By courtesy of Alain Brieux, Paris.

Inset: Photograph of the inventor sent to the author from the inventor personally

9.4 TRANSMISSIONS

The transmission of constant torque from the use of a gyroscope rotor has important applications in the 'space age' and this was appreciated by both PLOGER US 2877667 (1959) and KELLOGG Jr. US 3203644 (1965) who to some extent anticipate the recent transmission of HINDS US 4161889 (1979).

One of the earliest gyroscopic transmissions is that of BONN GB 25621/1911 who proposes the use of a gyroscope rotor to convert a reciprocatory motion into a direct or an alternating rotary motion (See Fig 9.4.1). The cycle of operations is as follows:-

During the first phase of the reciprocation the impressed force which acts in the line, or a line parallel to it, u v and through the medium of A or C displaces the axis of the gyroscope, which is normally in position o x, i.e. at or about right angles to u v and to w z but which at the beginning of the phase is o x', to a x" so as to revolve it about an axis w z which is at or about right angles to u v and at or about right angles to o x, the resulting gyroscopic reaction, provided the gyroscope be revolving in a certain sense or senses, tending to displace o x which at the beginning of the phase is at o y' to o y" so as to revolve it about the axis u v, should the gyroscope, however, revolve in the opposite sense or senses the gyroscopic reaction tends to rotate o x about u v in the opposite sense from that described. During the second phase of the reciprocation the axis o x is displaced from o x" to o x' the resulting gyroscopic reaction tending to displace the axis o x from o y" to o y'.

The cycle is now complete. The apparatus thus far is completely reversible inasmuch that if an alternating rotation about the axis u v displaces o x alternately from o y' to o y" to o y' the resulting gyroscopic reactions will tend to rotate o x alternately from o x' to o x" and o x" to o x' about the axis w z giving a reciprocating motion in the direction of the axis u v.

FIEUX GB 6424/1911 and GB 6428/1911 uses the gyroscope rotor when revolved in a suitable frame (See Fig 9.4.2) to impart intermittent motion to a driven shaft. In his most ingenious construction (shown) the gyroscope rotor is given a high rotational speed on its spin axis from the revolution in orbit of its spindle end which is provided with a bevel pinion 25.

SALOMON GB 225518 (1925)* is responsible for a transmission perfected in France in 1923/24 and presently refered to by historians as the 'transmission gyroscopique, systeme Bernard Salomon'.

The transmission uses a gyroscope rotor to provide a connection between a driving and a driven shaft disposed at right angles to one another. The gyroscopic system converts the rotation of the driving shaft into rotary motion of a frame carrying the gyroscope, the motion being such that the centre line of the said frame generates a cone of variable radii. This latter motion is transformed into movements of oscillation upon one or more axes, which are imparted to suitable devices of the free wheel type. Fig 9.4.3A represents a frame A-B which is adapted to oscillate upon an axis (a), mounted in suitable bearings said frame is provided with an axis (b) perpendicular to the axis (a), on which the frame B-C is adapted to oscillate. The latter frame carries - by suitable bearings - the shaft (c) which is perpendicular to the shaft (b) and is secured to the frame C-D of a gyroscope whose axis (d) is perpendicular to the shaft (c). If the gyroscope is actuated and if its frame is rotated on the axis (c) (for instance by rotating the shaft (c) by means of a flexible member f) said axis will generate a cone having as its apex the centre of the system, and this conical motion will be transformed in the following manner: The frame B C oscillates on the axis (b) while at the same time the frame A B oscillates on the axis (a), these two movements of oscillation having a phase displacement of a quarter of a period. The number of frames such as A-B or B-C and their disposition as well as that of the gyroscope may be suitably varied. The conical motion can be transformed into any number of movements representing the same number of phases. Instead of employing two frames A-B and C-D a single frame, or a plurality of frames, can be employed.

* See the 1981 transmission of Virgil A. HINDS US Patent Specification 4295381.

Fig 9.4.3B shows a gyroscopic transmission in which one utilises a system producing two-phase oscillations, according to the diagrammatic arrangement Fig 9.4.3A. the frame 1, which corresponds to the frame A-B is mounted by means of the journals 2 and 3 cooperating with the respective bearings 4 and 5; said frame is perpendicular to the plane of the figure, but it is herein represented in the dotted lines as turned down into the plane of the figure upon the axis X^1 X. The frame 1 is provided with two bearings 6 and 7 cooperating with the respective journals 8 and 9 of a frame 10 corresponding to the frame B-C. The bearings are situated at 90° from the journals 2, 3. The frame 10 has disposed thereon, at 90° from the journals 8 and 9, the respective bearings 11 and 12 of the shaft 13 which is connected to the driving shaft by a flexible member 15 or a system of universal (or like) joints, and is expanded into a frame 14; the said frame, which corresponds to the frame C-D is provided with the step bearings 16, 17 for the shaft 18 of a gyroscope, whereof the gyroscopic masses are disposed at 19. The shaft 18 is perpendicular to the shaft 13. The said gyroscope is driven by suitable means such as electric current, compressed air, mechanism or the like. An electric motor 20 is shown in the figure, it being mounted upon the frame 14 by means of the supports 21. Suitable rods 24 and 25 are connected to the bearings 6 and 7 by the respective ball-and-socket joints 22 and 23.

When the gyroscope is rotated and the frame 14 is set in rotation by the driving shaft, the frame 1 will oscillate on the journals 2 and 3, i.e. on an axis perpendicular to the plane of Fig 9.4.3B. At the same time, the frame 10 and the frame 26 secured thereto and in the perpendicular position, will oscillate on the axis X^1 X. These two movements of oscillation are substantially displaced in phase by a quarter of a period; the motion is imparted to the rings 31 by the rods 24, 25, 29 and 30; the said rings drive the actuated shaft 32 in the given direction whilst they rotate loosely thereupon in the opposite direction. It will be therefore observed that the gyroscopic system converts the rotation of the driving shaft into a rotary motion of the frame carrying the gyroscope the centre line of the frame generating a cone the apex of which is situated at the centre of the system; the said conical motion is then transformed into as oscillatory motion. (See also Fig. 9.4.3C)

The design of one powerful gyro-rotor suitable for such a transmission is that of MIDGLEY & VANDERVELL GB 119511/1918.

WILLIAMS US 2390341 (1945) proposes the gyroscope as a means for the transmission of an escapement to a timepiece, which escapement he believes to possess no backlash and one that can be used to advantage at any altitude and temperature.

HINDS US 4161889 (1979) proposes a gyroscopic transmission similar to that first proposed by BONN in GB 25621 of 1911 and to that of Societe ECA in French 997286 (1952) in which the gyroscope outer gimbal is reciprocated and the inner gimbal used to provide a reciprocation at right angles and at a constant torque for any given gyroscope rotor operating at fixed conditions. It is shown that a considerable torque can be transmitted to the output of the power transmission system of the invention, with very low input power. For a given power input, a given mass and angular velocity of the gyroscope, the torque at the output of the system can be calculated as follows:

Spinning gyroscopes develop a precessional force according to the following equation:

$$Rl = IW_1W_2, \text{ wherein} \qquad (1)$$

l is the length of the lever arm through which the precessional force, R, is applied.

I is the inertia of the gyroscope about its spinning axis;

W_1 is the angular velocity of the force applied to the gyroscope and tending to displace its spinning axis direction, therefore the angular velocity expressed in rad/sec of the input connecting link; and

W_2 is the angular velocity of the gyroscope wheel in rad/sec.

The inertia of the gyroscope about its spin axis is given by the equation:

$$I = Wr^2/g \qquad (2)$$

wherein W is the weight of the gyroscope, r is the effective radius of the gyroscope wheel, and g is the gravitational acceleration, or 32 ft/sec^2.

Assuming, for example in Fig 9.4.4 a gyroscope wheel 22 having a weight of 6 lb. and an effective radius of 6 in., and l being equal to 6 in., and assuming further that the gyroscope frame 20, together with the gimbal ring 14, is driven by the input connecting link 34 plus and minus 15° about the pivot axis 18, for a total of 30° deflection, the gyroscope enclosure or frame 20 is moved through 60° during a single revolution of the eccentric 40. 60° is equivalent to $\pi/3$ radians. If the input eccentric 40 is rotated at 3,000 rpm, or 50 rev/sec, W_1 is therefore $50\pi/3$ rad/sec. With a gyroscope 22 rotating at 18,000 rpm, or 300 rev/sec., W_2 equals 300 x 2π or 600π rad/sec.

By replacing I in equation (1) by its value obtained from equation (2), and by resolving equation (1) as a function of R, the following equation is obtained:

$$R = \frac{Wr^2\ \omega_1\ \omega_2}{l g} = \frac{6 \times (0.5)^2 \times 50 \times \frac{\pi}{3} \times 600\pi}{0.5 \times 32} = 9252.7525 \text{ lb.} \qquad (3)$$

With a crank pin 58 of the one-way clutches 60 having its axis 3 in., or 0.25 ft., from the axis of the output shaft 62, the torque applied to the output shaft is consequently equal to 9252.7525 lb. x 0.25 = 2313.1881 ft-lb.

As stated above KELLOGG Jr. US 3203644 (1965) has proposed a similar device for an inertial space drive and indeed this concept is seen to run through a number of earlier proposals such as those of STALKER US 2223745 (1940), OSGOOD US 2571159 (1951), TAYLOR US 2639631 (1953) and PRICHARD US 2811050 (1957).

9.5 DIFFERENTIAL FLUID COUPLER AND HYDRO-KINETIC COUPLINGS

FRY US 3267770 (1966) discloses a differential fluid coupler able to deal with a wide range of torque inputs. Its effectiveness is said to reside in its ability to utilize a precessional force created by gyroscopic action.
The coupler has a rotor incorporating a plurality of liquid tubes that may contain a heavy liquid tubes that may contain a heavy liquid such as mercury. The liquid reverses its direction of flow at each half rotation of the rotor about its spin axis and in consequence the flow velocity passes through zero twice for each rotation of the rotor (Fig 9.5.1). FRY's explanation of the modus operandi is as follows:-

Rotation of the shaft 38 end-for-end about the axis of the shaft 24 produces oscillatory movement of fluid in an inner circuit comprising the sub-chambers 56a and 58a and the tubes 63 and an outer circuit including the sub-chambers 56b, 58b and the tubes 64. The velocity amplitude of the fluid in such a circuit is equal in each direction. According to the present invention, however, the fluid flow is modified by either retarding or accelarating forces in such a manner that the phase of the oscillating flow is adjusted to provide a non-zero average torque and power transfer. By this arrangement, the fluid movement produces a non-zero time average precessional force which provides torque to the shaft 38, upon rotation of the drive shaft 24, as required by the load on the output shaft 48. In other words, the torque due to precessional forces occasioned by the fluid movement within the several circuits is provided in proportion to the amount of resistance afforded by the shaft 48. It will be appreciated that the coupling of the present invention would be operative with either the inner fluid circuit alone or the outer fluid circuit alone.

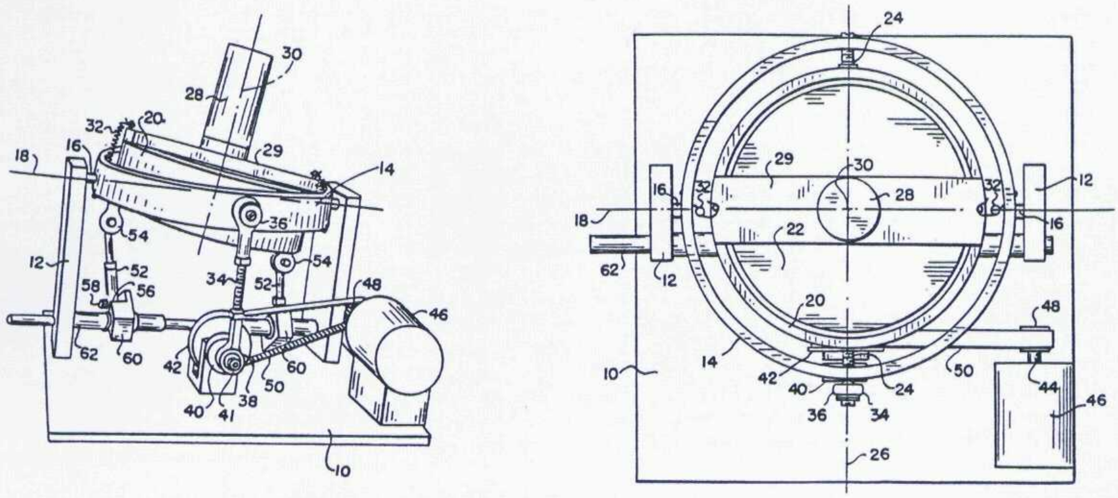


Figure 9.4.4

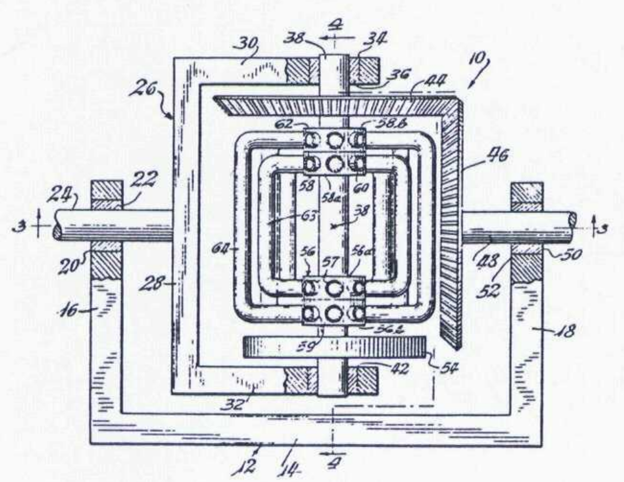


Figure 9.5.1

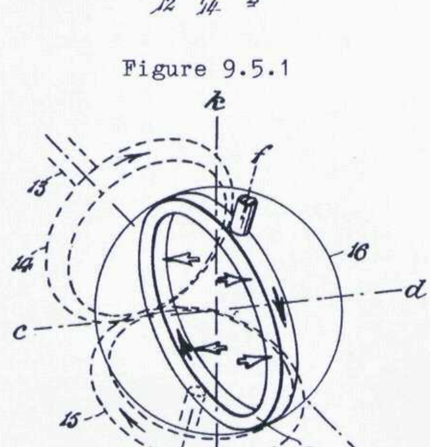


Figure 9.5.2

In respect of Fig 9.5.2 CLOETE US 2453684 (1948) offers the following:- Now, when the fluid flows along, during the operation of the device, it is evident that particles of fluid which have high instantaneous tangential velocities around c-d, are being transferred to points in a which have lower instantaneous tangential velocities around c-d, whereas other particles of fluid which have low instantaneous tangential velocities are being transferred to points in a which have higher instantaneous tangential velocities. From this follows that, due to inertia the fluid will exert pressures or forces against the walls of a, and the directions of these pressures or forces which act perpendicular to the plane of a, will be in the different quadrants, as shown by the open arrows in the perspective diagram. From these open arrows it will be noticed that the direction of the above mentioned forces in the part of a which is on one side of the instantaneous equator g is opposite to the direction of the said forces in the other part of a which is on the other side of g, with the consequence that these forces act like a couple, tending to rotate the plane of a around an axis h-k which lie along the plane of g and passes through the center of the sphere. More specifically, this axis h-k, which may be called the instantaneous torque axis, will always be at right angles to the instantaneous resultant axis of rotation c-d, and pass through the center of the sphere and points g^1, g^2 where g and the plane of a intersect.

It is not wholly clear how FRY's device is an improvement over the earlier hydro kinetic coupling of CLOETE for which no claims are made to the generation of gyroscopic forces, yet even a cursory study of CLOETE's coupling would suggest that they may well be present. Two earlier liquid coupling devices are due to CASTILLO DIAZ & GRILLO GB 176326 (1923) and RATH FRENCH 723339 (1932). In the same year as CLOETE's coupling a not dissimilar coupling was proposed by ROMANOL & VACCARI ITALIAN 434093 (1948).

A thorough investigation of the problems associated with a gyroscopic type, infinitely variable, fully automatic mechanical power transmission speed changer or torque converter is given by SCHONBERGER US 4169391 (1979) strikingly similar to the device of SALOMON FRENCH 582444 (1924). Basically it comprises a rotatable main frame, two identical and separate sub-frame members which are able to contra-rotate an axes normal to the axis of rotation of the main frame with which they also rotate. Each sub-frame contains a pair of identical rotors which contra-rotate. In operation the rotors have their radii of gyrations varied automatically in a predetermined sequence in concert with the rotation of their sub-frame members. When the rotors are spinning and a precessional torque applied from an input power source such as to impart contra-rotation to the subframes the paired rotors generate an effective output torque to the output shaft via the main frame (see Figs 9.5.3A,B and C).

SCHONBERGER gives a useful mathematical analysis which is presented below:-

When a torque is applied to the spin axis of a spinning rotor, the following relationship exists:

$$T = I\omega\Omega$$

where T = torque,

ω = angular velocity of rotor spin,
Ω = angular velocity of the precession, and
I = the moment of inertia of the rotor.

Except for the moment of inertia I, the other symbols represent vectors and thus can be related vectorially by the righthand rule.

When torque is applied to the spinning rotors, said rotors react by precessing, thereby generating an effective torque about the X-X axis. The input of power for the applied torque is transmitted to the rotors via the input shaft 22 and the gear train designated in the drawings by parts numbered 58, 60, 66 and 68. Each spinning rotor will react in a plane of the spin axis A or B at right angles to the input torque by imposing a torque about its spin axis. Said imposed torque is, in turn, transmitted to the rotating axis A or B and thus to

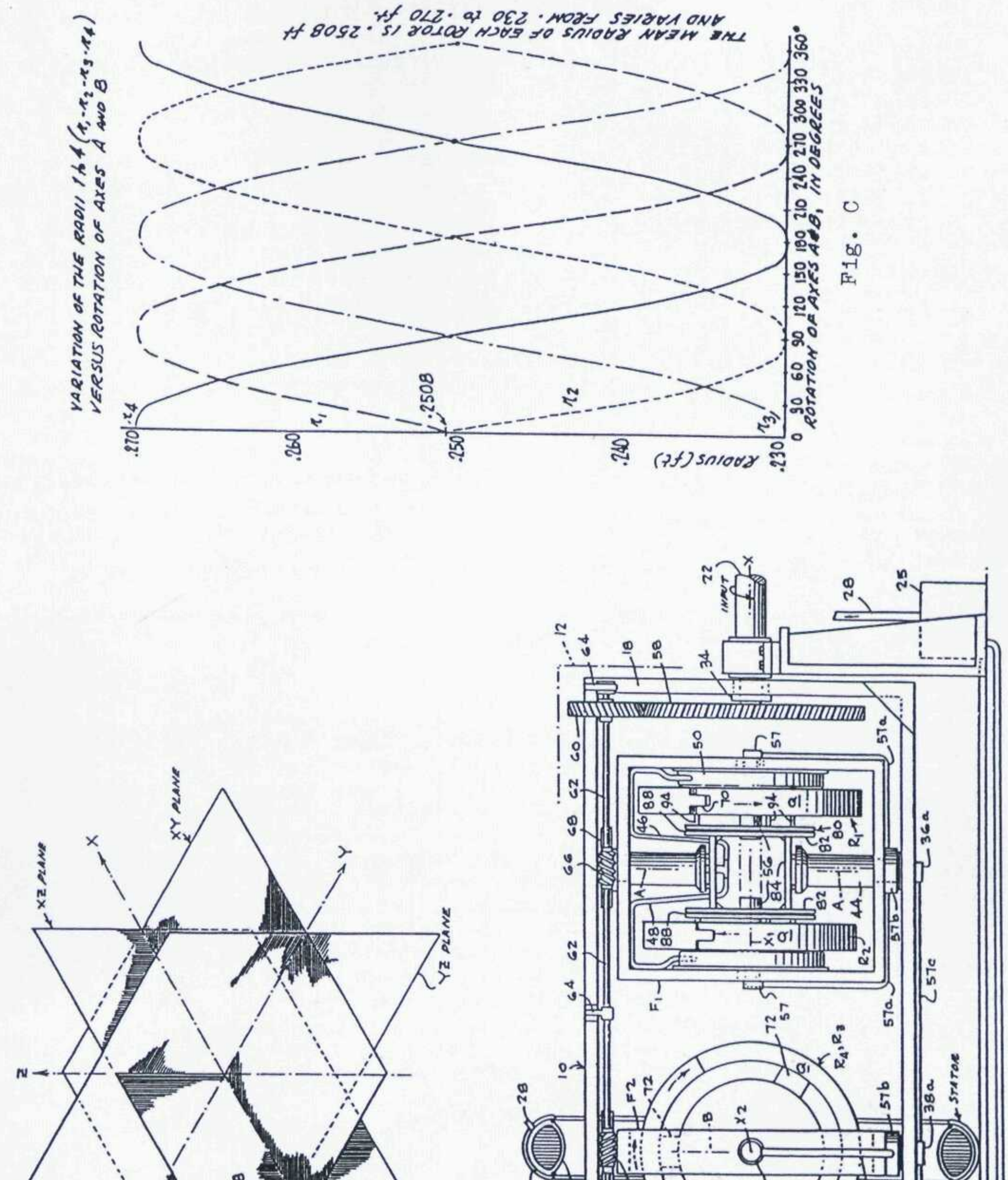


Fig. A

Fig. B

Fig. C

Figure 9.5.3

the main frame F, to which the output shaft 24 is connected. Since the main frame F rotates only about the X-X axis, it is apparent that in the initial position of the rotors, as shown in the Fig. 9.5.3A, only the rotor pair R3, R4 associated with axis A contributes effective torque to the frame F. Rotor pair R1, R2 will begin to contribute effective torque as it rotates away from its initial position where the aforestated angle $\phi = 0$. In view of the foregoing descriptions and illustrative drawings, it is apparent that rotors R3 and R4 produce torques that are opposite to each other. Thus, if they are made to spin at exactly the same angular velocity, and have equal moments of inertia, the torques will exactly nullify each other. The same rationale applies to rotors R1 and R2. Therefore, it is necessary to have an imbalance in order to develop a net torque, which is achievable by varying either the spin velocities or the moments of inertia of the rotors. For the purpose of this disclosure, it will be assumed that varying the moment of inertia is the more practical approach, and the mathematical analysis that follows takes said assumption into account. The following two basic equations applicable to rotor dynamics will be utilized:

$$T = I\omega\Omega, \text{ and}$$

$$I = mr^2,$$

where T = torque relating to output side due to precession of the rotor pairs;

I = moment of inertia of rotor;

ω = angular speed of rotor spin, which is the same for all rotors;

Ω = angular speed of rotation of the rotor pairs due to torque imposed at the input side, equal for both axes A and B;

m = mass of a rotor; and

r = radius of gyration of a given rotor. This is essentially equal to the distance from center of the rotor to the center of the cross-section of the outer rim, since most of the mass of the rotor is concentrated at the outer rim, as previously disclosed.

Considering torque T, let T_{yz} and T_{xz} be the torque in the YZ and XZ planes, respectively. There will be no torques in the XY plane. The main frame F knows no up or down and can only rotate about axis line X-X, and the three planes of reference rotate with it, keeping the same relative position. Since the axes A and B are always normal to the XY plane there can be no torques in this plane. The only useful torques are seen in line to the YZ plane. Torques in the XZ plane will merely provide a longitudinal twist to the main frame F which is useless and not needed or wanted. Therefore, the condition is imposed that the torque of the XZ plane is always by definition equal to zero, and represented as: $T_{xz} \equiv 0$. The following equations, based upon the geometry of the device, represent the adding of torques that lie in plane YZ, by using the relationships that apply to gyroscopes or anything that rotates in more than one plane.

$$T_{yz} = I_1\omega\Omega\sin\phi + I_2\omega\Omega\sin(180°+\phi)+I_4\omega\Omega\sin(90°+\phi)+I_3\omega\Omega\sin(270°+\phi) \quad (1)$$

which simplifies to

$$T_{yz} = \omega\,\Omega\,(I_1\sin\phi - I_2\sin\phi + I_4\cos\phi - I_3\cos\phi) \quad (2)$$

or

$$T_{yz} = \omega\,\Omega\,[(I_1-I_2)\sin\phi + (I_4-I_3)\cos\phi] \quad (3)$$

Similarly, adding the torques in the YZ plane, it follows:

$$T_{xz} = I_1\omega\Omega\cos\phi+I_2\omega\Omega\cos(180°+\phi)+I_4\omega\Omega\cos(90°+\phi)+I_3\omega\Omega\cos(270°+\phi) \quad (4)$$

Simplifying and applying the imposition that T_{xz} == 0, the following equation results:

$$I_1 - I_2 = \frac{(I_4 - I_3)\sin\phi}{\cos\phi} \qquad (5)$$

The discussion that follows indicates how to achieve the desired output torque by varying the radius of gyration. Reference is made to Fig 9.5.3C which graphically exemplifies how the four rotors vary as the axes A and B rotate through 360°.

Substituting equation (5) in (3), then

$$T_{yz} = \omega\Omega \left[\frac{(I_4-I_3)\sin^2\phi}{\cos\phi} + (I_4-I_3)\cos\phi\right] \qquad (6)$$

$$T_{yz} = \omega\Omega \frac{(I_4-I_3)}{\cos\phi} \qquad (7)$$

It is obvious that other than when the engine is stopped, or $\omega = 0$, T_{yz} must be greater than zero. Therefore neither $(I_4 - I_3)/\cos\phi$ nor $\omega\Omega$ can equal zero.

It will be seen later than $I_4 - I_3$ can be made to vary so that the value is zero at the null points, when $\phi = 90°$ or 270°, but at those points the spin axis X2 of rotor pair R3, R4 is collinear with the longitudinal axis X-X and therefore no torque can be contributed by that rotor pair.

Next, imposing the restriction that the spin energy of one pair of rotors is equal to that of the other pair and that this quantity is invariant so long as does not change, said equality can be expressed as

$$E_{1,2} = E_{3,4},$$

where

$E_{1,2}$ is the total spin-energy of the rotor pair R1, R1; and

$E_{3,4}$ is the total spin-energy of the rotor pair R3, R4. Further the total spin-energy of any one rotor, say rotor R1, may be expressed as

$E_1 = 1/2 m r_1^2 \omega^2$,

where the symbols are as before defined and

r_1 = radius of gyration of rotor R1.

Note: The same equation is applicable to the total spin-energy of the rotors R2, R3, R4, except that for each specific rotor, the radius of gyration for that specific rotor applies, i.e. r_2, r_3, r_4.

Utilizing the equations above, the total spin-energy of rotor pair R3, R4 is

$$E = 1/2 m\omega^2 (r_3^2 + r_4^2), \qquad (8)$$

$$2E/\omega^2 = m(r_3^2 + r_4^2) \qquad (9)$$

Equation (7) can be rewritten as

$$T_{yz} = \frac{\omega\,\Omega\,(mr_4^{\,2} - mr_3^{\,2})}{\cos\phi} \quad \text{or} \tag{10}$$

$$\frac{T_{yz}\cos\phi}{\omega\Omega} = m(r_4^{\,2} - r_3^{\,2})$$

Adding equations (9) and (10), the result is

$$2\,mr_4^{\,2} = \frac{T_{yz}\cos\phi}{\omega\Omega} + \frac{2E}{\omega^2} \tag{11}$$

$$= \frac{T_{yz}\ \ \omega\cos\phi\ + 2E\,\Omega}{\omega\,2\Omega}\text{, and}$$

$$r_4 = \frac{1}{\omega}\sqrt{\frac{T_{yz}\ \ \omega\cos\phi\ \ + 2E\Omega}{2m\Omega}}$$

The value of r_3 may be similarly obtained by subtracting equation (9) from equation (10). In like fashion, the equations governing the values of r_1 and r_2 are obtained by solving equation (5) for $I_4 - I_3$ and substituting in equation (3).

From the foregoing, the values of r are summarized as follows:

$$r_1 = \frac{1}{\omega}\sqrt{\frac{B+D}{C}}, \tag{12}$$

$$r_2 = \frac{1}{\omega}\sqrt{\frac{B-D}{C}}, \tag{13}$$

$$r_3 = \frac{1}{\omega}\sqrt{\frac{B-A}{C}} \quad \text{and} \tag{14}$$

$$r_4 = \frac{1}{\omega}\sqrt{\frac{A+B}{C}} \quad \text{where} \tag{15}$$

$A = T_{yz}\,\omega\cos\phi$,
$B = 2E\Omega$,
$C = 2m\Omega$ and
$D = T_{yz}\,\omega\sin\phi$.

A similar mathematical treatment, but one in which the changes with respect to time of the moments of inertia of the rotors are summed, proves the need for the stators as means for neutralizing an unwanted torque that is generated in the hydraulic fluid.

An example of the contemplated effectiveness follows, in terms of capability of transmit power. Power (P) equals the product of torque (T) and angular speed (Ω), or, $P = T\Omega$. As has been determined, the constant output torque T_{yz} is equal to $(I_4 - I_3)\omega\Omega$ when I_4 is at a maximum and I_3 is at a minimum. $(I_4 - I_3)$ is equal to $(mr_4^2 - mr_3^2)$, where m is equal to the mass of each rotor. Therefore power is equal to $(mr_4^2 - mr_3^2)\,\omega\Omega\Psi$ where ω is angular speed of the rotors, Ω is equal to the angular speed at which the rotors are rotated by the input shaft (engine), and Ψ is the angular speed of the output shaft. For said example, assume the following values:

m = 5 pounds (mass of each rotor);
r_4 = 0.33 feet (radius of gyration of one rotor at maximum);
r_3 = 0.30 feet (radius of gyration of other rotor at minimum);
ω = 1000 rpm = 104.7 radians per sec.;
Ω = 1000 rpm = 104.7 radians per sec.; and
Ψ = 1000 rpm = 104.7 radians per sec.

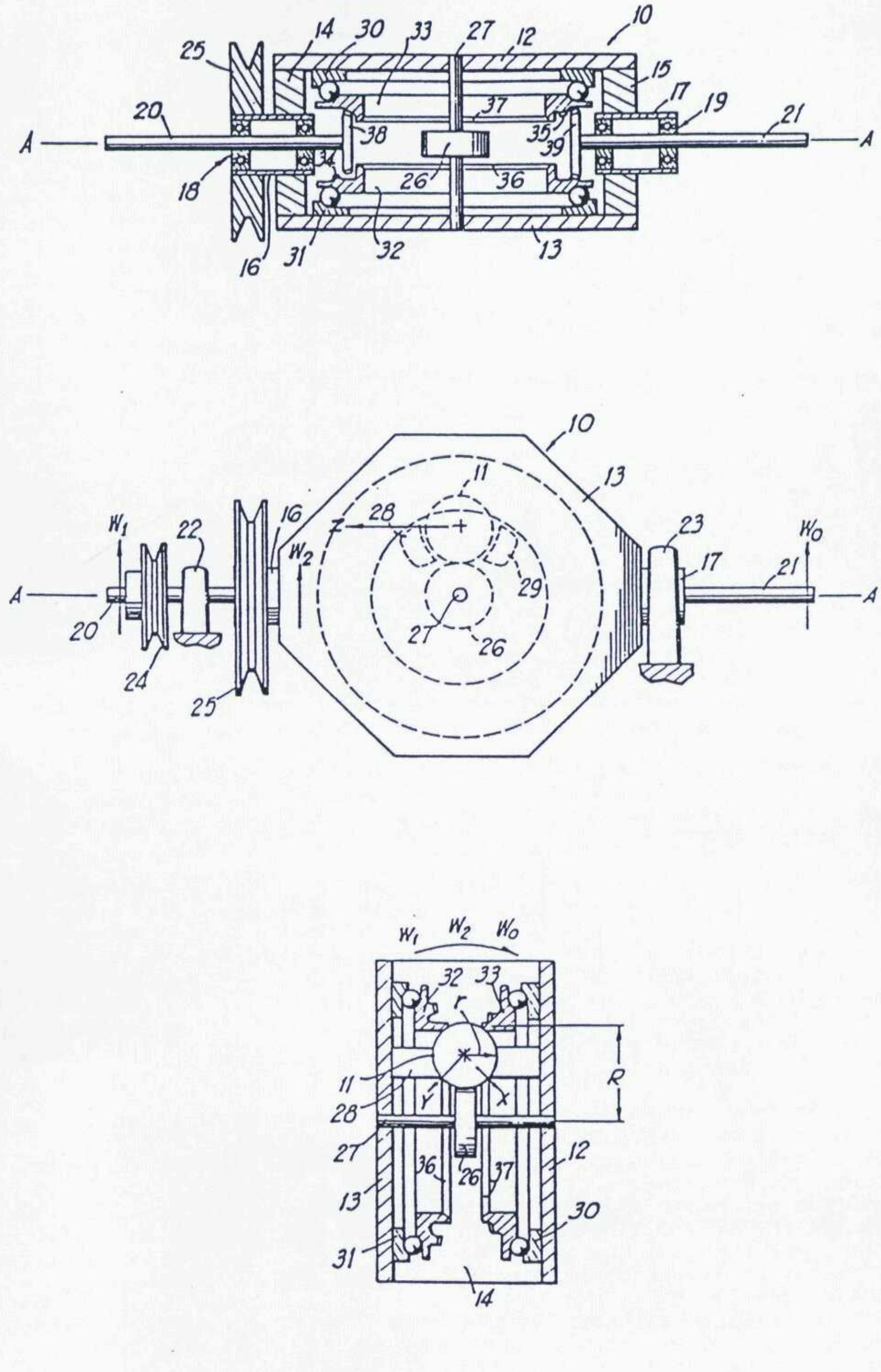

10
25
14
30
33
27
12
15
17
19
20
21
A
A
38
37
35
39
18
26
36
16
31
32
13
10
11
13
28
16
W1
22
W2
23
W0
17
29
21
A
A
20
27
26
24
25
W1
W2
W0
32
33
r
R
11
28
Y
X
27
26
37
12
36
13
30
31
14

Figure 9.5.4

Note that the maximum difference in radii of gyrations between the two rotors is quite small, 0.03 ft., or slightly more than one-third inch. The angular speeds are deemed to be on the low side.

Substituting the above numerical values in the formula for power, P, and converting foot pounds per sec. to horsepower, HP,

$$P = (5/32.2)(0.33^2 - 0.3^2)(104.7)^3, \text{ and}$$

$$P = 3368 \text{ ft. lb. per sec.} = 6 \text{ HP (approx.)}$$

Now if the angular speed of the rotor-spin is increased to 6000 rpm and the angular speed of the subframes to 6000 rpm by the engine, both being very reasonable figures, the power transmission value becomes about 220 Hp.

From the input data utilized, it is apparent that in comparison with other types of transmissions, a rotor-type mechanical power transmission will have a high power transmitting capacity per unit weight and size.

To facilitate numerical analyses, the above equation for power may be rewritten as

$$P = 2mr\ (\Delta r)\ \omega\Omega\Psi$$

where r is the average radius of gyration of the two rotors in a pair, and Δr is the difference of their values when one is at maximum and the other at minimum. It is evident from the above equation, as well as from the parameters used in the previous example, that rotor-type transmissions of widely varying capacities for all conceivable applications are possible and that small changes in sizes and rotational speeds will result in large changes in HP rating.
Using the parameters of the previous example as a reference, which resulted in 220 HP, by increasing the size of the rotors in all dimensions by 50%, and consequently increasing their masses approximately 3 1/3 times, and by doubling the Δr, say to .06 ft. (approximately 3/4 inch), the capacity will be increased to near 2200 HP. This demonstrates the capability of said inventive device to transmit high horsepower with a device of nominal size and weight.

Drive assemblies as disclosed by RASS US 3495479 (1970) and PRESTON US 3394619 (1968) are complex and expensive to manufacture. A much simpler gyroscopic mechanical torque converter is that shown by GLYNN M. in Australian patent specification 488057 and by ODELL E.I. US 4369673 (1983).

The invention of ODELL is directed to a gyroscopic traction drive assembly in which a spherical gyroscopic mass is caused to rotate about first and second orthogonal axes by input means which comprises a frame and a traction input member carried thereby. This combined motion causes the mass to develop gyroscopic output torque about a third orthogonal axis and this torque is imposed upon a traction output member also carried by the frame. In a preferred embodiment, the input axis of the frame and of a shaft connected with the traction input member and the axis of an output shaft connected to the traction output member are coaxial.

In the embodiment shown in Fig 9.5.4, the drive assembly comprises a suitable frame 10 which carries and locates a spherical gyroscopic mass 11. The frame may of course be constructed in any desired fashion but for the purpose of illustration, comprises a pair of opposite side plates 12 and 13 rigidly joined by opposite end members 14 and 15. The end members are provided with respective sleeves 16 and 17 which house the illustrated bearings 18 and 19 which receive the shafts 20 and 21 lying along the common axis A-A. As is illustrated, the frame is rotatably supported about the input axis A-A by means of suitable bearing blocks 22 and 23, the former of which supports the input shaft 20 and the latter of which supports the frame 10 directly through the sleeve 17. Drive input may be imparted to the input shaft 20 by suitable means such as the pulley 24 whereas drive input rotation is imparted to the frame 10 by suitable means such as the pulley 25 on the sleeve 16. The frame mounts a freely rotatable roller 26 by means of the cross shaft 27 which, in conjunction with the two locating members 28 and 29 and the traction members hereinafter described, positively locate the ball or mass 11 in offset relation to the axis A-A so that, as the frame rotates, the mass orbits along a path centered on the axis A-A and contained within a plane perpendicular thereto.

As shown for the member 28, the two members bridge between and are affixed to the side plates 12 and 13. Also fixed to the side plates are the annular bearing race members 30 and 31 which locate the respective traction output and input members 32 and 33. These two members are of annular form and have respective bevel gears 34 and 35 formed thereon and each is provided with an internal, bevelled end edge to define the torispherical traction surfaces 36 and 37. These traction surfaces engage and bear upon the mass 11 essentially with point contact and the mass 11 correspondingly is forced into essentially point contact with the roller 26. The two locating members 28 and 29 may be positioned to allow slight angular movement of the mass 11 about the axis of the roller 26. The input shaft 20 is provided with a bevel gear 38 in mesh with bevel gear 35 whereas the output shaft 21 is provided with a bevel gear 39 in mesh with bevel gear 34. Thus, whereas rotation of the frame 10 carries the mass 11 along the orbital path centered on the axis A-A and, in particular, at the point of intersection of the axis A-A and the transverse axis of the roller 26, the input shaft 20 imparts rotation to the input traction element 33. The rotation of the member 33, causes by virtue of its contact point with the mass 11 a rotation of that mass about the illustrated x axis, i.e., that axis passing through the center of the spherical mass 11 perpendicular to the y axis passing through the center and the contact point between the base and the traction surface 37. It will be appreciated that whereas rotational drive input must be imparted to the frame 10, the shaft 20 can be stationary or it can be rotated. In any event, the combined input motions imparted to the mass, namely, the axis A-A motion which is at the angular velocity ω_2 of the frame 10 and the rotation of the mass 11 about the x axis cause precessional motion of the mass about the orthogonal y axis. This precessional movement therefore develops gyroscopic output torque about the y axis and, correspondingly, of the output shaft 21, which may rotate at the angular rate ω_0. Using the notation N_1 and N_2 for the number of teeth of the respective gears 38 and 35 and ω_1 for the angular velocity of the input shaft 20, the angular velocity of the mass 11 about the x axis is:

$$\omega_x = \frac{N_1 R}{N_2 r} (\omega 2 - \omega 1) \text{ rad/sec}$$

where r is the radius of the mass 11 and R is the radius of the traction surface.

Similarly, the angular velocity of the sphere 11 about the y axis is:

$$\omega_y = \frac{N_4 R}{N_3 r} (\omega 0 - \omega 2) \text{ rad/sec}$$

where N_4 and N_3 are the notations for the numbers of gear teeth of the gears 39 and 34 respectively.
The angular velocity of the mass 11 about the z axis is,

$$\omega_z = \omega_2$$

Since the complete motion of the ball is known, as above, the angular momentum and time rate of change of angular momentum of the spherical mass or ball 11 can be computed.

Setting the time rate of change of the angular momentum equal to the moments that must be applied to the mass shows the output torque at the shaft 21 to be:

$$T_o = \frac{I\ N_1 N_4 R^2 \omega_2(\omega_1 - \omega_2)}{N_2 N_3 r^2}$$

where I is the mass moment of inertia of the ball 11. Since

$$\omega_2 = \omega_z \text{ and } (\omega_1 - \omega_2) = - \frac{N_2 r}{N_1 r} \omega_x$$

by substitution, the output torque expressed in terms of motion of the ball can be expressed;

$$T_o = -I\frac{N_4R}{N_3r}\omega x\ \omega z$$

which illustrates that the ball must possess motions about its x and z axes in order to produce an output torque. Since the motion ω_x is a function of $(\omega_2 - \omega_1)$ and since $\omega_z = \omega_2$, it is evident that in order to produce an output torque, rotary motion ω_2 must be imparted to the frame while the other input motion ω_1 may take any value, including zero, except that value where $(\omega_2 - \omega_1)$ is equal to zero. This output torque is of course limited by the maximum allowable normal force F and Hertz stress on the ball at the traction points and by the coefficient of traction μ as follows:

$$T_{max} = \mu FR\ \frac{N_4}{N_3}$$

As will be seen for the above equation for T_o, the output torque is wholly independent of the rotational speed of the output shaft 21 and can be controlled quite simply by variation in either one or both of the input speeds ω_1 and ω_2. It will be appreciated from the above that it is not essential that the gyroscopic mass be located, relative to axis A-A, such that the mass follows an orbital path, but that the center of the mass may, if desired, lie on such axis A-A. However, the most simple and straightforward arrangement is perhaps best realized by locating the mass center in offset relation to the axis A-A as shown in Fig 9.5.4.

IT IS OF INTEREST TO NOTE THAT PART TWO OF THIS REPORT DOES NOT MAKE A SINGLE REFERENCE TO THIS CHAPTER. THIS IS DUE TO THE FACT THAT NOT A SINGLE WORK DEVOTED TO THE PROBLEMS OF GYROSCOPIC GEARING, OUTSIDE OF PATENT LITERATURE, IS KNOWN TO ME. THIS IS IN THE NATURE OF A SURPRISE; BUT EVEN ERUDITE WORKS ON GEARING GENERALLY AND SIMILAR TEXTS ALL SUFFER FROM THIS SERIOUS OMISSION.

10. THE USE OF THE GYROSCOPE AND GYROSCOPIC FORCES IN OPTICAL AND RELATED DEVICES

It has long been a desideratum to improve the quality of vision for an observer in a vessel or vehicle that is constantly disturbed, such as for example a ship, an aircraft or a tank, more especially when the observer is using an optical aid that magnifies his field of view. The problem is one of stability and the gyroscope enters the subject in limine, since it possesses this property, as is extremely well known to anyone who has observed it in action.

In this chapter I deal with optical and related devices that make use of the gyroscope and/or gyroscopic forces; yet it is but one form of the broader subject of stability that is taken in Chapter 11.2. The question of sub-division is one of semantics; a telescope when aligned with a gun barrel is an optical sight; an accurate optical navigation aid to a specific destination depending what happens on arrival, may be a target seeker or a bomb sight; a laser beam with a beam riding missile may be outside of the visual range of the electromagentic spectrum yet sufficiently close in subject matter to demand inclusion within the term 'optics'. Here it is decided to exclude Serson's gyroscopic sextant, clinometers and target seekers.

PONTEVÈS and RAFAT (1972) have given a short dissertation on the subject of the stabilization of vision, but it is restricted to the work of de la Cierva, Alvarez, Bezu and Deramond and is in the French tongue.

Here it is proposed to explore the subject historically under ten heads:-

i. Telescopes (including binoculars and monocolars).
ii. Gunsights.
iii. Bombsights.
iv. Cameras.
v. Panoramic Sextants.
vi. Panoramic Film viewer.
vii. Periscopes.
viii. Navigational aids.
ix. Rangefinders.
x. Stabilization of image (missiles etc.).

Before we proceed it should be pointed out that it has been shown inter alia by ARNOLD & MAUNDER (1961) that there are two principal methods by which gyroscopes may produce a stable platform and both depend essentially on the stability of the axis of rotation of a high speed rotor of a gyroscope and the extremely slow rate at which it deviates from its original position if the applied torques are small, the stability is referred to correctly as direct stabilization, since it is now common practice in other fields to use the gyroscope purely as a sensor of changes in direction and link this via pick-offs to control motors that effect the stabilization which is called in contra-distinction to direct stabilization, servo-controlled stabilization.

10.1 TELESCOPES

The first stabilized telescopes appealed to the direct method referred to above and this was early advanced by KRELL US 940329 (1909) in Germany who assigned his rights to Siemens-Schuckertwerke, and by BARR. A. and STROUD. W. GB 17291/1910 in England. Krell was concerned with the stabilization of an optical device for determining the direction of travel of an airship or balloon. Two gyroscopes were deployed (Fig 10.1.1) but Krell gives no indication that he understood the problem in any depth. Barr & Stroud are more specific and desire to keep a telescope on target and they suggest that only the objective and fiducial mark need be carried on the single gyroscope.

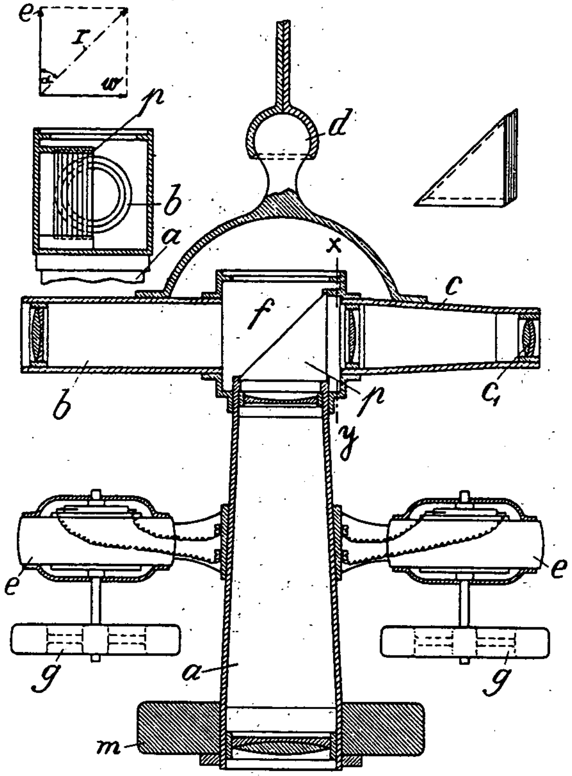


Figure 10.1.1

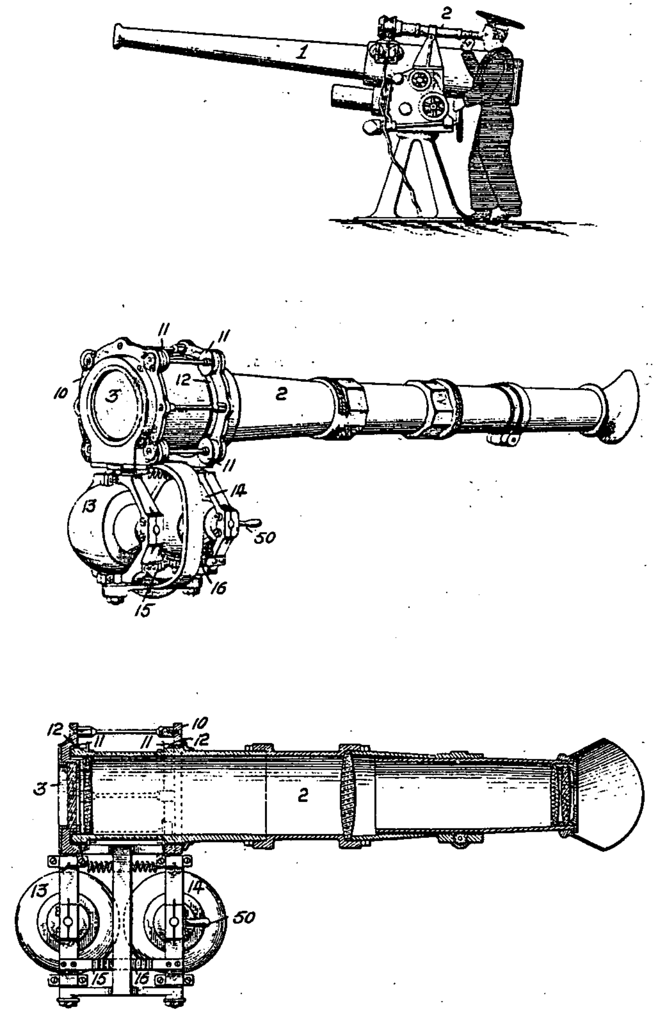


Figure 10.1.2

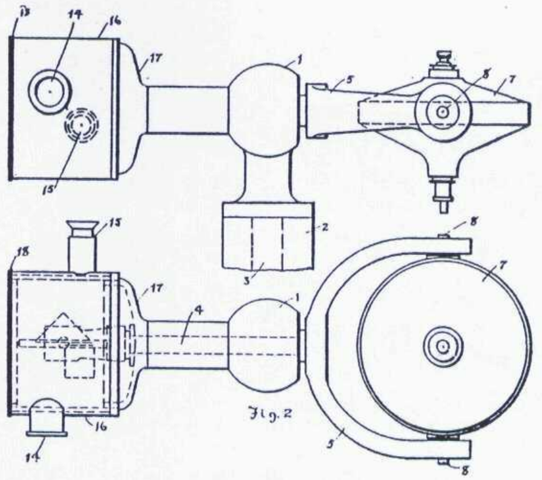


Figure 10.1.3

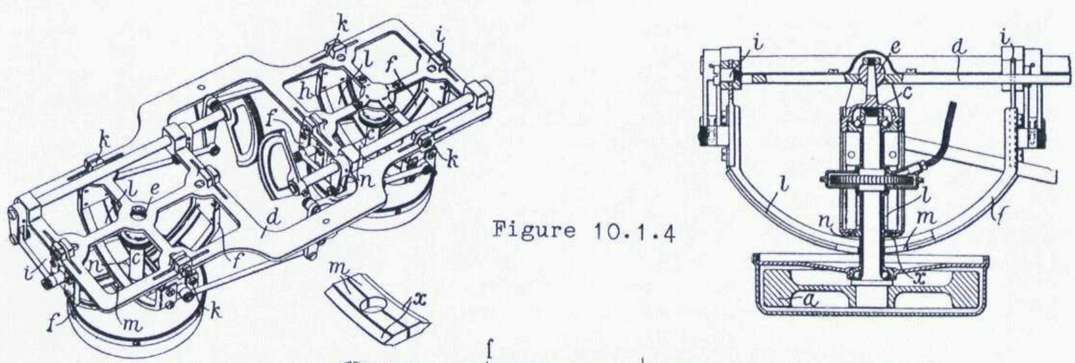


Figure 10.1.4

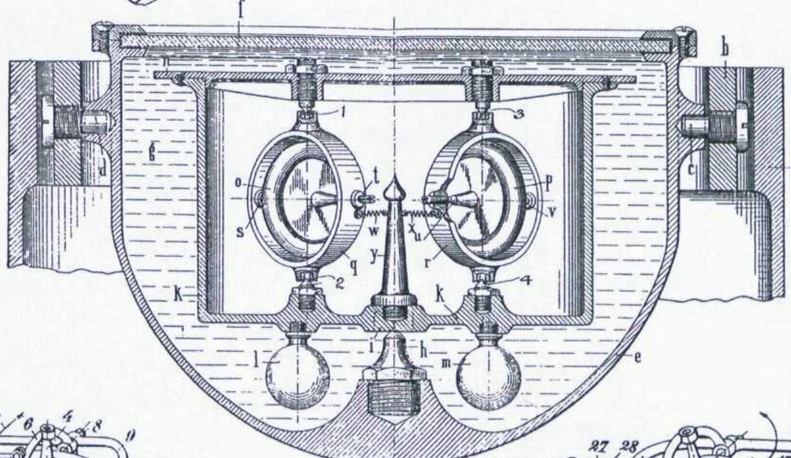


Figure 10.1.5

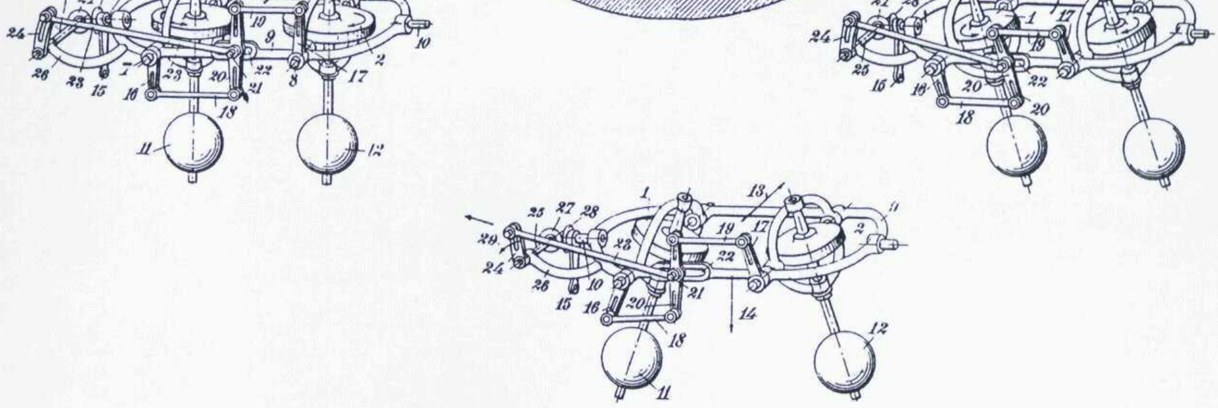


Figure 10.1.6

GARDNER GB 24647/1914 proposed a gyroscopically stabilized support to hold a telescope, but again little detail concerning the single gyroscope is given. A more useful device (Fig 10.1.3) is due to the work of HENDERSON. J.B. GB 6977/1915 professor of applied mechanics at the Royal Naval College, Greenwich, who later as Sir. J.B. HENDERSON made numerous contributions to this field of enquiry. It is however, salutary to remind ourselves before we advance further that de FERRANTI GB 24112/1909 ANSCHUTZ & CO. GB 10440/1911 and MARMONIER GB 23494/1911 had all proposed dual gyroscopic means for the stability of moving devices Fig 10.1.4, 5 and 6, yet FISKE US 1363861 (1920) is one of the first to couple two gyroscopes by means of sector gears and attach them to a telescope (Fig 10.1.2).

Some advances in the direct method are reported by KRUPP GB 146477 (1920) and ANCIENS ETABLISSEMENTS SAUTTER-HARLE GB 146847 (1920).

HENDERSON GB 165028 (1921) proposed a telescope in which the observer sits with his back to the object, the image of the object in the focal plane being steadied by means of two or more mirrors on the casing of a gyroscope rotor. Proposals were also advanced in which a prism was stabilized by the gyroscope and this was combined with a travelling achromatic prism to keep the object in the centre of the field when the gyroscope axis was tilted. In another construction HENDERSON uses an objective and an eyepiece that move with the ship and an inverting lens between them that is stabilized by the gyroscope to form an upright image in the second focal plane. KRUPP GB 178459 (1922) showed how auxiliary gyroscopes may be deployed to prevent deviations of the gyroscope axis that is used to stabilize the line of sight of a telescope.

GRAY. P.W. GB 232759 (1926) assigned to VICKERS LTD. a useful stabilized sighting device in which the gyroscope stabilized a prismatic element made of one piece of glass consisting of both a triangular and a roof prism portion so disposed that light entered the triangular portion normal to one surface and left it normal to the other surface after double reflection by the dihedral portion.

ZEISS & STEINLE GB 284871 (1928) appear to be the first to propose stabilizing the line of sight by servo-controlled means, the stabilizing apparatus being placed apart from the instruments to be stabilized.

NAAMLOOZE VENNOOTSCHAP NEDERLANDSCHE TECHNISCHE HANDEL MAATSCHAPPIJ GIRO GB 353137 (1931) place the sighting device for measuring the variations in direction of distant objects from a ship so that it is stabilized in azimuth by a gyroscopic compass repeater. SCHNEIDER & CIE with FIEUX GB 382253 (1932) prefer to use twin gyroscope rotors having a common axis of rotation but in contrary directions of rotation and precession. The gyroscopes are geared together by toothed sectors and pendulously supported, not unlike the construction due to FISKE.

ZARDECKI GB 559895 (1944) proposed the use of two rotors for stabilizing a telescope, the two rotors are mounted in a gimbal ring so that their axes are unidirectional, and so interconnected and driven that under the action of an external force each rotor has a different precession speed, the interconnection between the rotors being such that the rotor with the greater speed of precession imposes this speed on the other rotor to set up a second precession therein to eliminate the first mentioned precession in each rotor.

BENDIX AVIATION CORPORATION GB 623009 (1949) return to the use of gyroscopic stabilization by means of a servo-control as with ZEISS & STEINLE but they use a control signal that is a combination of two signals derived separately in response to the degree of pitch and bank of the vehicle to which the telescope is attached.

THE SPERRY CORPORATION GB 749987 & GB 749988 (1956) show how a star follower telescope may be stabilized in space by the use of three gyroscopes orthogonally placed in a stable element (see Fig 10.1.7).

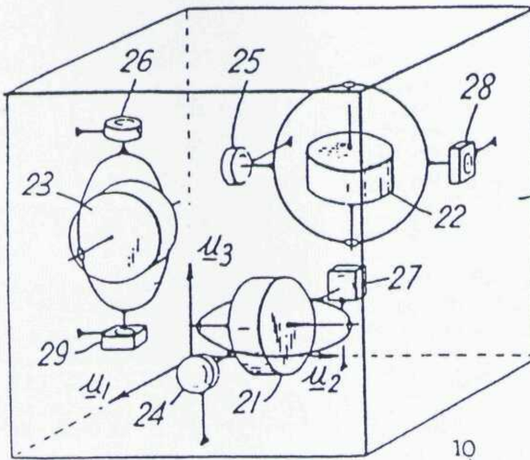


Figure 10.1.7

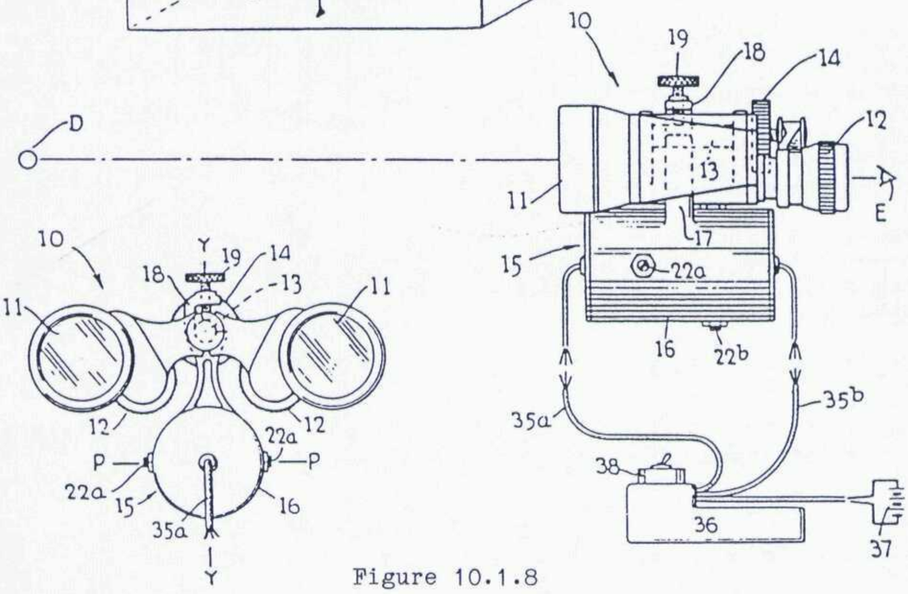


Figure 10.1.8

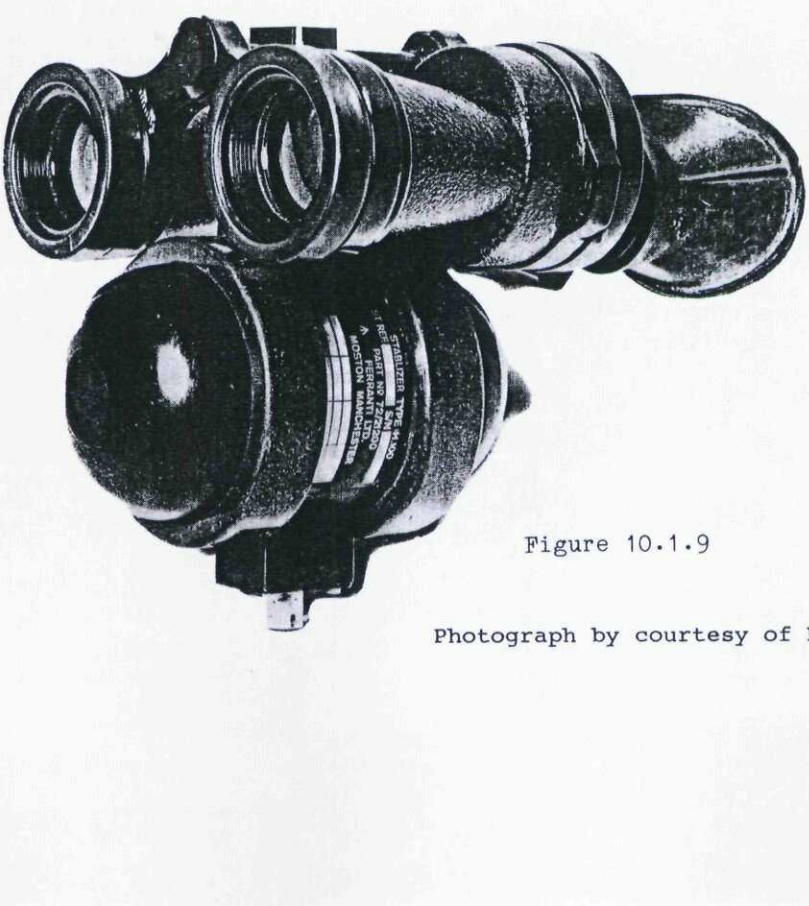


Figure 10.1.9

Photograph by courtesy of Ferranti Ltd.

The art now divides broadly into those who wish to stabilize the whole optical device and those like BARR & STROUD before them who think that superior results may be obtained by the stabilization of one or more parts of the optical train. In the first of these divisions is the work of KENYON US 2570130 (1951) US 2811042 (1957) directed primarily to hand-held binoculars so that by means of a pair of spring restrained coaxial rotors similarly rotatable and with their gimbal axes orthogonally disposed in the vertical and horizontal direction or disposed preferably at 75 degrees of arc, the rotors respectively stabilize the binocular about the yaw (YY) and pitch (PP) axes. (See Fig 10.1.8.)

A not dissimilar instrument in wide use is that of FERRANTI, the H-100 Hand-Held Stabilizer, Fig 10.1.9. The optical resolution of the Ferranti instrument is improved by a factor of about 2 1/2 when the gyroscopes are in operation from a 28 volt source. Another gyroscopic stabilizer to be clamped to a binocular is that of KUIPER US 2871707 (1959). He suspends beneath the instrument two rotors in frames that are orthogonal to one another but inclined at 45° to the vertical. The rotors are made to rotate in opposite directions.

BABAYEV and GRISHMANOVA (1968) have described the theory of a gyroscopic damper for binoculars according to KENYON'S proposals in 1957. Taking the equations of motion and using the method of Ishlinskiy based on Euler's equations and examining the stability of the system by means of Hurwitz polynomials* they show that all of the roots of the characteristic equation have negative real parts and that in consequence the motion of the system is stable. They conclude that the gyroscope damper of KENYON for hand-held instruments such as a binocular effectively reduces the angular vibrations of the hands of the user by a factor of at least a thousand in a frequency range of a fraction of a cycle per second to several tens of cycles per second. They go on to show that the angle of divergence of the precession axes of the two gyroscopes is defined by the expression

$$X = \arctan \sqrt{\frac{J2}{J1}}$$

where J1 is the moment about the outer axes.
J2 is the moment about the inner axes.

so that in the case of a load with different moments of inertia along the vertical and horizontal axes, the gyrostabilizer is designed in such a manner as to exert different damping forces along the two axes.

BABAYEV and SUKHOPAROV (1972) have set out the design parameters of the gyroscopic stabilizer, referred to above, including the measured dependence of the resolving power of the human eye on the relative velocity of the object in the view finder. Their findings are of considerable importance and since they are short I give them below in-extenso. I may at this point refer the reader to the work of OSTROVSKAYA (1972) who proposes an empirical formula for determining the daytime efficiency of visual instruments, such as binoculars, over the field of view.

In justifying the need for stabilization of viewing instruments and in the design of an appropriate gyroscopic unit to achieve this, BABAYEV & SUKHOPAROV think that it is necessary to take into consideration both the reduction in visual contrast and the deterioration in the resolving power of the eye in the case of relative movement of the object.

The displacement of the image during the lag time θ of the eye, while the position of the latter is fixed, is equal to $\omega_o\theta$, where ω_o is the relative angular velocity of the object.

* See SNEDDON's Encyclopaedic Dictionary of Mathematics for Engineers and Applied Scientists (1976) p.343.

Since eye movement lags behind the object movement there exists some difference $\omega_o - \omega_e$ in the object velocity and the tracking velocity of the eye, so that there is an image displacement $(\omega_o - \omega_e)\theta$ on the retina. If, in viewing a stationary object, the limit of resolution of the eye is δ_e then this resolution increases when there is image movement and amounts to

$$\delta_v = \delta_e + (\omega_o - \omega_e)\theta$$

in the direction of motion. It is assumed that the object is tracked by the eye only; this is true for instruments in which the eye is fixed with respect to the eyepiece by means of a forehead or eye-level grip, or fixed support, and in which there are no other devices for moving the line of sight to follow the object.

In instruments with no such fixation, such as in hand-held instruments (binoculars, still and motion-picture cameras, sextants, etc.), the object is tracked not only by the eye, but also by turning the head and even the body of the observer.

A moving image of a target, the angular velocity of which could be varied over a wide range, was formed in the field of view of a telescope with a 15x magnification. The experiment was performed for two cases: fixation of the eye on stationary cross-hairs in the field of view (a) and tracking of a moving object by the eye (b). See Fig 10.1.10. In both cases the head of the observer was fixed in position by a forehead grip and an eye peephole, and the viewing telescope was stationary. The variation of the resolution, i.e., $(\delta_v - \delta_e)/\delta_e$, and the resolution of the eye δ_v during object motion in the field of view are plotted in absolute units along the ordinate. The angular velocity ω_o of the object (target) and the relative angular velocity of the object in the eyepiece field of view (in deg/sec) are plotted along the abscissa.

It is seen that the resolving power starts to deteriorate when the object velocity in the field is 1.5 deg/sec. Thus, if one must count on the resolution of the eye in the design of a visual instrument, the allowable angular velocity of the object in the eye's field of view should be limited to 1.5 deg/sec.

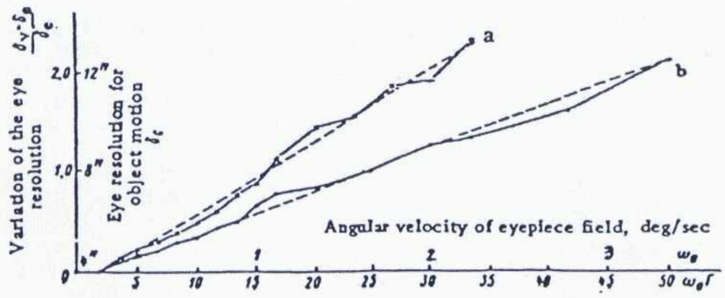


Figure 10.1.10

The required value of the gyroscopic damping factor should be no less than

$$W_1 \geqslant \frac{500}{0{,}8} \approx 625.$$

To design a gyrodamper it is also necessary to choose the damping frequency range. The lower limit of the range can be derived both from the low-frequency components of the platform rolling and from the conditions for the shift of the binocular field of view from one object to another. Tentatively, ω_1=1-3 sec^{-1}. The upper limit for a nonvibrating platform will be determined by the hand tremor, and amounts to $\omega_2 = 2\pi f_{tr} = 60$ sec^{-1}.

The angular velocity of the object in the eye's angular field also depends on system magnification Γ and is equal to $\omega_o \Gamma$.

As an example consider hand-held binoculars with a magnification of 10x, with the observer on a stationary base, and a stationary object. The tremor of the hands of the observer, for simplicity, will be considered sinusoidal, with an amplitude of 15 arc min and a frequency of 10 Hz; other spectrum frequencies are neglected. The maximum angular velocity of the hand tremor is ω_o = 15 deg/sec, the maximum angular tracking velocity of the eye will be $\omega_o \Gamma$ = 150 deg/sec, and the angular resolution will be reduced by a factor of about 5 compared to the case of no tremor. In this case the resolving power with binoculars is 5 times smaller than that of the unaided eye (see figure).

As a result, the observation with such binoculars is actually done at an instant when the hand tremor is relatively low, by straining the eye muscles; therefore it is not very suitable for use from a moving platform. Mass production is limited to binoculars with a magnification of 6 to 8x, in which the resolving power at the maximum angular velocity of hand tremor is reduced by no more than a factor of 2 to 3.

In practice, it is desirable to have binoculars with a magnification of 30 to 50x for observing floating objects from shipboard, for identifying the license numbers of speeding vehicles, and in a number of other cases.

Given the lack of a gyroscopic damper these binoculars obviously cannot be used and therefore are not made. One can assume a priori, in the design of such binoculars with a gyrodamper, that the actual resolving power will be half the theoretical. Then, according to our figure, the relative angular image velocity must be limited to $\omega_o \Gamma \leq 25$. For naval binoculars with a magnification of 33x, the angular velocity of the object in the eyepiece field is 32 times greater and the allowable angular velocity of the object amounts to $25/(33 - 1) \approx 0.8$ deg/sec.

Assuming that the low-frequency rolling of a large ship and its vibration do not significantly increase the 15-deg/sec angular velocity of hand tremor, the angular velocity $\omega_o \Gamma_1$ of an object being examined from a ship should be taken equal to 500 deg/sec in the field of view of a stationary eye.

The foregoing discussion applies to hand-held binoculars. With respect to other visual instruments, the choice of the input parameter for the design of a gyroscopic device will be somewhat different although the reasoning will be analogous. For still and motion-picture cameras the maximum exposure time, and not the lag behaviour of the eye becomes a factor in the design.

Another device that seeks to stabilize the whole binocular is that of FLANNELLY US 3742770 (1973). It uses two gyroscope rotors each overhung and contrarotating, its precursors go back to the ideas of MARMONIER GB 23494/1911.

We turn now to the stabilization of parts of the optical train by gyroscopic means; one of the first is that of JENSEN US 2829557 (1958) who proposes a Porro-prism binocular telescope using a low-pass mechanical filter which may be a gyroscope not wholly dissimilar to that of DOIGNON GB 9737/1911 and HARTMANN & BRAUN German patent 2403696 (1911) or his own stabilization device JENSEN US 2688456 (1954). The 'DYNALENS' of de la CIERVA US 3212420 (1965) and DYNASCIENCES CORPORATION GB 1056528 (1967) is better known; it deploys a liquid prism or lens of a variable geometry that is determined by sensitive gyroscopic means. The liquid or fluid lens is further developed by de la CIREVA as disclosed in his U.S. Patent Specification 3514192 of 1970. The lens has transparent plates juxtaposed to encapsulate a liquid such as benzene bromine, ethyl alcohol, ethyl ether, glycerol and combinations thereof.

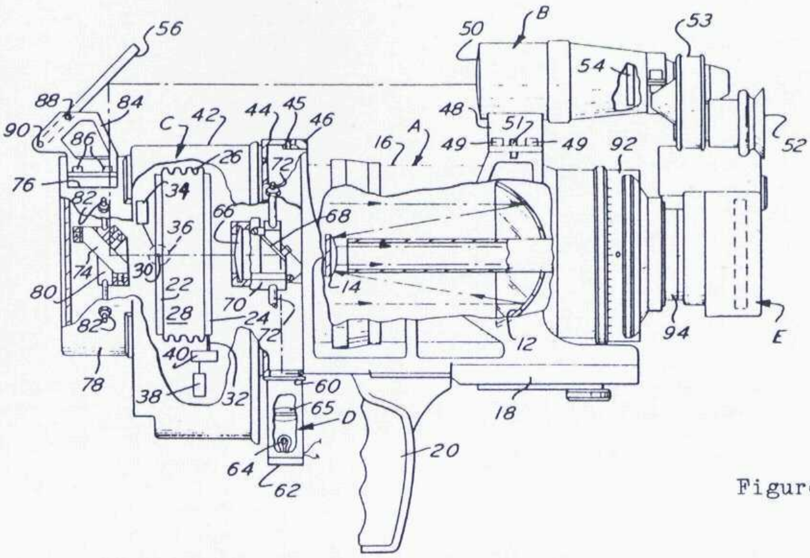


Figure 10.1.11

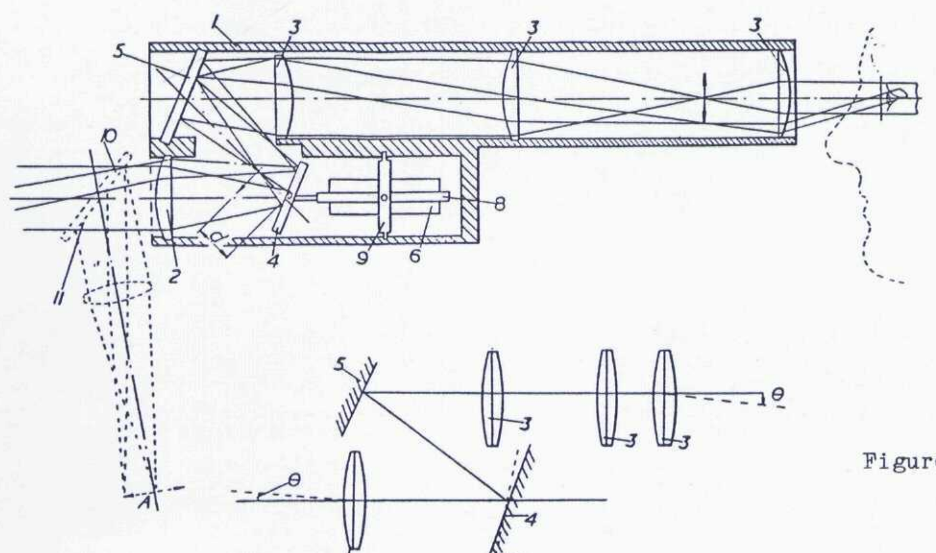


Figure 10.1.12

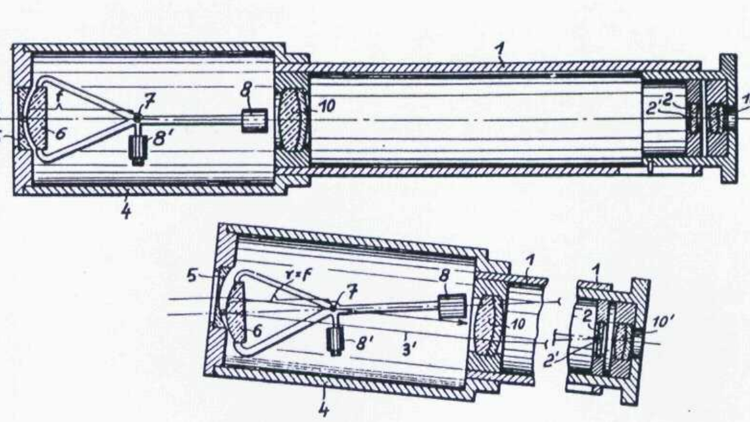


Figure 10.1.13

More recently the gyroscopically controlled variable geometry lens appears in the high power optical device of de la CIERVA US 3503318 (1970) (See Fig 10.1.11) and the dual gyro image motion compensator of SHIN and RICHARDS US 3756687 (1973).

BARR & STROUD GB 1093131 (1967) disclose a stabilized telescope which seeks to mitigate the translational movement of the image that is produced by pitching and yawing movements of the body to which the telescope is attached (See Fig 10.1.12). The telescope includes an objective lens system 2 and an erecting eyepiece lens system $3_1.3_2.3_3$. optically interposed between the two lens systems is a gyro-stabilized reflector in the form of a plane mirror 4. The mirror is secured to a gyro comprising a rotor 6 in two gimbals 8. and 9. The reflecting surface of the mirror is so positioned that the following relation hold $d \nless \frac{f1-f2}{2}$ where d is the distance between the reflecting surface and the focal point of the objective lens system measured along the optical axis and f1 and f2 are the focal lengths of the objective and eyepiece lens system respectively.

In a practical refinement

$$\frac{f1-f2}{2} < d < \frac{f1-f2}{2 \cos i}$$

where i is the angle of incidence of the optical axis on the reflecting surface.

In the interesting system of ALVAREZ GB 1099026 (1968)+ mating plano-concave and flano-convex lens are arranged to be rotatable with one another, one component being fixed and the other supported by a substantially 'free' gyroscope. The idea of compensating for small inclinations in an optical instrument using a pair of lens, one fixed and one pendulously suspended so that it swings about an axis perpendicular to the optical axis appears from the earlier work of RANTSCH* US 2959088 (1960) Fig 10.1.13 the pendulous lens thereby forming an optical wedge with the fixed lens and satisfying the condition

$$\sum_{i=1}^{n} r_1 \Phi_1 = 1$$

where r_1 represents the distance of the principal point of the ith pendulously supported lens from the axis, Φ_1, being the refractive power of this lens and n being the number of lens pairs. Some other works toward steadying a telescope by the use of pendulous means are due to DRODOFSKY US 2741940 (1956) US 2779231 (1957) his double Schmidt system (Fig 10.1.14) being most ingenious.

ALVAREZ is fully aware of the drawbacks associated with pendulum suspensions and provides relative lens rotation to compensate in two dimensions thereby offering to the user an instrument adapted for compensation in respect of accidental motions, to this end he mounts each second lens of each set to a gyroscope. Fig 10.1.15.

ALVAREZ not only concerns himself with stabilization but also with the correction of chromatic and coma aberrations in the optical system. His detailed dissertation on these recondite problems is as follows:-

In the case of a telescope, field glasses, etc., having an associated field of view in which the image is to be stabilized, the total deflection angle of light rays through the respective doublets is modified by an additional compensation, $\frac{\theta}{M}$, and is given by the expression: $\theta(1 \pm \frac{1}{M})$ where M is the magnification power of the telescope.

* This has developed into the STEDI-EYE U.S. Patent Specification 3728948 of D.B. FRASER (1973) with references to numerous earlier patent specifications.

\+ See US Patent Specification 3378326.

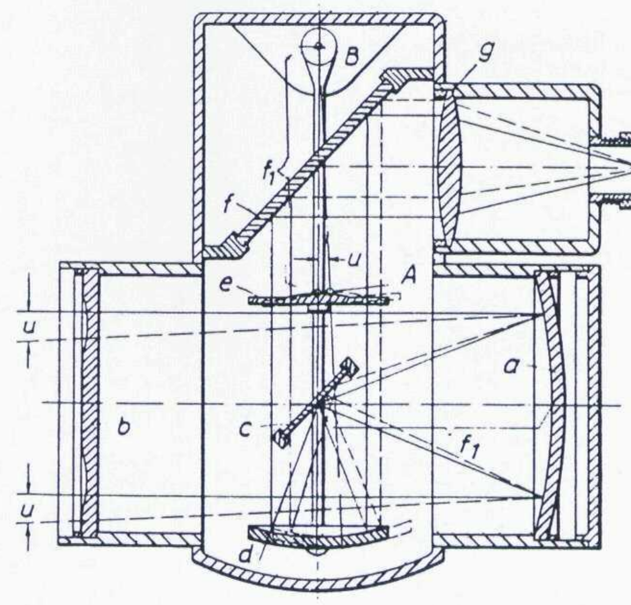


Figure 10.1.14

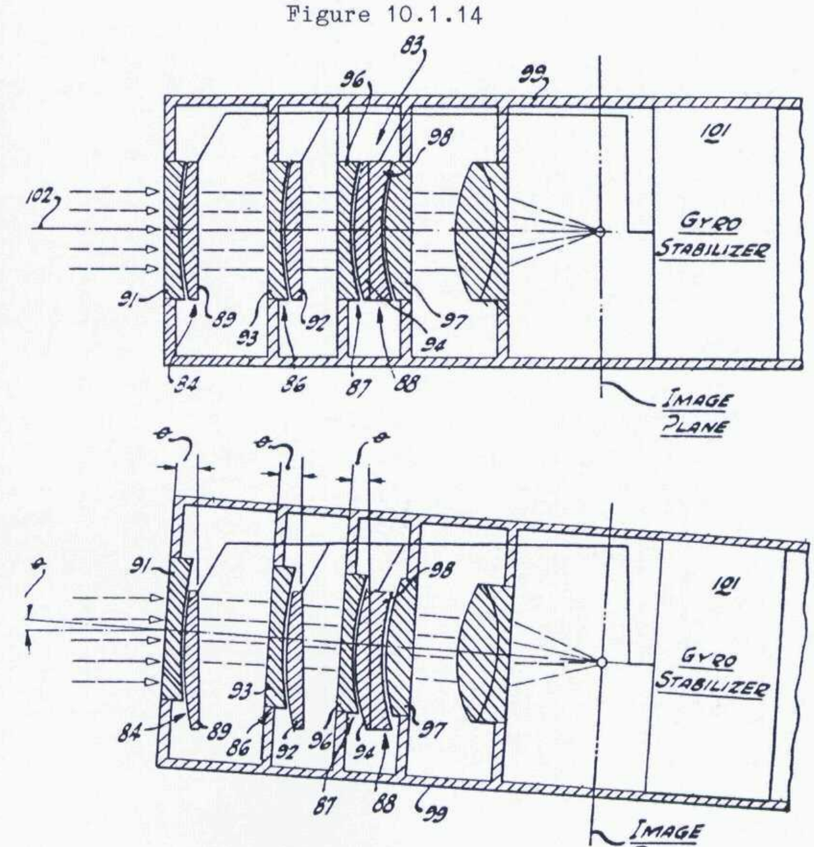


Figure 10.1.15

Chromatic abberation is overcome in the compound lens by arranging the doublets 84, 86, and 87 to generate positive prisms and the doublet 88 to generate a negative prism. In this regard, the concave lenses 91, 93, and 96 of doublets 84, 86 and 88 are fixedly secured to the optical housing 99 while the convex lenses 89, 92, and 94 of these doublets are secured to the gyro stabilizer 101. However, the positive, or convex, lens 97 of doublet 88 is secured in fixed relation to the housing while the negative, or concave, lens 98 of this doublet is secured to the gyro stabilizer. Preferably, the convex lens 94 of doublet 87 and the concave lens 98 of doublet 88 are cemented together and secured as a unit to the gyro stabilizer. It is by virtue of this arrangement that the doublet 88 generates a prism of opposite sign to the prisms generated by the other doublets 84, 86, and 87 of the compound lens. Furthermore, the lenses of the doublets 84, 86, and 87 are formed of crown glass, while the lenses of the invert prism generating doublet 88 are formed of flint glass. Various crown and flint glasses may be selected for employment in the doublets of the compound lens to provide a total dispersion through the crown glass doublets which is equal to the dispersion through the flint glass doublets. More particularly, assuming that the crown glass doublets provide dispersions, D_1, D_2, ... D_n, while the fling glass doublet provides a dispersion, D' such that $D' = \Sigma D$, then by virtue of the prism generated by the flint glass doublet being of inverse angle to the prisms generated by the crown glass doublets, the net dispersion through all the doublets of the compound lens is zero and such compound lens is hence achromatic, i.e., no chromatic abberations are produced by the compound lenses. Expressed in another manner, the condition for achromatization is given by the relation $\Sigma \frac{\Delta_c}{\gamma c} = \frac{\Delta_f}{\gamma f}$, where: Δ_c is a measure of the deviation of the FraunhoferD line in the respective crown glass doubles, Δ_f is a measure of the deviation of the D line in the respective flint glass doublets, γ_c is the reciprocal of the dispersive power of the crown glasses of the respective doublets for two standard wavelengths such as the F and C Fraunhofer lines which are to be achromatized, γ_f is the reciprocal of the dispersive power of the flint glasses of the respective flint doublets for the same two standard wavelengths; $\Delta_c = n_d - 1$, where n_d is the index of refraction for the D line of the particular crown glass employed in each doublet; $\Delta_f = n'_d - 1$, where n'_d is the index of refraction for the D line for the particular flint glass employed in each doublet;

$\gamma c = \frac{\Delta c}{n_1 - n_2}$, where

n_1 and n_2 are the indices of refraction of the crown glass of a given doublet for the two standard wavelengths being achromatized; and $\gamma_f = \frac{\Delta_f}{n'_1 - n'_2}$, where n'_1 and n'_2 are the indices of refraction of the flint glass of a given doublet for the two standard wavelengths being achromatized. Thus, by selecting crown and flint glasses having characteristics satisfying the above equation, the compound lens is made achromatic.

It should be noted that the conditions for total deflection of the doublets of the compounds lens commensurate with stabilization of the image against deviations of the housing from the line of sight axis, may be expressed in terms of the measures of deviations of the D line in the respective crown and flint glass doublets, i.e., in terms of Δ_c and Δ_f for the respective doublets. More particularly, considering image stabilization first in the instance of a camera, the total deflection D of the D line through the compound lens is equal to the deviation angle θ of the instrument housing from the line of sight. Thus, taking the sum of the deflections through the individual prisms generated by the respective doublets, the foregoing expands to: $\Sigma \phi_c - \Sigma \phi_f = \theta$, ϕ_c being the deflection of the D line by each crown glass doublet, and ϕ_f being the deflection of the D line by each flint glass doublet. As noted hereinbefore, the deflection, ϕ, of a thin prism in air is given by: $\phi = (n-1)\alpha$, where n is the index of refraction of the prism for a particular wavelength and α is the prism angle. In the present instance, $\alpha = \theta$, and for the D line, $n_d - 1 = \Delta_c$, in the instance of the crown glass elements, and $n'_d - 1 = \Delta_f$, in the instance of the flint glass element. Accordingly, the equation for stabilization of the image noted above may be expanded as follows: $\Sigma \Delta_c \theta - \Sigma \Delta_f \theta = \theta$. Dividing both sides

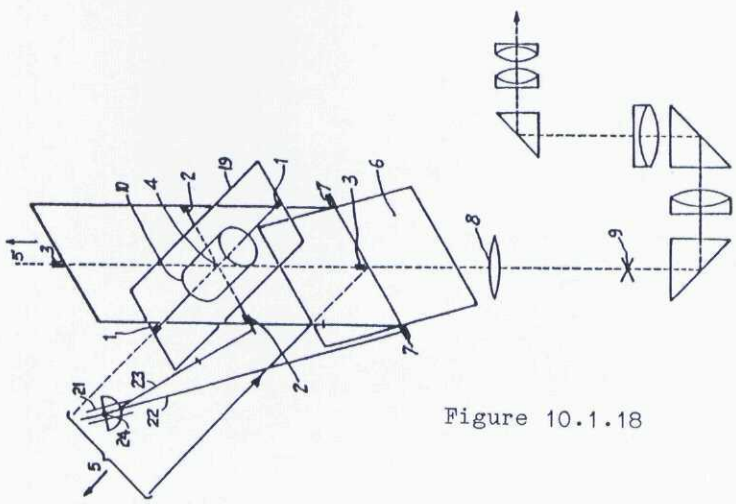


Figure 10.1.18

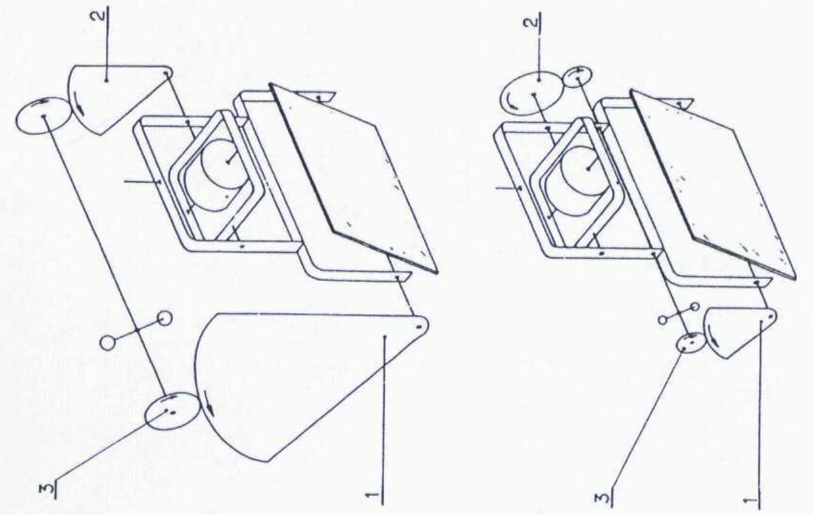


Figure 10.1.17

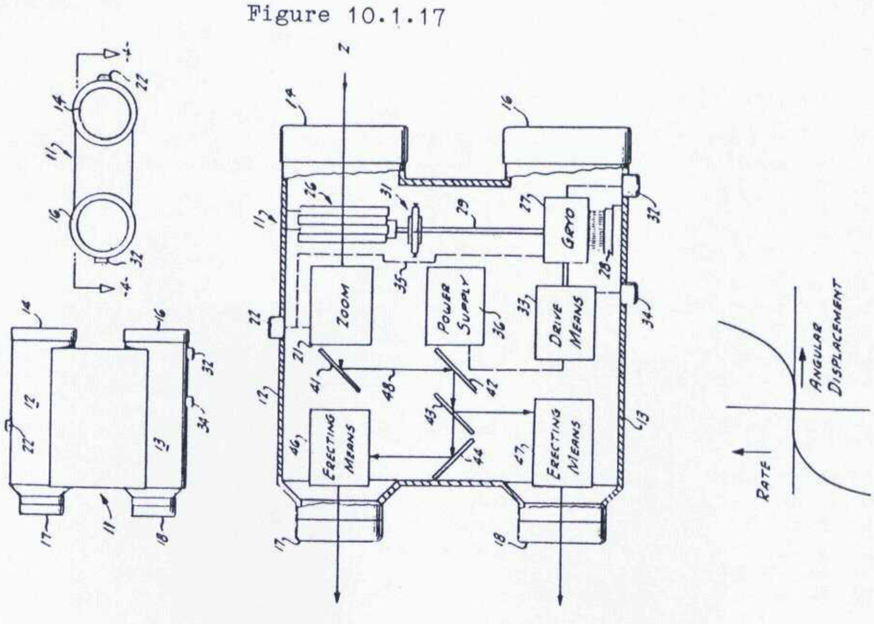


Figure 10.1.16

of the equation by θ , the equation reduces to: $\Sigma \Delta_c - \Sigma \Delta_f = 1$, which is the condition for image stabilization which must be satisfied simultaneously with the equation for achromatization noted hereinbefore for a camera. In the instance of a telescope, field glasses, or other optical instruments having an associated field of view, the foregoing equation is expanded to include the additional compensation $\frac{\theta}{M}$, and for these instruments the equation hence becomes:

$$\Sigma \Delta c - \Sigma \Delta f = 1 \pm \frac{1}{M}.$$

The equations for achromatization and image stabilization noted above pertain to situations where the doublets of the compound lens are comprised of plano-concave and plano-convex lenses. It should be noted that the equations still hold where the outer faces of the doublets have a slight curvature, provided small correction factors δ_c and δ_f are added to the equation to account for the fact that the outside surfaces of the doublets are not planar. These correction factors may be evaluated in terms of the common radius of curvature of the meshing surfaces of each doublet. This is accomplished by noting that $\delta c = \frac{Rc}{\rho c}$ and $\delta f = \frac{Rf}{\rho f}$, R_c and R_f being respectively the radii of curvature of

of the meshing surfaces of the respective crown glass doublets and flint glass doublets, and ρ_c and ρ_f being the very large radii of curvature of the outer surfaces of the crown glass and flint glass doublets, respectively. Taking these correction factors into account, the condition for achromaticity becomes

$$\Sigma \frac{\Delta_c (1+\delta_c)}{\gamma_c} = \Sigma \frac{\Delta_f (1+\delta_f)}{\gamma_f}$$

Likewise, the condition for stabilization of the image becomes:

$$\Sigma\Delta_c (1+\delta_c) - \Sigma\Delta_f (1+\delta_f) = 1,$$

in the case of a camera, and

$$\Sigma\Delta_c (1+\delta_c) - \Sigma\Delta_f (1+\delta_f) = 1 \pm \frac{1}{M},$$

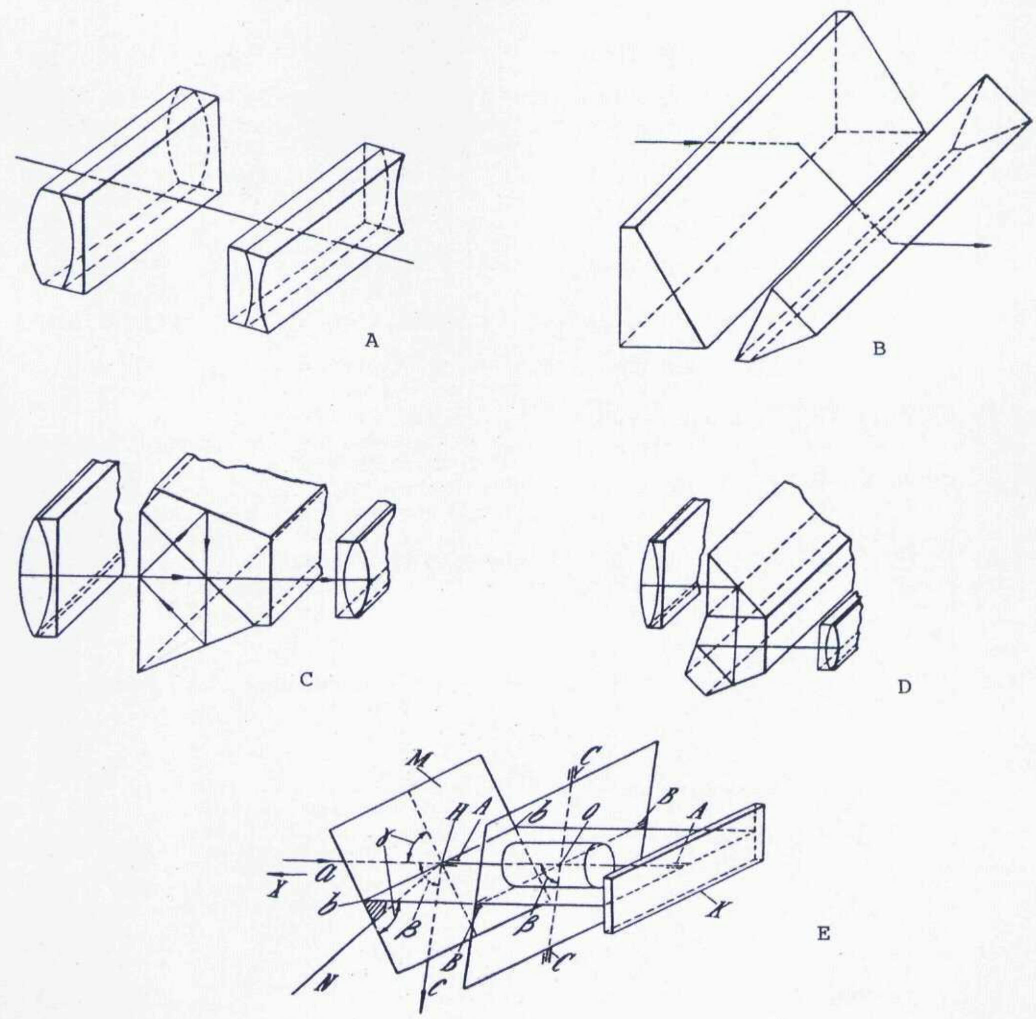
in the case of an optical instrument having an image field of view.

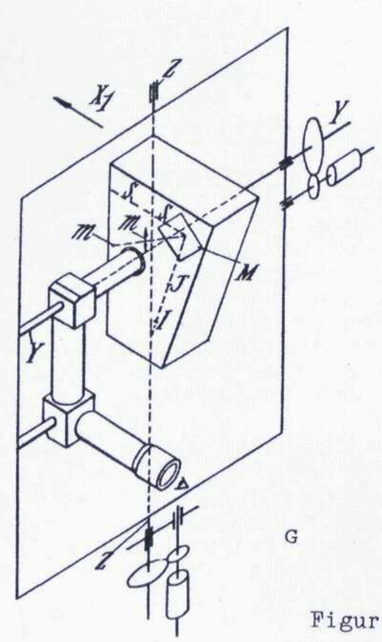
Considering now one specific example of an image stabilizing achromatic compound lens which may be designed in accordance with the foregoing considerations, such lens may advantageoulsy employ Schott Crown, SSK 9, as the crown glass doublets of the lens. "Schott" is a Registered Trade Mark.

SSK 9 has constants of $\Delta_c = 0.62$, and $\frac{\Delta c}{\gamma c} = 1.245 \times 10^{-2}$, and therefore assuming the use of three identical crown glass doublets in the design of an achromatic lens for a 20-power telescope, or the like, the equation for zero net dispersion becomes:

$$3.735 \times 10^{-2} (1+\delta_c) = \frac{\Delta_f (1+\delta_f)}{\gamma_f}$$

To satisfy this equation, a flint glass must be found having a constant $\frac{\Delta_f}{\gamma f}$ which is substantially equal to 3.733×10^{-2}. It will be found that Kodak Flint F has a constant $\frac{\Delta f}{\gamma f}$ which is equal to 3.74×10^{-2}, and thus relatively close to the constant $\frac{3\Delta c}{\gamma c} = 3.733 \times 10^{-2}$. "Kodak" is a Registered Trade Mark. As a further criterion in the selection of suitable crown and flint glasses for employment in the compound lens, the correction factors δ_c and δ_f should be practicably no greater than 0.2 in order that the coma aberration of the lens will be negligible. Inasmuch as the constants of the selected crown and flint glasses substantially satisfy the dispersion equation, the correction factors δ_c and δ_f requisite to complete satisfaction thereof will be relatively small.





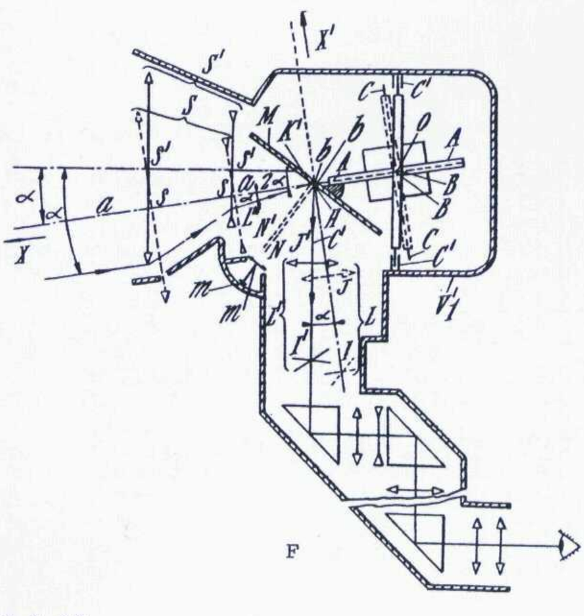


Figure 10.1.19

However, the stabilization equation:

$$\Sigma\Delta_c\ (1+\delta_c) - \Sigma\Delta_f\ (1+\delta_f) = 1 \pm \frac{1}{M},$$

must be simultaneously satisfied with correction factors δ_f and δ_c of relatively small values less than 0.2. Kodak Flint F has a constant $\Delta_f = 0.995$, and it will be found that upon substituting this value, as well as the value $\Delta_c = 0.62$ for the crown glass elements, into the stabilization equation, that the correction factors δ_c and δ_f necessary to satisfy same will be of negligibly small value. More particularly, we have:

$$1.86\ (1+\delta_c)-.995\ (1+\delta_f) = 0.95.$$

Through the simultaneous solution of this equation and the dispersion equation,

$$3.733 \times 10^{-2}\ (1+\delta_c) = 3.74 \times 10^{-2}\ (1+\delta_f),$$

δ_f is calculated as being 0.089 and δ_c is 0.091. thus, the correction factors are of substantially negligible order for a compound lens comprising three crown glass doublets of Schott Crown SSK 9 and one flint glass doublet of Kodak Flint F which may be employed in a 20-power erecting telescope, or the like. Such a lens provides image stabilization with negligible chromatic aberration, as well as negligible coma aberration. It will be appreciated that various other specific systems may be designed in the manner set forth hereinbefore using other combinations of glasses and numbers of doublets to provide achromatization and image stabilization for substantially any desired magnification power of a given optical instrument.

ALVAREZ US 3468596 (1969) is responsible for a gyroscopically stabilized zoom mono-binocular shown in Fig 10.1.16.

BEZU French 1372585 (1963) French 1435872 (1965) has proposed a telescopic stabilization means (Fig 10.1.17) using a mirror stabilized by a gyroscope when supported by the outer gimbal. An unusual mechanical linkage is employed using mating sectors and pulleys of which one gives a double reversal (pulley 3 in the Figure). The following relation is said by BEZU to be essential -

$$\frac{I}{J} = n\ (n+2)$$

where I is the moment of inertia of the mirror.
J is the moment of inertia of pulley 3.
n is the demultiplication ratio.

These proposals are not, to my mind, fully developed and one should consult the later work of BEZU GB 1114094 (1968) GB 1269811 (1972) GB 1340212 (1973) for elucidation of his ideas which are taken here under the heading of Gunsights.

We turn now to the complex gyroscopic devices of DERAMOND French 1453857 (1966).

The devices are divided, in the corresponding United Kingdom patent specifications, to distinguish between those devices in which the first reflecting means (the mirror) is on the outer gimbal or the inner gimbal of the gyroscope. A first arrangement DERAMOND GB 1149164 (1969) is shown at Fig 10.1.18) in which a single mirror is pivotally mounted on the outer gimbal. DERAMOND GB 1150699 (1969) shows inter alia a double mirror device with the first mirror on the inner gimbal and the second mirror on the outer gimbal, to give an optical arrangement in which the line of sight when directed parallel to the gyroscope rotor axis is, after successive reflections on the two mirrors, parallel to the pivotal axis of the outer gimbal.

DERAMOND GB 1150700 (1969) in a further viewing device places an anamorphic optical system in front of the mirror that is connected to the inner gimbal of a gyroscope having two degrees of freedom. The anomorphic lens has been known since the work of CHRÉTIEN GB 282078 (1928)and may be studied to advantage from LEVI (1968) (p429 et seq) and one may also look with profit at the zoom anomorphoser of LUBOSHEZ US 2780140 (1957). As shown in Fig 10.1.19 DERAMOND prefers an anamorphic Galilean system with cylindrical magnifying lenses Fig A or an anamorphic system formed of Brewster-Amici prisms Fig B. Two other possible systems Figs C and D use hemi-symmetric afocal arrangements with converging elements combined with a mirror or a prism having an odd number of reflections. Fig E is a diagram showing the basic geometry of DERAMOND's optical viewing device and Fig F shows a complete device in section. The device gives an anamorphosed image of the landscape in the plane of the reticle I. By interposing between the mirror M and the telescope L a cylindrical afocal system with magnification of 1/2,suitably arranged, this disadvantage can be overcome. It is also possible to compensate this anamorphic behaviour beyond the telescope L, in the part of the sighting device which leads to the eyepiece, behind which is located the eye of the observer.

Some earlier sighting devices stabilize the sighting line strictly but nevertheless the image of the landscape generally shows obliquity about this sighting line in relation to the crossing lines of the reticle, and in order to overcome this disadvantage there can be interposed between the mirror M and reticle I a rotating prism of the Wollaston or Pechan type (with uneven number of reflections and with direct vision), which corrects the image of the landscape by a suitable angle, taking into account the rotation of the sighting device. It is also possible to cause the reticle to rotate and subsequently to correct the unit reticle/landscape by the rotation of a prism of the above type, arranged between the reticle and the eyepiece; the observer, placed behind the latter, will see a corrected landscape. This latter arrangement makes is possible to take into account any out-of-plumb introduced beyond the reticle, for example in the case where it is desired to keep the eyepiece of the sighting device fixed (Fig G).

It can accordingly be said that the sighting device has in front of the mirror M an anamorphic afocal system having two magnifications for two perpendicular directions one of the said magnifications being always 1, whereas the other can have any value greater than two. For all the sighting devices the sighting line will be stabilized for any rotation of these sighting devices, on the condition that:

1. The "image" sighting line, located in the rear of the mirror M and materialized through the optical centre of the objective lens and the cross lines of the reticle, is parallel to the external axis of the gyroscope.

2. The "object" sighting line defined by inverting the direction of the light, as coupled with the "image" sighting line through the optical system formed of optical elements connected to the gyroscope, or placed in front of them, is parallel to the axis of the rotor of the gyroscope.

This second condition can also be expressed in the following way:

The optical system formed in the first place of the optical elements connected to the sighting device and placed in front of those that are connected to the gyroscope, then of the optical elements connected to the gyroscope, should give an image of a line parallel to the axis of the rotor of the gyroscope which is parallel to the outer gimbal axis of the gyroscope.

The sighting device has the following advantages:

(a) The connection gyroscope/mirror is of extreme simplicity.

(b) The inertia of the mirror M does not hinder the stabilization of the sighting line when the sighting device undergoes rotary movements about an axis parallel to the inner axis BB of the gyroscope.

(c) The mirror M is smaller than with many earlier sighting devices from which it follows that there is less vulnerability when it is a question of military application, the mirror has less mass, with the result that the gyroscope obtained has less extreme characteristics for the same degree of stabilization and there is greater facility in the piloting of the mirror, particularly when it is desired to impart to it rapid precession speeds: for example in the case of visual pursuit of objects moving at high angular velocities.

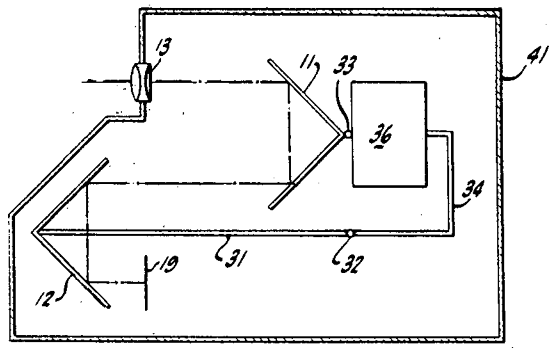


Figure 10.1.20

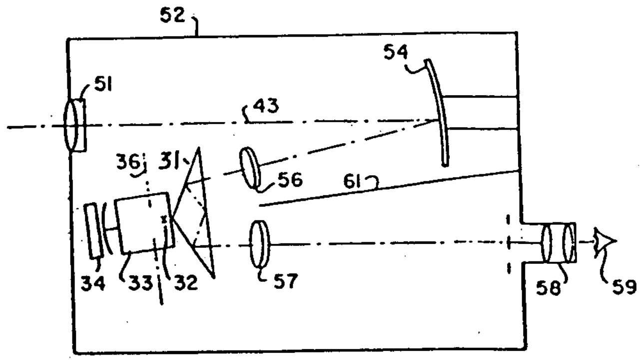


Figure 10.1.21

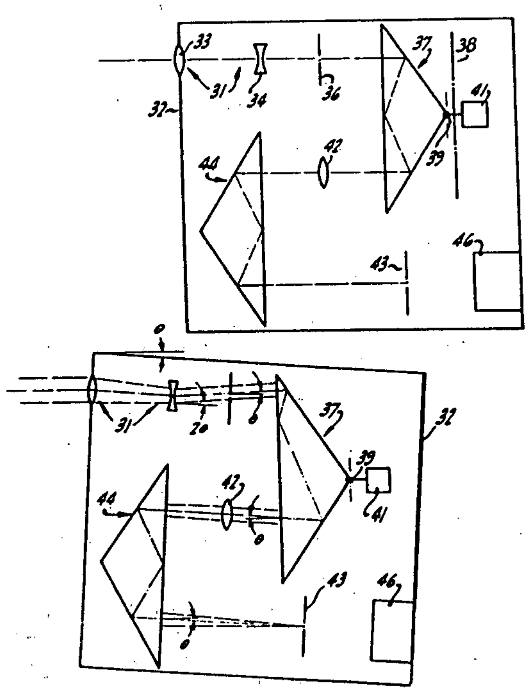


Figure 10.1.22

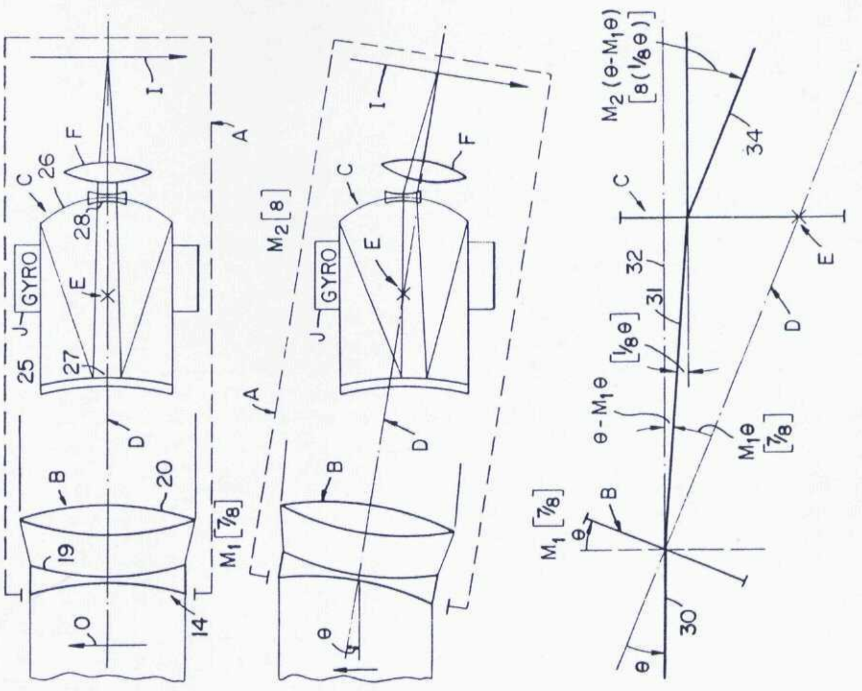


Figure 10.1.23

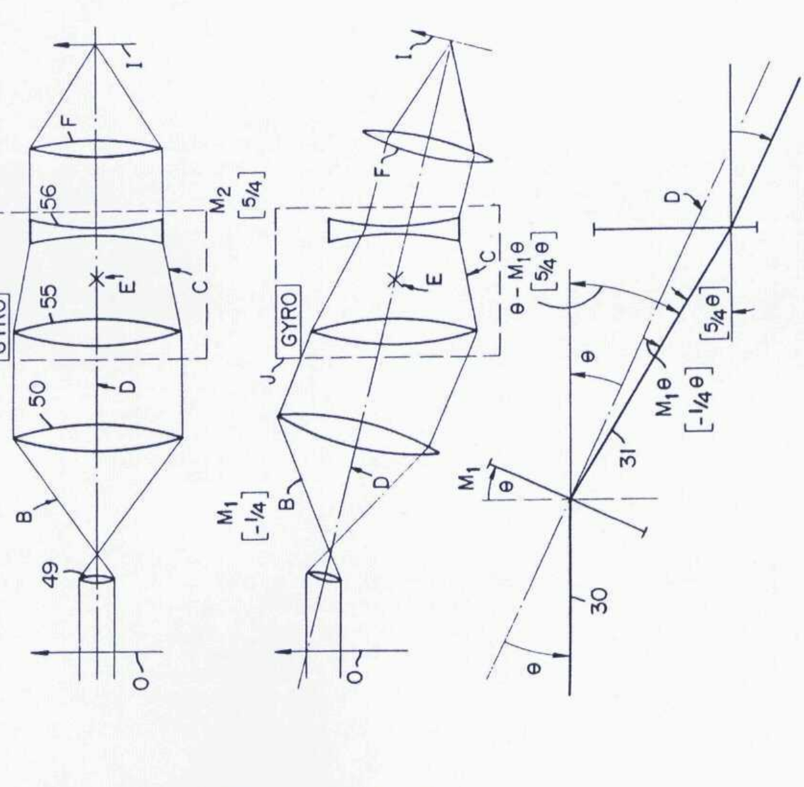


Figure 10.1.24

DERAMOND'S United States patent specification No. 3556632 (Original French patent 1453857) attracted a large number of citations during its prosecution and some of these, are worthy of further study, see for example the disclosures of

SCHNEIDER & CIE GB 226163 (1926)
NEWELL. W.H. US 2684007 (1954)
JENSEN. H. US 2829557 (1958) and
BOUWERS. A. US 3200250 (1965)

In a virtuoso display over a short period of about four years HUMPHREY. W.E. has given several different optical arrangements for the telescope; all of which may be gyroscopically stabilized; the accent is upon the stabilization of a part of the optical train, but the exact manner of the gyroscopic coupling is not always clear. I give below eleven arrangements taken seriatim.

HUMPHREY US 3437396 (1969) discloses an optical stabilizer (Fig 10.1.20) having an objective lens for focusing received light, a plane for displaying an image and two displaced parallel roof mirrors that each displace and redirect the focusing light on the display plane. Each reflecting element is stabilized by a gyroscope and each pivots on gimbals about an arm extending parallel to the optic axis with a distance along the optic axis between apex lines substantially equal to one half of the focal length of the objective. For a telescope or binocular the total pivot arm distance for the two mirrors is reduced from f/2 and is made $f/2\ (1 \pm \frac{1}{m})$ where m is the magnification of the system and the minus sign employed for an erect image. This system is in part anticipated by that of POWLEY GB 1015916 (1966) and HENDERSON US 1628777 (1927).

HUMPHREY US 3437397 (1969) discloses an optical stabilizer using first and second stabilized pairs of mutually perpendicular roof mirrors for intercepting and displacing the focusing light there between. Each of the roof mirrors has an apex line and is pivoted about a pivot axis parallel to the apex line and at a distance from the apex substantially equal to one half of the focal length of the lens. Single degree of freedom gyroscopes are used each having its spin axis perpendicular to the pivot about which the roof mirror is free to pivot in a plane normal to said pivot.

HUMPHREY US 3468595 (1969) shows a corner cube reflecting surface for use with a telescope or binocular, whilst in US 3473861 (1969) he proposes a two power inverting telescope having optics within the telescope for retrodirecting the light beam to project a retrodirected collimated beam of light in which stabilization of the image occurs by means of a reflecting element mounted within the collimated beam (Fig 10.1.21). The stabilized reflecting element is seen at 31 and is stabilized by a small gyroscope 33.

HUMPHREY US 3475074 (1969) advances a two power erect image telescope followed by a stabilized triple reflecting element. The gyroscope Fig 10.1.22 is shown at 41 and is said to be a free gyroscope. The triple reflecting element 37 is balanced at 39.

HUMPHREY US 3531176 (1970) discloses two optically aligned telescopes, the first having a magnification power of M_1 and the second a magnification power of M_2. The second telescope is adapted to receive light from the first telescope and the second is inertially stabilized, the magnification of the two telescopes being related such that $\pm M_1 = (1 - \frac{1}{M_2})$. See Figs 10.1.23 and 24.

Fig 10.1.23 shows a fixed telescope B having a positive magnification of 7/8 and stabilized telescope C having a positive magnification of 8. This combination of telescopes produces stabilizer optics having a magnification of 7 power at objective lens F and provides a convenient mathematical example. In substituting other magnification values within the derived stabilization formula, it will become apparent that relatively high or large overall magnifications can be obtained by permitting the magnification M_1 of fixed telescope B to approach a positive value of one (+1).

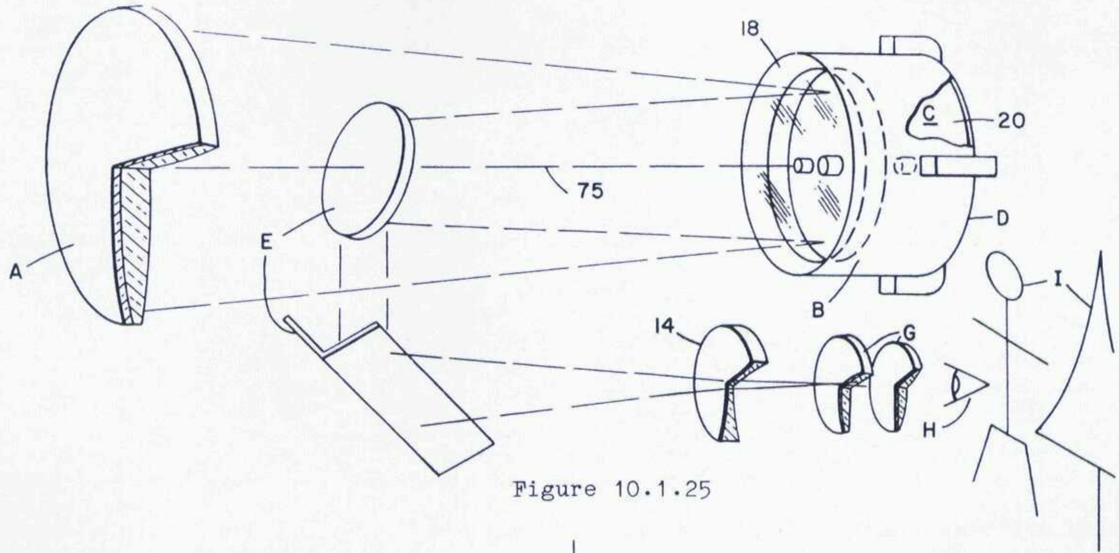


Figure 10.1.25

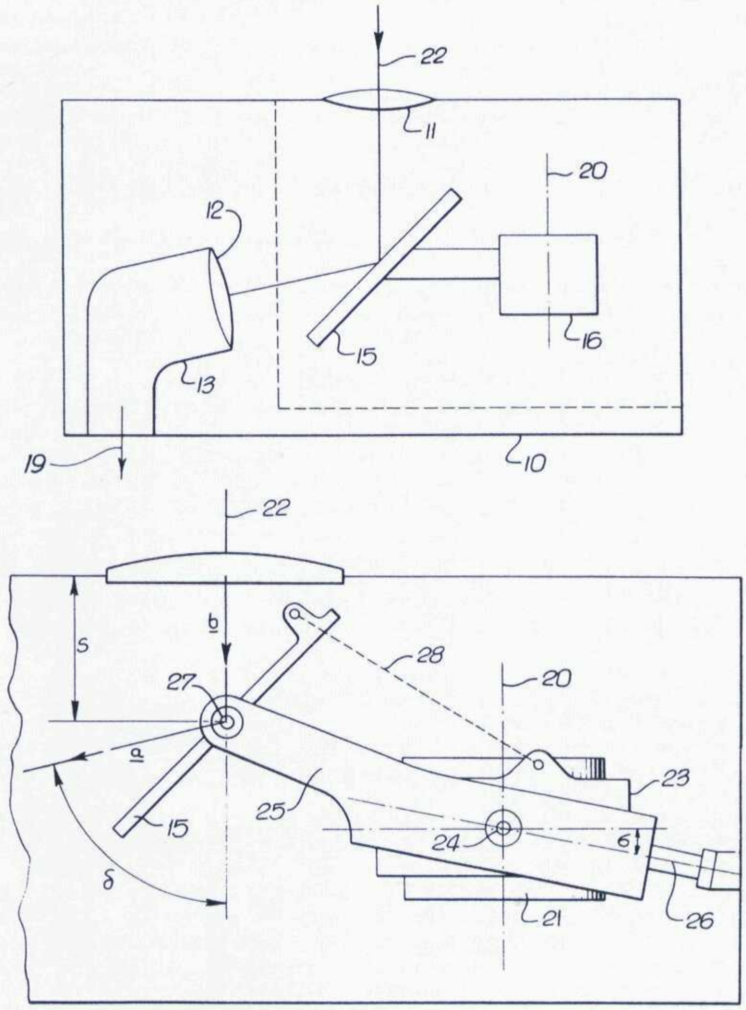


Figure 10.1.26

It is necessary that fixed telescope B angularly deviate deflected light E at some small angle from the line of sight. If there is no deviation (as when the magnification M_1 of fixed telescope B is a positive value of one (+1), no stabilization will be achieved.

The stabilizer of this invention compensates only for angular deviation of the optical axis D with respect to the viewed object O. If casing A is moved in other than a pitch or yaw relation, no stabilization will occur nor is it ordinarily desirable.

With reference to Fig 10.1.24, an alternate embodiment of the stabilizer is illustrated particularly suited for stabilizing wide angle camera images. This stabilizer has a fixed and reversed inverting telescope B of negative magnification combined with a stabilized erect image telescope C (enclosed within broken lines) of positive magnification, casing A being omitted for purposes of clarity. Inverting image telescope B is of the Keplerian variety and comprises a first biconvex lens 49 and a second biconvex lens 50. Stabilized erect image telescope C is of the Galilean variety and includes a first biconvex lens 55 and a second biconcave lens 56. Similar to the stabilizer previously described, fixed telescope B has its axis on optical axis D and stabilized telescope C is mounted on gimbals E also near axis D.

In the angularly deviated position, it is seen the deflection of light interior of the illustrated stabilizer is analogous to the stabilizer of Fig 10.1.23. Received ray 30 impinges upon fixed telescope B and is deflected clockwise below optical axis D due to the negative magnification M_1 of telescope B. Emanating as the deflected ray, the light passes through stabilized telescope C and is deflected therein so as to emanate parallel optical axis D. As is apparent in traversing the subject stabilizer, the light from an object O when focused through an objective lens F will produce an erect and reduced image I. The stabilization formulae for this stabilizer is precisely identical to that specific formulae derived above.

A mathematical example of stabilizer optics having an overall negative magnification 5/16 power at objective lens F is illustrated in Fig 10.1.24. Fixed telescope B has a negative magnification of 1/4 while stabilized telescope C has a positive magnification of 5/4 power.

HUMPHREY US 3564931 (1971) shows in considerable detail a system for damping the nutational motion of a gyroscope used with an optical image stabilization system for a telescope. If the gyroscope rotor is subjected to a sudden mechanical impulse the rotor axle tends to undergo nutational motion and displaces resilient biasing springs that tend to maintain the gyroscope and the optical stabilized element in alignment while the energy is dissipated in a damping dash-pot.

HUMPHREY US 3608995 (1971) shows a telescope that incorporates an optical relay comprising a pair of spaced negative lens with an intermediate erecting prism. The relay is inertially stabilized and nutationally damped as in HUMPHREY (1971_1).

HUMPHREY US 3608996 (1971) discloses a telescope with an optical train that receives an image from an objective lens, the optical train is inertially stabilized from a gyroscopic platform; and in HUMPHREY US 3608997 (1971) there is shown a Cassegrain or Maksutov telescope with gyroscopically stabilized objective lens.

HUMPHREY US 3756686 (1973) discloses a hand-held twenty power optical telescope having an objective lens mounted to focus light onto a gyroscopically stabilized mirror (B) in a fluid bath. Stabilized light from mirror (B) is directed onto an inverting mirror complex (E) and viewed at eyepiece (G) (Fig 10.1.25). The input torques to the gyroscope are said to require an orthogonal alignment and a dynamic performance integrating stabilizer system. HUMPHREY examines the amount of integration that can be incorporated into the torquing system mathematically. His dissertation is given below:-

Beyond a certain level of integrating contribution, the stabilizer enters into a realm of uncontrolled oscillation. This theoretical prediction has been born out in experiments with models having a variable integrating contribution to the total torque on the stabilized component. The region of acceptable operation can be predicted on the basis of a mathematical model corresponding in a reasonable degree with the performance of the actual embodiment of a servo integrating stabilizer although no manageable model is a perfect analog of the real mechanical embodiment. For example, bearings may have small unpredictable "stiction " electronics may saturate at certain signal levels, or fluid flow may be complicated by small local vortices. Nevertheless, these practical perturbations notwithstanding, useful limitations on the practical embodiments can be established for the gross performance of a servo integrating stabilizer.

The basic equation to consider is shown below:

$$\ddot{x} + A\dot{x} + Bx + C \int x dt = f(x)$$

This is a third order differential equation relating acceleration ($\ddot{x}$) of a body whose position (x), is subject to acting forces proportional to velocity ($\dot{x}$), position (x) and position integrated over time ($\int x\, dt$). In the case of a floated mirror, x would represent angular orientation of the mirror and A, B and C could be represented more specifically as follows:

$$A = k_1/I$$
$$B = k_2/I$$
$$C = k_3/I$$

where I = effective movement of inertia of mirror-fluid system.

k_1 = torque per velocity (viscous type drag torque from fluid, proportional to viscosity and float-to-case geometrical coupling, as well as electrically generated torques proportional to velocity).

k_2 = spring like restoring torque per angular position.

k_3 = torque per time integrated angular position.

$f(_t)$ = time varying torque inducing motion of float and including coupling of case vibrations to the mirror through the fluid and restoring forces as well as long term effects such as torques generated with change in temperature, etc.

For the purpose of the present discussion, the exact nature of f(t) is not important. The solution of such a differential equation involves a particular solution describing the response to the perturbing function f(t) as well as a more general homogeneous solution for the equation. This homogeneous solution is of prime importance in regard to stability of the stabilizer as it is composed essentially of exponential terms of the form:

$$X = \alpha_1 e^{\alpha_1 t} + \alpha_2 e^{\alpha_2 t} + \alpha_3 e^{\alpha_3 t}$$

where represents roots of the equation:

$$\alpha^3 + A\,\alpha^2 + B\alpha + C = 0$$

These roots may be complex numbers; however, the real part of α is of critical importance because a positive real part indicates an exponential increase with time in the motion of the stabilizing element. Therefore, all combinations of A, B and C which result in a positive real part of α must be excluded. Fortunately, this limitation can be expressed in the relatively simple form:

I. AB must be greater than C in addition -

II. The coefficients, A, B and C are restricted to positive values. These conditions define a region of useful stabilizer constructions.

In contradistinction to the work of HUMPHREY discussed above, PARKER GB 1399121 (1975) pays great attention to the gyroscopic coupling. He is concerned to perfect a hand held binocular or telescope in which the vibration component derived from linear tremor transverse to the line of sight is ignored and that from angular tremor about horizontal and vertical axes transverse to the line of sight substantially removed by gyroscopic means.

The construction of one optical viewing device according to PARKER is shown at Fig 10.1.26. A reflecting surface (mirror 15) placed between two objective lenses (11.12) is controlled by a gyroscope (16). The line of sight (22) represents the optical axis when the gimbals are undeflected and the undeflected spin axis (20) of the gyroscope rotor (21) is parallel to the line of sight (22) and to the viewing direction (19) of the said device. The rotor (21) is mounted in an inner gimbal (23) which in turn is mounted about an axis (24) perpendicular to the line of sight in an outer gimbal (25).

The outer gimbal (25) is mounted on the frame of the device about an axis (26) that lies in a plane containing the axis (20) and perpendicular to axis (24) and is inclined at a small angle to the axis orthogonal to axes (20) and (24). The outer gimbal (25) supports the mirror (15) about an axis (27) parallel to the axis (24). A mechanical link (28) interconnects the mirror (15) and the inner gimbal (23) so that their angular movements relative to the outer gimbal (25) are related. This need gyroscopically so to connect a reflecting surface to both inner and outer gimbals was seen earlier by HENDERSON US 1709314 (1929) but it was not then appreciated how critical was the nature of the linkage. I give below PARKER's analysis using the following symbols and appealing to the larger figure of Fig 10.1.26.

- s the distance along the optical axis between the mirror 15 and the second principal plane of the lens 11, i.e. that part of the lens system that lies in front of the mirror 15;
- δ is the deviation from a straight line of the optical axis by the reflecting surface with the gimbals in their undeflected positions;
- σ the skew angle of the axis 26;
- $\Delta\theta$ is a small deflection of the device about a vertical axis, i.e. a small yaw;
- $\Delta\phi$ a small deflection of the device about a horizontal axis, i.e. a small tilt;
- m the linear magnification of the complete optical system;
- r the ratio of angular movement with respect to gimbal 25 of the inner gimbal 23 to that of mirror 15 due to the coupling link link 28; and
- f is the focal length of the lens 11.

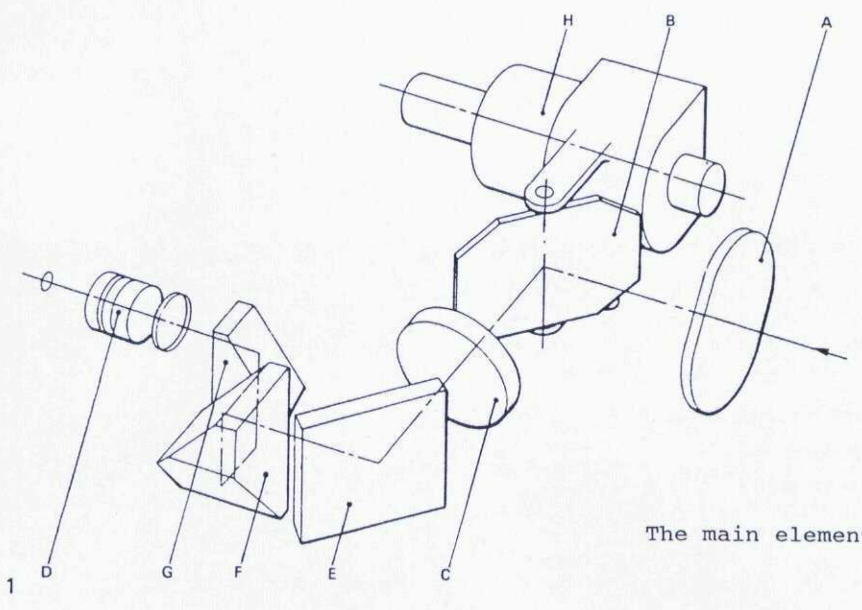
For stabilization in the horizontal plane the arrangement has to meet the conditions expressed in equation (1).

$$1 - \frac{2}{r} + \frac{2s}{f.r} = \frac{1}{m} \qquad (1)$$

For stabilization in the vertical plane the displacement is tripartite and has to meet the conditions expressed in equation (2).

$$\frac{s}{f} + \left(1 - \frac{s}{f}\right)\left(2 \sin \frac{\delta}{2} \sin \sigma \left[\cos\left(\frac{\delta}{2} - \sigma\right) - \tan \sigma \sin\left(\frac{\delta}{2} - \sigma\right)\right] + \cos \delta\right) = \frac{1}{m} \qquad (2)$$

The sightline entering window A is reflected by the stabilising mirror B into the monocular telescope formed by lenses C and D. The fixed mirror E and prisms F and G serve to fold the optics into a compact shape and provide an erect image. The angle of the stabilising mirror B is controlled by the gyroscope H.



The main elements of Steadyscope

Figure 10.1.27

Photograph by courtesy of British Aerospace Dynamic Group.

There are three Special Cases:-

Case (I)

$$f = \infty \ , \ \delta = 90°$$

Equations (1) and (2) reduce to:

$$1 - \frac{2}{r} = \frac{1}{m} \qquad \text{or} \qquad r = \frac{2m}{m-1}$$

and

$$\sqrt{2}\,\sin\sigma[\cos(45° - \sigma) - \tan\sigma\,\sin(45° - \sigma)] = \frac{1}{m}$$

with solutions:

σ°	4	5	6	7
m	14.4	11.4	9.5	8.1

Case (II)

$$f = \infty \quad \sigma = 0$$

Equations (1) and (2) reduce to:

$$r = \frac{2m}{m-1}$$

and

$$\cos\delta = \frac{1}{m}$$

Case (III)

$$\delta = 90° \quad \sigma = 0$$

Equations (2) reduces to:

$$\frac{s}{f} = \frac{1}{m}$$

and thus equation (1) becomes

$$r = 2.$$

PARKER's ideas are now incorporated in the STEADYSCOPE type GS907 of British Aerospace Dynamic Group. The Steadyscope is a monocular with two eyepieces and a magnification of seven. The field of view is 7.4 degrees of arc. The weight is 2.0 kg (4.4 lb) and the electric power for the gyroscope provided from a single Manganese Alkaline 1.5 volt cell giving eight hours of running time. See Fig 10.1.27 and BRAKE (1980).

Another contribution to the art of stabilized optical devices is that of the French MINISTRE DE L'AIR French 1329532 (1963) to be found in a document filed in Paris some twenty-four years before its publication, which suggests that it was the subject of a secrecy order. The arrangement is shown at Fig 10.1.28. A stirrup (1) able to turn about a vertical axis (2) supports on pivots (3) normal to the axis (2) a first gimbal (4) to which is rigidly attached a telescope (5) and prism (6). On the first gimbal (4) is mounted via pivots (7) a second gimbal (8) which gimbal carries a plane mirror (9) and a gyroscope (10). The gyroscope spin axis (11) is carried by a gimbal (12) able to turn about pivots (14). The axis (14) of the gyroscope is kinematically linked to the axis (16) of the mirror (9) by spur gears (17.18.19) such that if axis (14) rotates through α degrees of arc, the plane of mirror (9) turns through $\frac{\alpha}{2}$ degrees of arc.

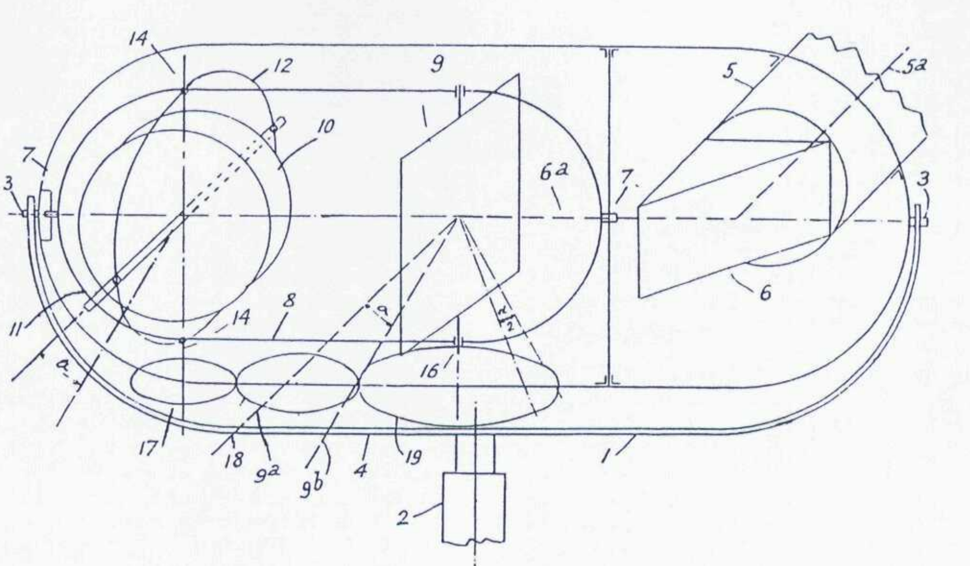


Figure 10.1.28

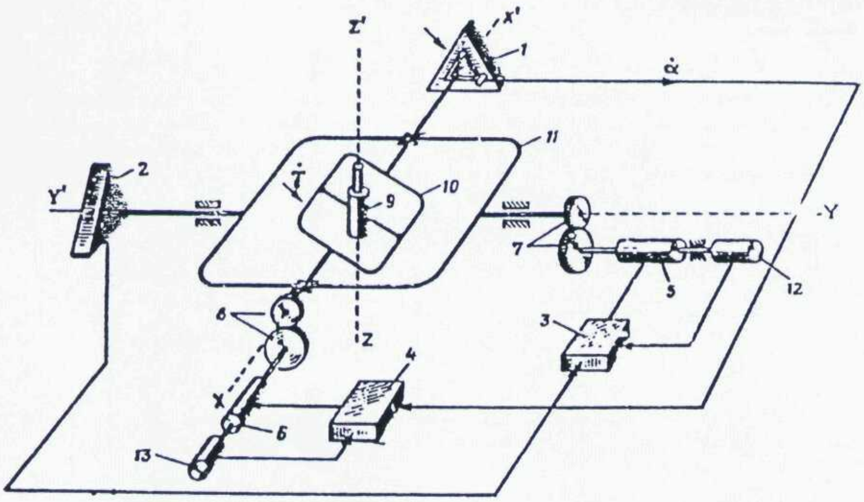


Figure 10.1.29

Yet another contribution in the field of binoculars is that of TAKEMI, KENJIRO, and KUNIO EP 0001204 (1979) who refer inter alia to the earlier works of HUMPHREY, KAESTNER US 2939363 (1960) and JENSEN US 2829557 (1958). We have already referred above to the ideas of JENSEN using Porro-prisms but we have neglected until now to refer to the work of KAESTNER who using porro-prisms and counterweights with cross linkages provides an image that is reduced in its vibrations, not wholly dissimilar to that of TAKEMI et al who deploy gyroscopic means to stabilize a pair of Schmidt prisms, or erect prisms such as those of Abbe or Bauern Fend, each capable of having its incident light optical axis and its emanating light optical axis aligned with each other. The gyroscopic gimbals are so associated with the optical system that only the vibration in the vertical direction is completely eliminated since it is mainly the vertical component that is imparted to the optical device from a moving vehicle, the horizontal component being in comparison very small.

It is not without interest to close this account of the gyroscopically stabilized telescope with a reference to DIENSTBACH (1915) who wrote with enthusiasm concerning the work of Mr. E. SPERRY to give aviators such as Lieutenant SCOTT a workable gyrotelescope; what would he say today of the sophisticated Large Space Telescope (LST) of three metres aperture for launch by the space-shuttle into an orbit some 500 miles above the Earth with its accurate pointing determined by control moment gyros with reference from precision gyros and star trackers as described by O'DELL (1973) SCHIEHLEN. W. (1974) and SELTZER S.M. (1975). For a detailed consideration of the control moment gyro see MAGNUS. K. (1971) at pages 435 to 438.

Finally the gyroscope has found itself in the interior of the astronomers tool, the Cassegrain* telescope. In the proposals of CUTHRIE et al GB 958415 (1964) the primary mirror is rotated and in the proposals of McCAFFREY US 3158676 (1964) both primary and secondary mirrors are rotated at a speed of 4200 revolutions per minute, but the purpose is not the altruistic persuits of the astronomer but the acquisition of a moving target.

The laser gyroscope is proposed by BOGDANOV (1977) for use in a stabilizer attached to a ship's telescope (See Fig 10.1.29). In this stabilizer laser angular velocity pickups generate control signals proportional to the absolute angular velocities of rotation of the stage about axes XX^1. YY^1. Since rotation of the telescope about the direction to a star (about the ZZ^1 axis) is unimportant, stabilization is carried out with respect to but two axes. Consider axis XX_1.

When the ship rolls at angular velocity θ the stage on which telescope 9 is mounted (in this case the inner frame of gimbals 10) turns relative to the rolling vessel at angular velocity γ. The absolute angular velocity of the stage relative to the inertial space, which velocity is measured by laser angular velocity pickup 1, is:

$$\alpha = \theta - \gamma$$

A signal corresponding to this velocity enters amplifier-converter device 4. An amplified signal proportional to the angle of rotation of the stage relative to the XX^1 axis is taken from the output of this device. The signal then enters motor 6 which moves the stage by means of reducing gear 8 to the original position.

Tachogenerator 13, which is rigidly connected to the motor, is used to compensate the signal coming from angular velocity pickup 1 to the amplifier-converter unit on movement of the stage to the original position.

* Virtually all that is known of CASSEGRAIN (c1672) is that he conceived the arrangement of mirrors that bears his name. A convex secondary mirror reflects the light from the primary back through a hole in the primary where it is brought to a focus. See Journal des Savants 3 (1672-1674) p.121-123.

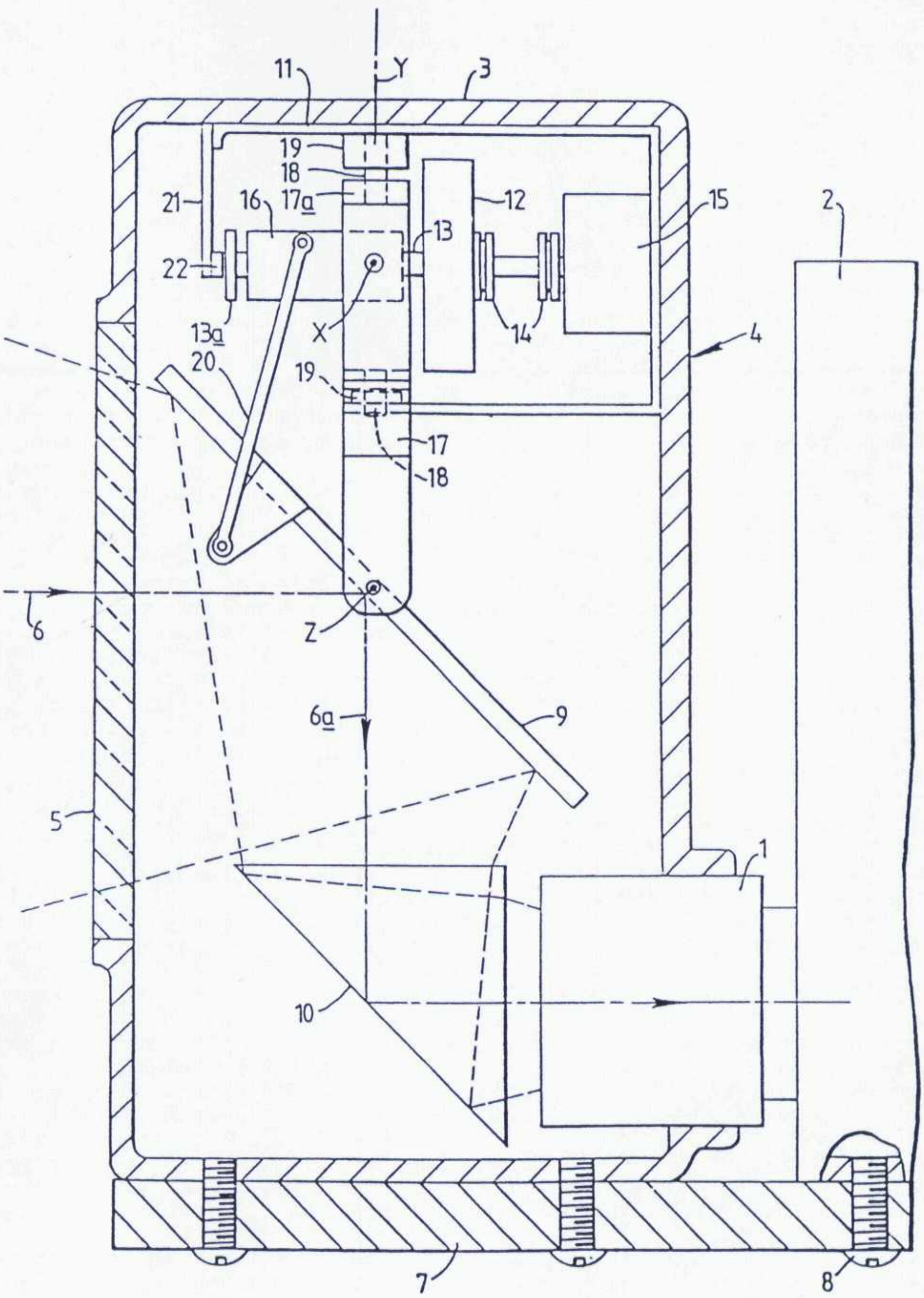


Figure 10.1.30

I close with a contribution by an authority on the subject. I refer to the work of BRAKE. D.G. assigned to BRITISH AEROSPACE GB 2036998 (1980) in which he proposes a device that overcomes certain disadvantages to be found in the earlier yet sophisticated devices of BEZU, DERAMOND and PARKER.

It can be shown that generally the stabilising mirror must receive light directly from the object (in order to avoid reverse coupling between the gyroscope rotor or inertial mass and the stabilising mirror, which would degrade dynamic performance) so that the instrument must either point in a direction substantially at right angles to the direction of the object or a second reflecting surface must be interposed. The former method makes it difficult for the user to acquire and follow objects because, in a practical arrangement, elevation of the pointing direction requires a rotation of the instrument about its optical axis. The latter method, while being satisfactory in some cases, embodies a disadvantage particularly where the instrument accepts a substantial field of view. In such cases where, for example, the field of view exceeds 20 degrees, the separation of the stabilising mirror from the instrument lens along the optical axis due to the introduction of a second mirror, necessitates an inconveniently large stabilising mirror, so degrading its performance due to increased inertia.

As shown in Fig 10.1.30 the case (3) of the image stabilizing device is a mirror (9) that reflects a light ray (6) through a right angle toward a prism (10) mounted in front of the lens (1). The prism (10) reflects the light through a further right angle into the lens (1). The mirror (9) is mounted such that it can turn about a horizontal axis X perpendicular to the plane of the figure. These movements are controlled by a stabilizing mechanism such that despite vibration or tremor the light ray (6) remains fixed in position in relation to the case (3) so that the scene viewed remains fixed relative to the frame. The stabilising mechanism comprises a gyroscope rotor 12 fixed to a shaft 13 one end of which is coupled by way of two universal joints 14 to the output shaft of an electric motor 15. The motor 15 is fixed to a frame 11 and is energised by a battery (not shown). The other end of the shaft 13 is supported via bearings (not shown) within a mounting block 16. The block 16 is mounted between the arms of forked gimbal member 17 such that the block 16, along with the gyroscope rotor 12, can turn about the axis X with respect to the gimbal member 17. The member 17 has two stub-shafts 18 fixed thereto, which stub-shafts are supported by bearing blocks 19 fixed to the frame 11 such that the member 17, along with the block 16 and the gyroscope rotor 12, can turn about the axis Y with respect to the housing 11. The ends of the arms of the forked gimbal member 17 support the mirror 9 between them such that the mirror can turn about an axis Z parallel to axis X. A link member 20 is pivotably coupled to the mirror 9 and the mounting block 16 so as to control movements of the mirror about axis Z in dependance upon the angular movement of the block 16 relative to the member 17. If the stabilising device 4 moves, the gyroscope rotor 12 tends to maintain its position in space, i.e. it moves about axes Y and X in relation to the case 3. This relative movement is transmitted to the mirror 9 such that the light ray portion 6a remains fixed relative to the case 3 as described earlier.

The shaft 13 projects from the forward end of the block 16 and carries a disc 13a. A leaf spring member 21 is fixed to the frame 11 and extends in front of the disc 13a and carries there a friction pad 22.

When the case is rotated intentionally in order to steer the sightline i.e. to change the direction in which the camera is pointing, the friction pad 22, which in its central position is just clear of the rotating disc 13a rotates about the axes X and/or Y and touches the disc. For example, in the perspective drawing at Fig 10.1.30 the case and pad are shown tilted downward relative to the disc which is touching the pad at a position below the disc's centre. Because the disc is rotating clockwise, the pad will apply a frictional force in the direction of the arrow N to the surface of the disc. This force results in a torque being applied to the rotor about the axis X which begins to precess the rotor downward about the axis Y to follow the movement of the case, according to the known rules of gyroscope motion. Because of the angular momentum of the rotor and the characteristics of the pad and spring, the system forms a low-pass frequency filter which reacts to slow steering actions but not to tremor and vibration and only if the case's movement is sustained will there be time for the rotor to catch up with the case.

Thus, the pad 22 and disc 13a form a steering device which allows movements of the camera and stabilising device greater than those associated with vibration and tremor i.e. intentional movements of the camera to change the scene viewed. Alternative steering means may be employed instead of the disc and pad. For example, a steering device is known which operates magnetically to apply a precessing torque to the rotor.

The described manner of operation in which the light ray remains fixed relative to the case of the stabilizing device is known as "case stabilization" or "frame stabilization" and is the most desirable manner of operation where, as with the described embodiment, the intention is to stabilise the scene viewed. There is another kind of stabilisation which is known as "space stabilization" and which is more desirable when the stabilizing device is to be used in conjunction with a telescope or the like through which a scene is viewed by eye. Information about the construction of the stabilizing device with a view to obtaining one or the other form of stabilisation may be found in the aforementioned Patent Specifications of BEZU. DERAMOND and PARKER.

The lens 1 accepts an angular field of view of 30 degrees of arc and, accordingly, the angle of the convergent envelope of useful light rays reflected from the mirror 9 is also 30 degrees. However, on entering the prism 10, this angle is reduced according to Snell's law of refraction so that, by the use of this prism as opposed to another mirror, the mirror 9 can be substantially smaller in area for the same field of view of the camera. The reduction in the angle depends upon the refractive index of the prism material, which index should therefore be as great as possible. In this embodiment, the prism is made of glass having a refractive index of approximately 1.75 and the angle of convergence of the light, while within the prism, is approximately 17 degrees. The use of such glass for prisms is not usual - normal glass has a refractive index of about 1.5. The glass used for the prism in this invention preferably has an index substantially greater than 1.55, for example the aforementioned 1.75. A range of specialised glasses including some having refractive indexes of the order mentioned are manufactured by the Schott Company, one such glass being that referred to by this Company as "LaSF 18".

Where the angular field of view is such that any of the ray angles within the prism are less than the critical angle for total internal reflection, the hypotenuse surface of the prism may be coated with silver, aluminium or other reflecting material.

10.2 GUNSIGHTS

We are indebted to MITKEWITSCH GB 2077/1909, GB 30583/1909 of St. Petersburg for two basic yet sublime observations that a gyroscopic flywheel will give a datum line for a telescopic sight substantially fixed in direction; observations soon to be elaborated by the sight of RHEINISCHE METALLWAAREN UND MASCHINENFABRIK GB 9466/1909.

One of the first gun sights to make a more full use of the inherent properties of the gyroscope is that of BARR & STROUD GB 17291/1910 and GB 20373/1910 of England who proposed a sight in which only the objective and fiducial mark of the telescope are carried by the gyroscope and correction allowed for change of range and the temperature of the explosive in use.

A gunsight in which the action of the gyroscope was modified to give the lead ignition in firing of the gun was advanced by SCHIER GB 308184/1910 of Vienna and even more sophisticated sights relying upon the special characteristics of the gyroscope were proposed by HENDERSON J.B. GB 6977/1915, GB 16669/1915, GB 156870 (1921). ANCIENS ETABLISSEMENTS SAUTTER-HARLE GB 146847 (1920) and BREWER G. for KRUPP GB 154933 (1922) GB 306764 (1929).

In the gyroscopic sight of SCHNEIDER & CIE GB 177146 (1923) (Fig 10.2.1) for naval guns a correction equal to j tan i is applied where j and i respectively are the inclination to the horizontal of the trunnions and the gun barrel. The axis of the gyroscope (34) retains an invariable direction and it is obvious that the rod (74) will move transversely in relation to the telescope (4) through amounts that are proportional to the inclination j. The distance d is made to be proportional to $\frac{1}{\tan i}$.

The sight of FISKE discussed earlier under "Telescopes" used a prism (3) (Fig 10.1.2) to deflect the line of sight of a gyrostabilized telescope thereby offering a method of target practice that did not destroy the the target, but juxtaposed the target and the image of the field at which the shell once fired lands. Later GRAY assigned to VICKERS LTD GB 232759 (1926) a sight incorporating a gyroscopically stabilized prism to be followed within two years by a sight due to SPERRY US 1688559 (1928) based on the realisation that a fully movable sight may be adequately corrected in its line of sight by moving a gyroscopically controlled mirror in front of it.

With the rapid advances that were being made in the special discipline of aeronautics it was soon apparent that fast moving aircraft provided a difficult target and much thought was given in the early part of the second quarter of the 20th century to the perfecting of anti-aircraft guns. WILLARD US 1936442 (1933) saw that a predictor gun sight was possible and be proposed means associated with a sighting device for introducing corrections that are dependent upon the speed of the target in order to anticipate the movement of the target during the time of flight of the projectile. To do this he made a direct appeal to the gyroscope. He proposed, in effect, the precession of a gyroscope in a predetermined relation with the movements of the sighting device of the gun. Since the precessional force is proportional to the angular velocity of displacement of its axis, and since the axis displacement is proportional to the velocity of the target, it follows that for a given range the precessional force is proportional to the velocity of the target and hence becomes a measure of the velocity of the target. WILLARD also included corrections for the time of flight of the projectile for the observed range of the target.

At the same time as WILLARD's disclosure BARR & STROUD & FRENCH GB 387848 (1933) proposed a gyroscopic anti-aircraft sight, for use from a moving ship, that made use of a 'stabilization plane' to which the target was referred.

An improved artificial horizon or vertical using a ball like rotor on an air film bearing was proposed by GILLMOR & WITTKUHNS US 1984874 (1934) to enable the gun crew to calculate the exact angle of elevation of the gun at the time of firing when the ship is both rolling and pitching.

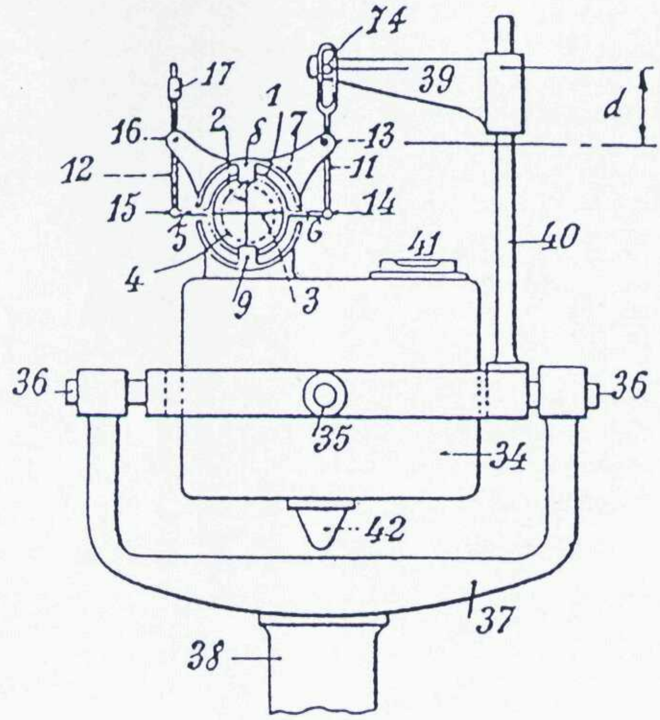


Figure 10.2.1

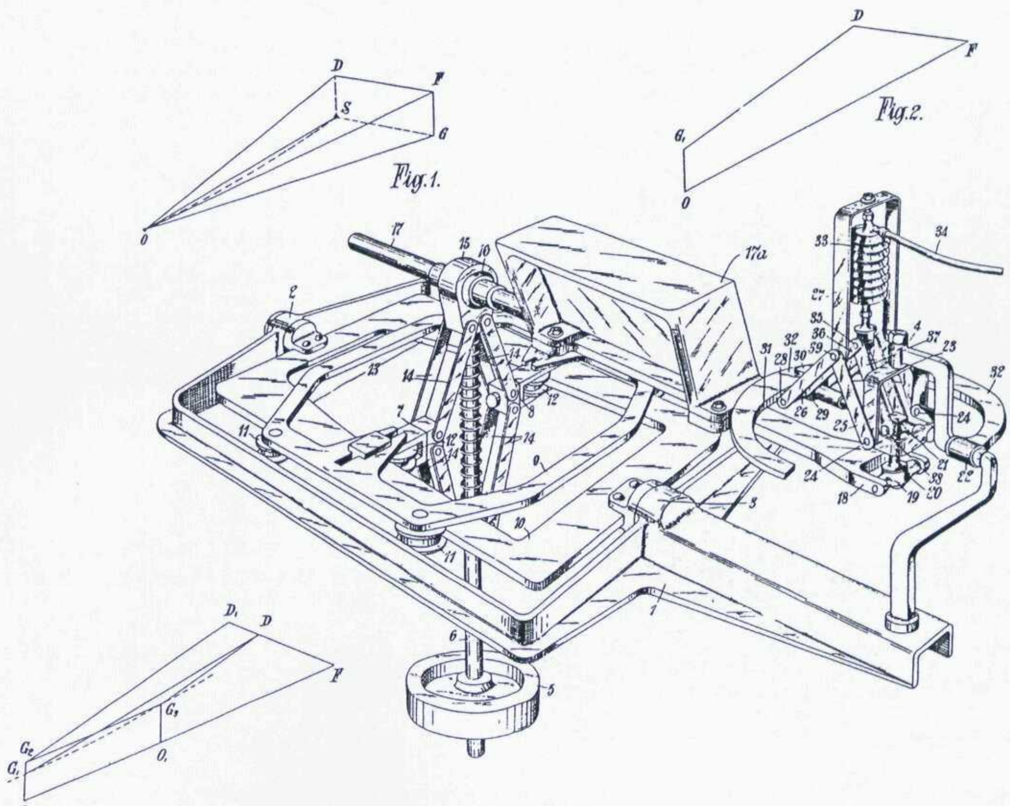


Figure 10.2.2

The problem of tracking a fast moving target proved to be complex and one that required devices operated by a plurality of attendants upon the gun who corrected the input data transmitted to them and so produced a final correction to a tachymetric device. Such a sighting device, much improved by the inclusion of gyroscopic means was proposed by SCHNEIDER & CIE & FIEUX GB 453744 (1936).

VOROBIOFF & HERCKELBOUT GB 480185 (1938) disclose a sight for both a light or heavy machine gun and use a gyroscope to control the sight when the gun is deployed for offensive firing from one aircraft at another. Their gyroscopic sight determines the angular correction to be made in training a gun due to the relative angular velocity between the gun and its target. They deploy a grid that divides the field into sections and has means for spacing its elements according to the relative angular velocity of gun and target per unit of range at the specific angular velocity that is demanded.

MEREDITH & GARDNER GB 576359 (1945) propose a stabilized gyroscopic sight for use in an aeroplane such that when a turn is required the sight is rotated; the aeroplane being turned at a rate depending upon the extent of the original movement of the sight.

It is to FRAZER-NASH & WHITAKER GB 574704 (1945) and to FRAZER-NASH alone GB 577129 (1945) that we must turn for cognizance of the basic geometry for guns mode air to air which is stated by various authorities in the following form.

To allow a sufficiently long burst of fire against a rapidly moving target and still stay within the dynamic capabilities of the airframe, all air-to-air attacks with guns are started from aft of the beam.

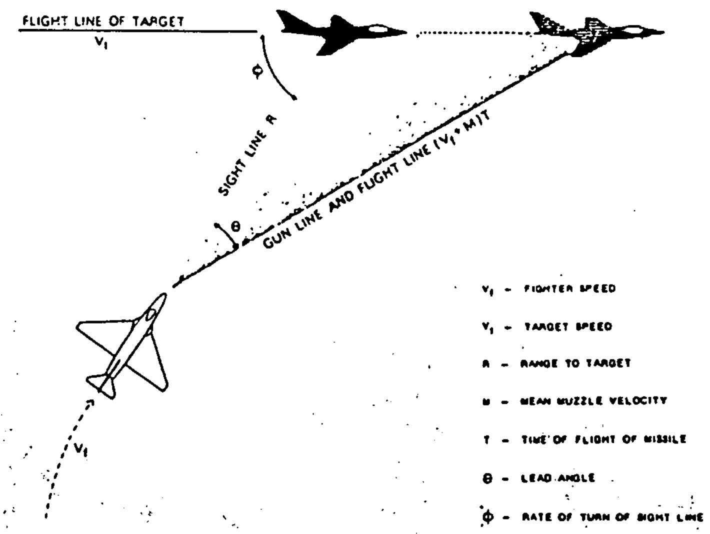


Figure A

Fig. A shows a typical attack profile which is applicable to level, climbing or diving attacks. A fixed-gun fighter is assumed, the gun line being collinear with the fore-and-aft axis of the aircraft.

Fig. A shows the situation at the moment of firing when the aiming mark (sight line) has been continuously maintained on the target for long enough to obtain steady conditions i.e. perfect tracking.

The sighting equation is

1\. ..... $\mathrm{SIN}\ \theta = \dfrac{V_t\ \mathrm{SIN}\ \phi}{M + V_f}$

Where V_t, V_f are target and fighter speeds and M the mean speed of the missile relative to the gun.

The rate of turn of the sight line is

2. ..... $\dot{\phi} = \frac{1}{R}(V_t \sin\phi - V_f \sin\theta)$

and combining (1) and (2) using the small angle approximation gives

3. ..... $(M + V_f)\ \theta = R\dot{\phi} + V_f\ \theta$ or $\dot{\phi} = \frac{M\theta}{R}$

The sight equation is $\dot{\phi} = \theta/S$ and, comparing this equation with (3), if the sight sensitivity $S = \frac{R}{M}$ equation (1) is satisfied.

Therefore, the sight sensitivity required for automatic prediction of lead angle is of the order of the time of flight of the bullet and should decrease as range decreases.

By keeping the target calipered by these arcs, the pilot automatically feeds the correct range to the sight which then computes lead angle.

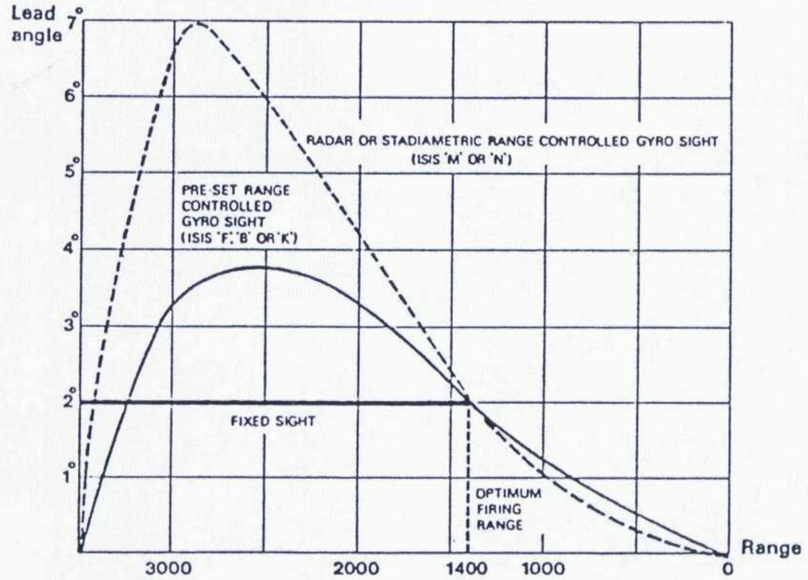


Figure B

Fig. B illustrates these statements and clearly demonstrates the inflexible nature of a fixed sight line compared with a gyro controlled sight line.

FRAZER-NASH & WHITAKER propose a sight having two gyroscopes one adapted to precess about a vertical axis and the other about a horizontal axis. FRAZER-NASH alone (See Fig 10.2.2) uses a gyroscope (5) to keep a rod (6) accurately vertical in combination with a mechanism that inter alia takes into account the effects on the projectile of gravity and the air flow relative to the gun. The sight includes means whereby two separate displacements relative to the gun are imparted to it, one in the direction of gravity and of a magnitude controlled by the factors affecting the gravity drop of the shot and the other displacement being in the direction of air flow relative, to the gun and of a magnitude controlled by the factors affecting the drift of the shot due to the air flow relative to the gun. The sight is characterised by its ability to secure correction for gravity drop that is applied by movement of a member in a rectilinear path that is parallel to the direction of gravity and which cooperates with a member slidable along a sight bar. The displacement for drift is of the form $I\sqrt{ph}f(r\Phi)$ where:

I. is the indicated air speed.
ph. the air density.
r. the range and
Φ the angle between the direction of motion of the gun carrying aircraft and the line of five of the gun and
$f(r\Phi)$ a known function of r and Φ.

GRIMSHAW US 2412453 (1946) discloses a nutationally damped gyroscopic sight in which the connection between the gyroscope and the reflector is such that the reflector is moved but one half of the angle of movement of the gyroscope a feature found to be necessitated by this form of instrument and found in other sights to be described below.

For an important advance in this art we have to wait for the proposals of CUNNINGHAM, FORD, BARNES SYKES HANCOCK & ROBINSON GB 578958 (1945) all of the Royal Aircraft Establishment Franborough in Hampshire and later assigned to FERRANTI LIMITED. The gentlemen are well aware of the earlier predictor gun sights of WILLARD and VOROBIOFF & HERCKELBOUT and they state the position in these words:-

"Whenever relative motion takes place between a gun and its target, or, when relative motion takes place between the gun and the surrounding air having a component in a direction at right angles to the gun barrel, giving rise to "bullet trail", or when the gun barrel is not parallel to the direction of the earth's gravitational field, giving rise to "bullet drop", it is necessary to point the gun in a direction different from that of the straight line joining it to the target in order to hit the target. When these various conditions exist simultaneously, the required angle between the gun barrel and the straight line joining the gun to the target is the algebraic sum of the angles necessitated by the said separate factors. This resultant angle between the gun barrel (the line of aim) and the line joining the gun to the target (the sighting line) is termed the deflection angle.

In conventional practice, when using non-automatic gunsights, the gunner, estimates the required deflection angle approximately by the application of simple standardised rules and aims accordingly. The exact evaluation of the required deflection angle involves consideration of the following factors:-

(a) the velocity of the target relative to the gun;
(b) the range of the target;
(c) the velocity and direction of the air relative to the gun;
(d) the density of the air along the trajectory of the projectile;
(e) the direction of the earth's gravitational field relative to the gun;
(f) the ballastic characteristics of the gun and the projectiles.

It is therefore a matter of considerable complexity. The approximate estimate of the rquired deflection angle obtained by the application of said standard rules is therefore frequently in considerable error, and is known to be responsible for much ineffective shooting.

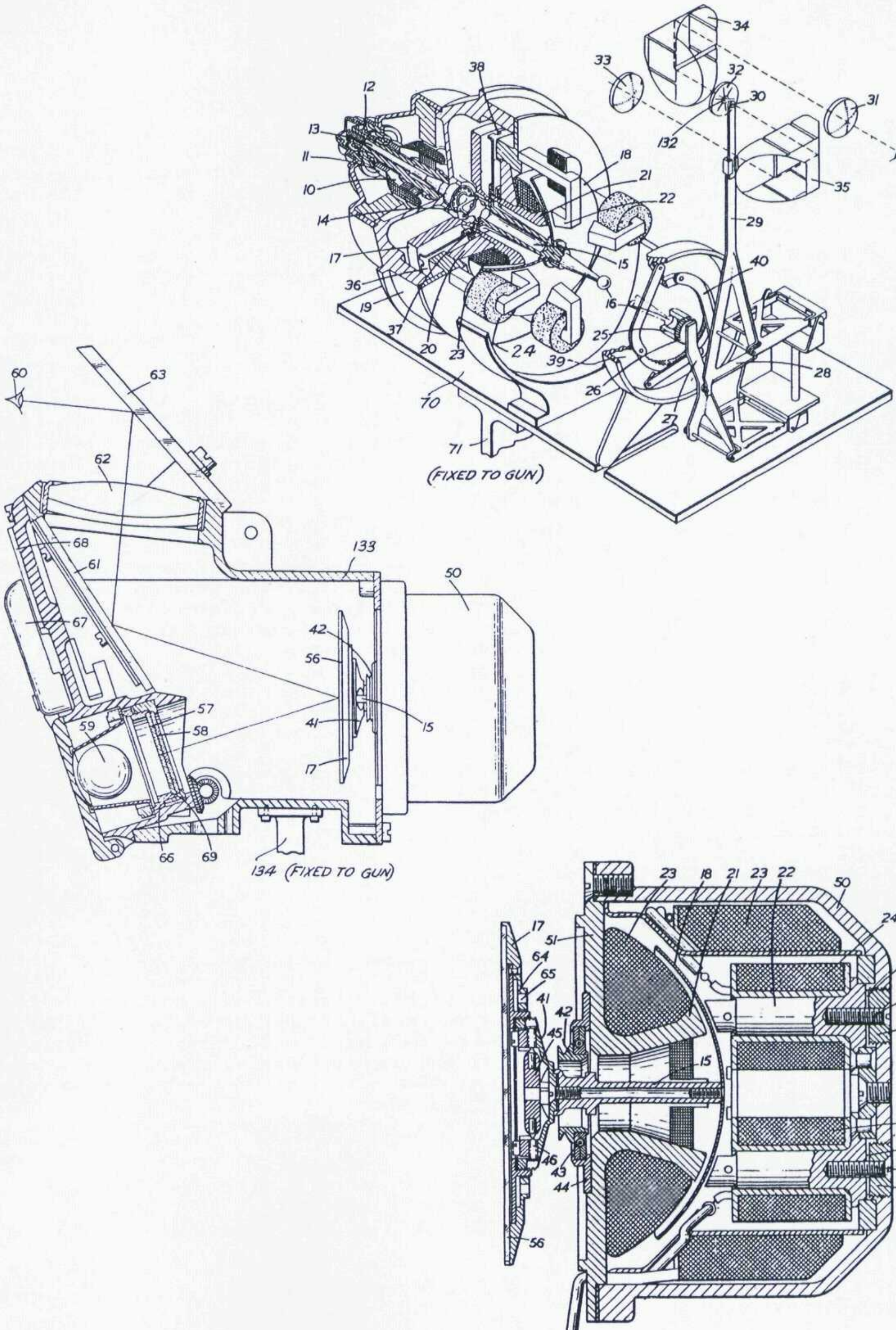


Figure 10.2.3

Our invention has the object of enabling such aiming allowances to be automatically made by interposing a variable deflection angle between the sighting line and the line of aim by automatic adjustment of the gunsight, which may be termed a predictor gunsight, since it enables the required deflection angle to be automatically computed and maintained."

Their solution is a predictor gun sight in which the spin axis of a gyroscope controls the direction of a sight line and is constrained to follow angular movement of the associated gun or groups of guns with a lag which indicates the required deflection angle by eddy current drag applied by spinning dome of copper and the gyroscope given a desired freedom from mechanical constraint by the use of a Hooke's joint.* The first of what has become known as the Hooke's joint gyro mounting. A preferred arrangement of this sight was to form the renowned FERRANTI ISIS gyroscopic gun sight shown at Fig 10.2.3 which was licensed to United States manufacturers for use in certain aircraft of the US Air Force and to SADIR CARPENTIER of France for use in the DASSAULT and MIRAGE aircraft.

In regard to the coercing means for the gyroscope used by CUNNINGHAM et al it is of interest to review briefly the proposals of BROWN GB 359071 (1931) and ESVAL et al US 2229645 (1941) US 2270876 (1942). A not totally dissimilar gyroscopic gunsight to that of CUNNINGHAM et al is that of JOHNSON US 2467831 (1949).

ISSERSTEDT US 2559435 (1951) JOHNSON US 2756625 (1956) US 2859655 (1958) and GRIMSHAW US 2859526 (1958) all advance tachymetric sights that seek to use the basic ideas of CUNNINGHAM et al and to improve them by means of an eddy current disc that coerces the gyroscope.

In the gun laying apparatus of CARTER GB 581966 (1946) one or more gyrostats co-operate with a pair of right angled bell crank levers of which the longer arms intersect to provide at the intersection one of the gun sights so that the sight is moved in relation to the gun axis and the gun directed ahead of the target in the line of motion, by an amount proportional to the angular velocity of the target across the line of sight.

WHEELER US 2570298 (1951) has set out clearly the geometry of lead angle and gravity drop and uses twin gyroscopes to actuate contacts on potentiometers to vary an image on a cathode ray screen via its deflecting plates to provide a proper lead angle when optically tracking a target. A little earlier to this LANDSTAD GB 587951 (1947) had proposed and had assigned to Morris Motors Limited a stabilized sight using two gyroscopes, one for pitching and one for rolling, the gyroscopes cooperating with three radial arms to provide automatic compensation for the pitching & and one for rolling, the gyroscopes cooperating with three radial arms to provide automatic compensation for the pitching and rolling motion.

SPERRY GYROSCOPE CO INC. GB 603389 (1948) disclosed the work of HAMMOND who advanced a computing gun sight in which a gyroscope is used to solve for the prediction angle (Fig 10.2.4). It can be shown that a small scale prediction triangle ABC may be constructed within the sight per se where the length AB is proportional to the reciprocal of the target velocity, the length AC is proportional to the ratio of present range to the product of future range and target velocity and the length BC is proportional to the ratio of the product of target velocity and time of flight to the product of future range and target velocity. This is simply the ratio of time of flight of the bullet to future range and is assumed to be a constant for short range use.

* Hooke's joint known in Europe as a CARDAN JOINT a universal joint due to ROBERT HOOKE (1635-1702).

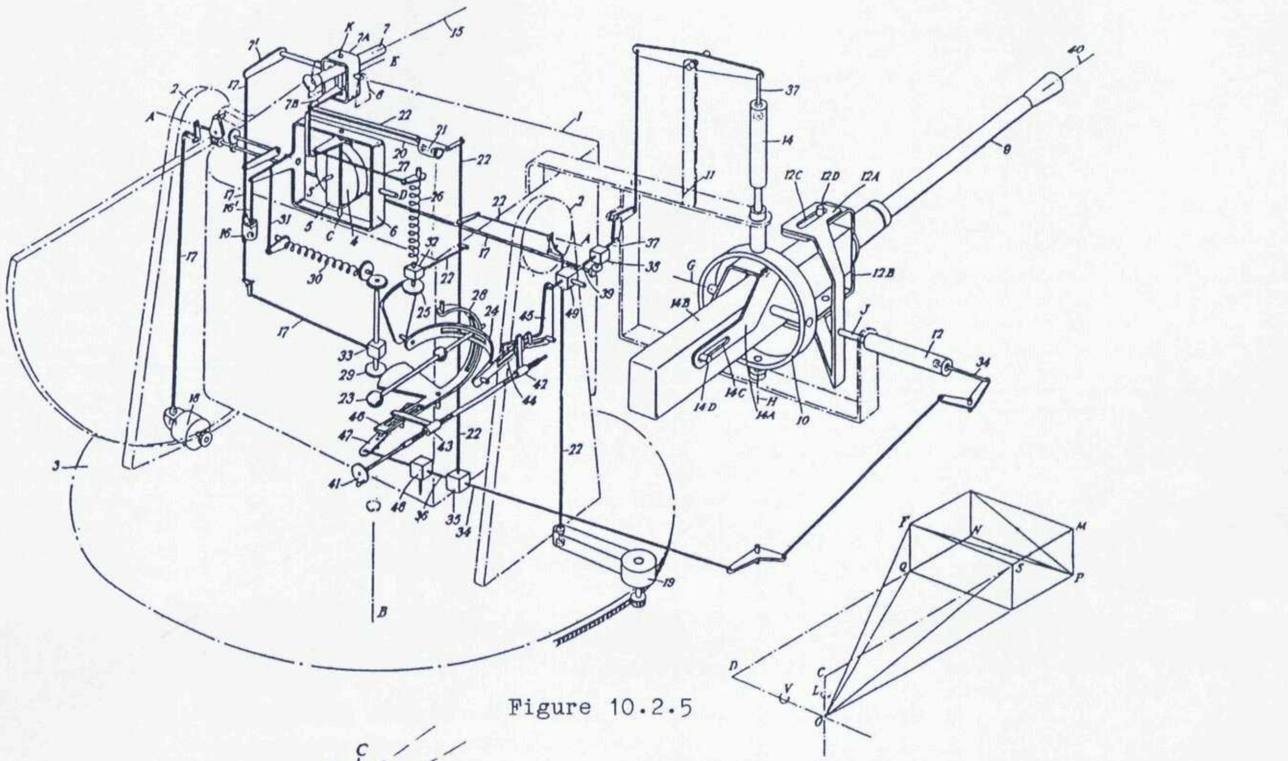


Figure 10.2.5

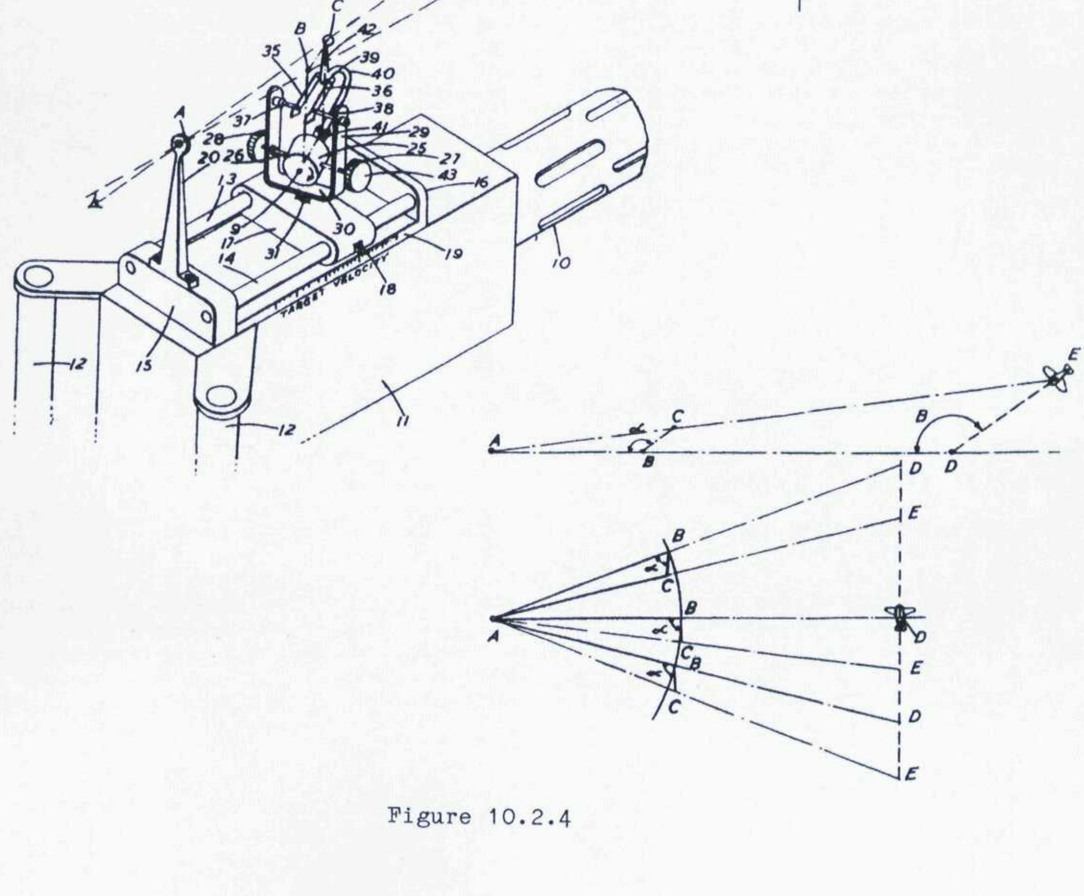


Figure 10.2.4

BURLEY US 2583815 (1952) discloses a nice gyroscopic sight, kept secret from 1940, in which twin gyroscopes orthogonally mounted rotate twin mirrors proportionally to the rate of precession and displace a reticle in the line of sight of the telescope of the sight. A cam surface provides a correction for the 'droop' of the trajectory of the projectile, ideas put forward by BACON GB 5903/1912 as long ago as 1912 and explored with consumate mathematical skill by WATSON GB 498745 (1939) who proposed a sight without gyroscopic assistance and utilized to some extent by VOROBIOFF & HERCKELBOUT GB 480185 (1938).

GLENNY GB 606635 (1948) assigned to SPERRY GYROSCOPE CO LTD. a prior computing gun sight in which the correct deflection angle between the line of sight and the gun is obtained from input quantities fed into a computing mechanism, the said input quantities are supplied by an azimuth input shaft, an elevation input shaft and a range setting, the shafts are driven differentially from the movements of the gun and from a gyroscopic control means that ensures that the computation proceeds on the absolute rate of turn of the gun. A nice piece of mechanism incorporating an almost classical gyroscopic control is due to LANDUCCI, NICHOLLS and MARESCAUX GB 633866 (1949) (Figure 10.2.5). A 'free' gyro is provided with means to operate power controls to stabilize the gun and means for applying precession torques to the gyro to maintain its spin axis parallel to a line of sight to a target and means for superimposing on the stabilization of the gun angular deflection to aim the gun to a future target position. Referring to the geometric figure in Fig 10.2.5. O represents the point of observation, P and F represent the present and future positions of the target respectively, and PF represents the path of the target during the time of flight of the projectile. CO and DO are axes corresponding with the axes C and D respectively in the main figure.

The lateral and vertical deflections depend basically upon the angular difference POF between the positions of the line of sight at the moment of firing and at the moment when the target should be hit. This angular difference may be resolved into two components $\angle POQ$ and $\angle POS$ measured about axes C and D respectively, and the most convenient way of effecting this resolution is to resolve the said angular difference in the first instance into two component angles FOS and FOQ measured normal to the two planes COPM and DOPN. If L represents the present angular rate of movement about axis C and V represents the present angular rate of movement about axis D, then NP = FS = OP.tL, and similarly MP = FQ = OP.tV, where t is the time of flight of the projectile for range OF. We can therefore write

$$\sin \angle FOS = \frac{OP}{OF}\, tL \qquad (1)$$

also

$$\sin \angle FOQ = \frac{OP}{OF}\, tV \qquad (2)$$

Suitable gear may be provided for computing a fictitious range with a time of flight equal to $\frac{OP}{OF}\,t$ which is fed to a range setting spindle 41 as hereinafter described, this fictitious range being calculated from present range OP, present rate of change of range, L and V.

Now, since the spin axis S is caused to follow the target continuously, the two rates L and V are equal to the rates of precession of the gyro about the axes C and D respectively. These precessional rates are proportional to the degrees of torsion in the springs 30 and 26 respectively, which are themselves proportional to the angular displacements of the members 28 and 24 respectively, these two displacements are thus an instantaneous measure of the two component rates of target movement, and are accordingly used to transmit these rates to the mechanism for calculating the lateral and vertical deflections.

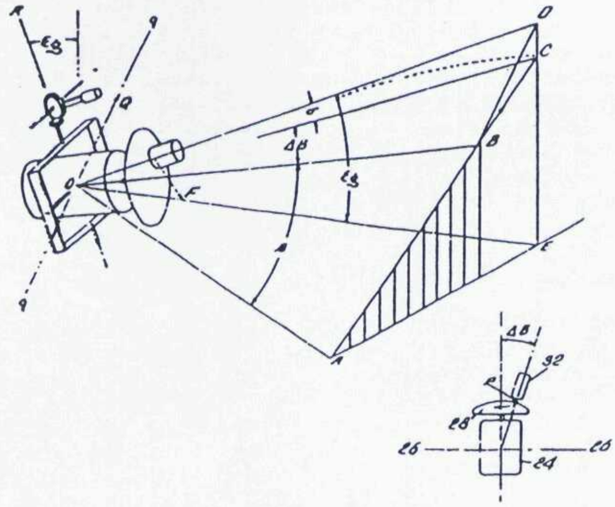


Figure 10.2.6A

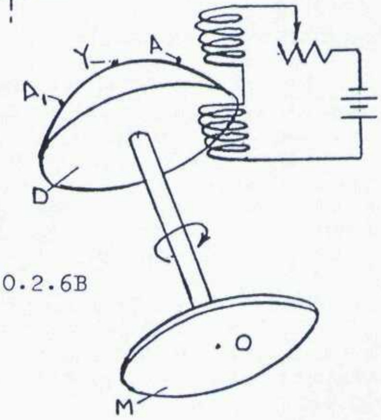


Figure 10.2.6B

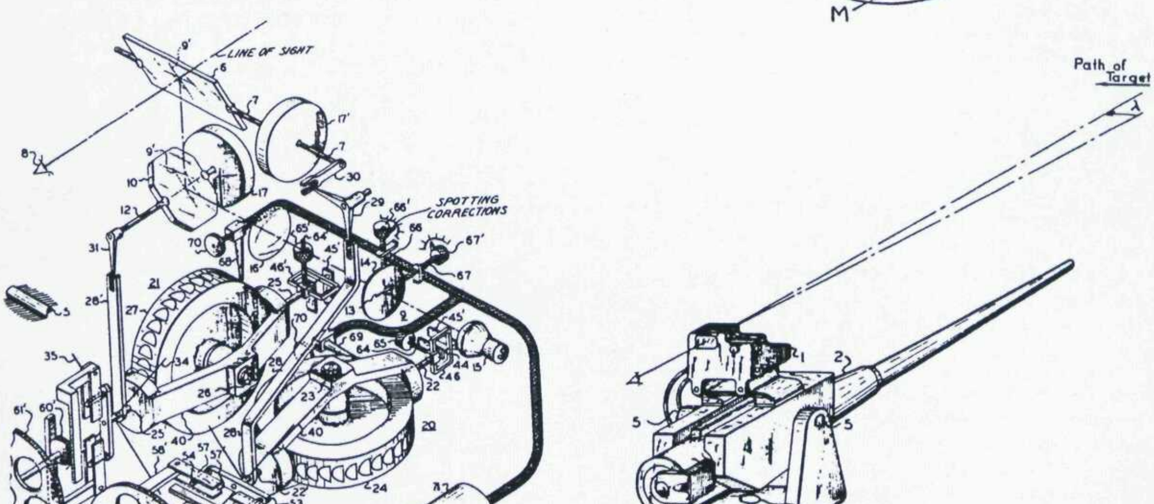


Figure 10.2.7

This mechanism is shown in outline as comprising a spindle 41 capable of rotational movement but not of axial movement, this spindle is given a and rotational movement proportional to the fictitious range referred to above, nuts 42, 43 located on screwed parts of unequal pitch of the spindle are thus positioned axially to produce the multiplying effect of time of flight for the range input. The nut 42 controls the position of a pin engaging slots in a bar 44 attached to the member 24 and a bar 45, the arrangement being such that angular movement of 44 produces an angular displacement of 45 proportional both to its own angular displacement (i.e., angular rate of movement of target about axis D) and also to the displacement of the nut (i.e., time of flight); the resulting displacement of 45 thus represents the product of the two quantities $\frac{OP_t}{OF}$ and V, which (by equation 2) equals sin $\angle$FOQ (i.e., sin vertical deflection): this is applied by the ram 14 to the gun, producing a movement of the gun equal to $\angle$FOQ about axis G and thus automatically producing a movement equal to $\angle$POS about axis J (parallel to D). The nut 43 is associated in a similar manner with members 46, 47 giving sin $\angle$FOS (sin lateral deflection - see equation I) which is applied by the ram 12 to produce movement of the gun equal to $\angle$FOS in the plane containing this angle, this plane having been defined by the movement equal to $\angle$POS about axis J as described above; a movement equal to $\angle$POQ of axis G about axis H (substantially parallel to C) is thus automatically achieved.

BRITISH THOMSON HOUSTON GB 653610 (1951) acting as communicatee for the General Electric Company (GE) of Schenectady disclosed a gyroscopic predictor gun sight for use with machine guns against rapidly moving targets, not wholly dissimilar to the famous predictor sight of CUNNINGHAM et al, but incorporating a weight that exerts via a lever a torque for precessing the gyroscope according to the elevation of the gun barrel. The modus operandi of the sight is made more clear from the field of battle set out at Fig 10.2.6A. Some improvements to this sight were subsequently proposed by G.E. in GB 930207 (1963).

The line of sight is shown in the figure making an angle β with a fixed base line OA in the plane of target travel OAC. $\Delta\beta$ is the lead angle made necessary by the velocity of the target along ABC, and σ is the superelevation angle. The elevation of the gun is denoted εg. The pivot axis of the gyroscope gimbal lies in vertical plane OED, and is inclined at an angle g from the vertical. The magnet axis lies along the bore of the gun.
The lead angle $\Delta\beta$ can be found with good accuracy from the expression

$$\Delta\beta = T\,\frac{d\beta}{dt}$$

where T is time of flight of the projectile corresponding to present range, and $\frac{d\beta}{dt}$ is the angular velocity of the target.

It can also be shown that the superelevation correction is closely represented by

$$\sigma = KT \cos \varepsilon g$$

where K is a constant depending on the ballistics of the projectile. That is, σ varies with the cosine of the elevation of the gun and is nearly a linear function of the time of flight corresponding to the present range of the target. Torques from two sources act to precess the gyroscope in following the target. The first of these arises in the eddy current coupling device. The eddy current disc 28 is shown turning counter clockwise as viewed from the magnet. The portion of the disc directly under the centre pole at the magnet is travelling in a direction perpendicular to plane OBD. The eddy currents induced in the disc by its motion in the magnetic field interact with the field to produce a retarding force, shown as F, which is also directed perpendicular to plane OBD; but in a direction opposite the motion. This force constitutes a torque about the spin axis of the gyroscope and also about axis q-q, in plane OBD, and passing through the axis of suspension of the gyroscope. The torque about the spin axis tends to slow the spin of the gyroscope, but such slowing is prevented by the electrical power supplied to the motor. The torque about axis q-q is shown as vector Q, the direction of which indicates the axis about which the torque acts, and the length of which indicates the magnitude of the torque. The speed with which the disc passes under the magnet depends on the magnitude of angle DOB, so that force F increases as angle DOB increases. Since the moment arm of F about axis q-q remains essentially constant, the torque Q also increases as angle DOB increases. By properly proportioning the thickness of the conducting sheet 31 on the eddy current disc 28, the torque Q is made to vary directly with angle DOB. Torque Q also varies with the square of the flux density under the magnet, and this is controlled by the rheostat 36 which is set in accordance with present range to the target. The torque Q is therefore

$$Q = C \text{ (angle DOB) (square of flux density)}$$

where C is a constant depending on the design of the sight.
The second source of torque acting on the gyroscope is the superelevation weight. This, through the combination of face gear 55 and pinion 58 applies a torque R to the gyroscope about the pivot axis of the gyroscope gimbal, which axis is perpendicular to the gun bore and lies in a vertical plane OED. The magnitude of R varies with the cosine of the elevation angle εg of the gun. This will be seen from the fact that when the gun bore is horizontal, the weight projects horizontally and exerts its maximum torque on the gimbal, whereas when the gun bore is vertical the weight is pointed upward and exerts no torque on the gimbal. The torque applied by the weight may thus be expressed as

$$R = M \cos \varepsilon g$$

where M is a constant depending on the weight and the gear ratio at the face gear and pinion combination.
It is well known that under the action of a torque such as Q, the gyroscope will precess in such a direction as to tend to align itself with the torque; the further that if two torques such as Q and R act simultaneously the rate at which the gyroscope moves can be found by vectorial addition of the rates due to the torques separately. Since the gyroscope precesses in plane OBC to follow the target it is evident that the resultant of torques Q and R must lie in plane OBC. In other words, the superelevation torque R tends to precess the gyroscope downward out of plane OBC, and therefore the magnet must be elevated to line OD above plane OBC to apply an equal and opposite precession rate upward.

The precession rate produced by torque Q, taking place in plane OBD, is proportional to Q and inversely proportional to the angular momentum of spin of the gyroscope. It is thus

$$\frac{C \text{ (angle DOB) (square of flux density)}}{\text{angular momentum of spin}}$$

The scale of a rheostat is calibrated to make the magnet flux such that

$$\frac{C \text{ (square of flux density)}}{\text{angular momentum of spin}} = \frac{1}{T}$$

when range R is set on the scale. Then the angular rate produced by torque Q is

$$\frac{\text{angle DOB}}{T}$$

This may be resolved into two components of velocity,

$$\frac{\Delta \beta}{T} \text{ in plane OBC}$$

and

$$\frac{\sigma}{T} \text{ in plane OCD}$$

But the velocity of the gyroscope in plane OBC is $\frac{d\beta}{dt}$ whence

$$\frac{d\beta}{dt} = \frac{\Delta\beta}{T}$$

or

$$\Delta\beta = T\,\frac{d\beta}{dt}$$

This is the desired solution for the lead angle. As has previously been described, the rate $\frac{\sigma}{T}$ in plane OCD is exactly equal to and opposite that produced by the superelevation weight, which is

$$\frac{M \text{ Cos } \varepsilon g}{\text{angular momentum of spin}}$$

Thus

$$\frac{\sigma}{T} = \frac{M \text{ Cos } \varepsilon g}{\text{angular momentum of spin}}$$

or

$$\sigma = T \text{ Cos } \varepsilon g \; \frac{M}{\text{angular momentum of spin}}$$

However, the desired value is

$$\sigma = KT \text{ Cos } \varepsilon g$$

which is obtained if the moment of the superelevation weight is made

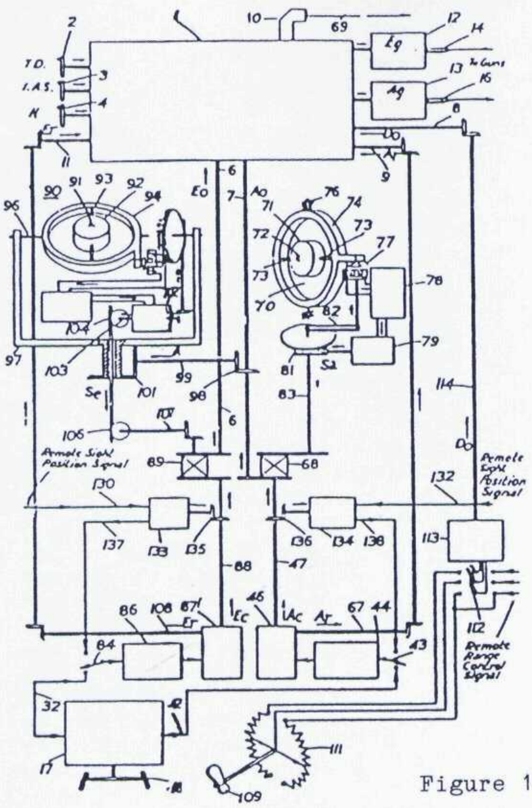
$$M = K \text{ (angular momentum of spin)} = \text{constant.}$$

The physics of the copper dome may be understood as follows. The actual gyro of the sight consists of a copper dome, a flat circular mirror and a short connecting axle. this sytem often revolves at some 3000 r.p.m. about a universal mounting. The cap of the toadstool-like dome moves between the two poles of an electromagnet the current of which can be varied through a variable resistance. (See Fig 10.2.6B). The strength H of the magnetic field between the poles is H = ki where $\underline{k}$ is a constant and $\underline{i}$ is the current in the coils. The lines of magnetic force pass through the dome intersecting it in a circular area centred at A. The area of one instant is immediately replaced by another since the dome D is spinning. Eddy currents are induced in the dome, since a circular annulus of the dome can be thought of as a coil of wire moving across a magnetic field. The current $\underline{j}$ induced is $j = k_1Hv$ where $\underline{k_1}$ is a constant and _ is the velocity with which the coil moves relative to the dome. The induced eddy currents set up four magnets in effect in the instantaneous dome segment producing four forces all of which opposed the motion of the dome with a force F = k_2Hj where k_2 is a constant. The force F tries to slow the dome down, but the constant speed motor driving the dome supplies a couple with a force F/2 at A opposing F and force F/2 at A^1 in the same direction as F. Hence the speed of rotation is maintained and an unbalanced force (F-F/2) + F/2 = F is left which is directed down into the page of the figure. Hence precession will occur in the plane of the spin axis and the axis of the bore of the gun. Under tracking, then, as the bore axis is displaced the spin axis tries to remain fixed, but the displacement causes a force F which leads to precession of the spin axis toward the bore axis. If the gun keeps rotating, the spin axis does not catch-up with it and the two axes revolve with an angular displacement between them. The applied torque, assuming that the moment arm OY = OA = $\underline{l}$ is

$$T = l\, k_2 k_1 k^2 \, \frac{(V)^2}{R^2} v$$

where V is the voltage of the power supply to the driving motor.

A gyroscopic sight known as K-8 was used in the Martin upper turret of the B.24 bomber during World War II; and the major German gyroscopic sights were known as the EZ40. EZ41. EZ42 and EZ25. The EZ40 is a single gyro sight with a mirror on



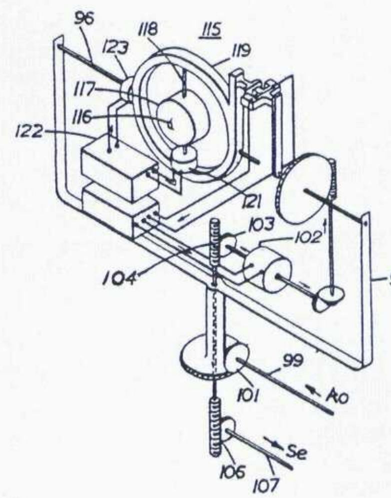


Figure 10.2.8

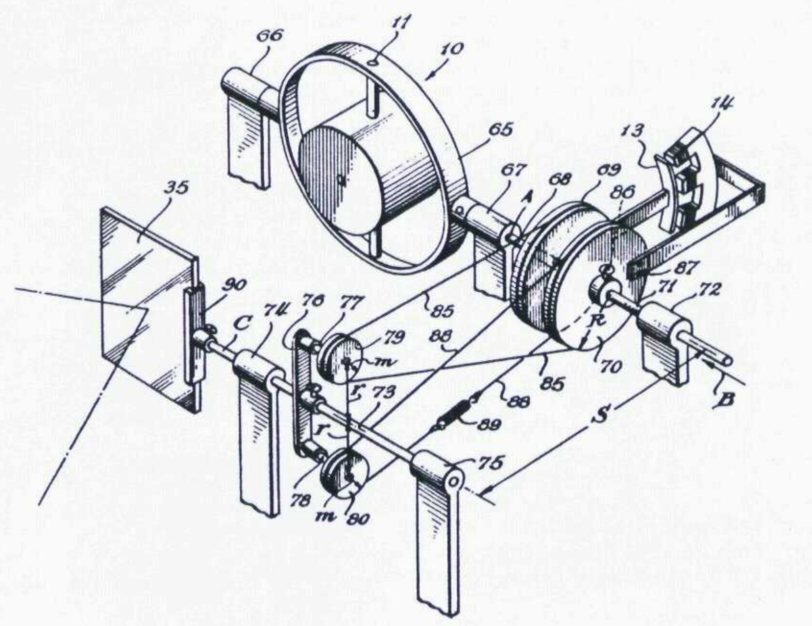


Figure 10.2.9

the gyroscope to deflect the line of sight and the ray from the reticle hits the mirror perpendicularly in the neutral position and this avoids what is known as optical dip. The EZ41 is a single gyro sight with the gyro unit remote from the sight head. The optical system is actuated by motors controlled by a follow-up system. The EZ42 is a twin-gyro sight installed in fighter aircraft such as the FW190 and Me262. The two gyros are placed aft, one being mounted with its axis parallel to the longitudinal axis of the aircraft and the other with its axis parallel to the vertical axis. Friction dashpots are provided for smoothing and the gyros are further restrained with springs. Gyro deflection is picked-off by tiny potentiometers and the lead is electrically computed. A servo drive, employing a very small two-phase motor, supplies motion to the optical system. The EZ45 is a remote single gyro sight. The gyro is controlled by erecting coils and its supporting case is driven by a small motor. Through a time-of-flight potentiometer the angle through which the case turns is made proportional to the lead. The sight parameter is 0.33 and the position of the case is transmitted by a servo mechanism to the sight head and a polarized relay of high sensitivity is used.

STEVENS GB 684667 (1952) acting as communicatee to the Massachusetts Institute of Technology disclosed a gyroscopic lead computing sight (Fig 10.2.7) designed by Charles Stark Draper* et al that relied upon a computing mechanism having the ultimate effect to advance the axis of the gun ahead of the line of sight through the predicted angle . This desideratum is achieved by displacing the line of sight through an angle - with respect to the gun since the gunner always keeps the line of sight onto the target. Two rate-of-turn gyroscopes (20.21) one (20) responsive to the tilt of the gun in elevation and the other (21) to the turning of the gun each rocking, via a lever, one mirror or more placed in the optical train; the one to give a displacement proportional to the rate-of-turn in elevation modified by the time of flight of the shell and the other to give a displacement in the azimuth plane. The precessional movements of each gyroscope is damped by damping means having a damping constant c such that $\frac{c}{ks} > 1$ where s is the sensitivity of the device and k an elastic constant of a centralising means for the gyroscope. In the gunsight disclosed a preference is expressed for c to be between 1.05 and 1.5.

NEWELL US 2684007 (1954) has no serious precursor except BARR & STROUD (1910) and his gyroscopic sight has a collimator in which the rays are reflected parallel to the spin axis of the gyroscope, the mirror following the gyroscope in train at a ratio of unity and following it in elevation at a ratio of 1 to 2. The sight includes an adjustable diaphragm able to form an image of a size such that it exactly incompasses the target.

SPERRY GYROSCOPE COMPANY INC. GB 694850 (1953) GB 714670 (1954) disclosed a gyroscopic aiming device Fig 10.2.8 for a projectile-dispatching apparatus such as a gun mounted on a platform the angular attitude of which was liable to variation relative to a frame of reference regarded as stable. The device sought to remove the difficulty of aiming and to remove the errors associated with the determination of correct gun laying angles. The device included a computer that had set into it data corresponding to target dimensions, indicated air speed and altitude of the enemy craft, the present position of the target relative to the craft in terms of its elevation; azimuth and slant range co-ordinates and the rates of change of the azimuth and elevation co-ordinates to produce therefrom the proper gun elevation and azimuth. The device essentially included a directional gyroscope (70) able to keep the orientation of its spin axis in the horizontal plane and a gyro-vertical (90) each with suitable pick-offs to a computer that fed output voltages to the guns. In the improved device of 1954 the aiming equipment was characterised by the provision of means whereby a measure of the magnitude of the control quantity was combined additively with the measure of the instantaneous angular position or displacement of the line of aim that was served to the computer, so that its response to change of the magnitude of the control quantities was accelerated. A not wholly dissimilar servo-controlled gun stabilization and sighting means are disclosed by SIMMONS LIVERSIDGE and WRATHALL GB 724896/7/8 (1955) and

* This is the legendary gunsight used on the Oerlikon 20mm guns and referred to by Rear Admiral W.F. RABORN in his bibliographic obituary note to 'Stark' DRAPER. MASS HIGH TECH. Aug. 17-30 1987.

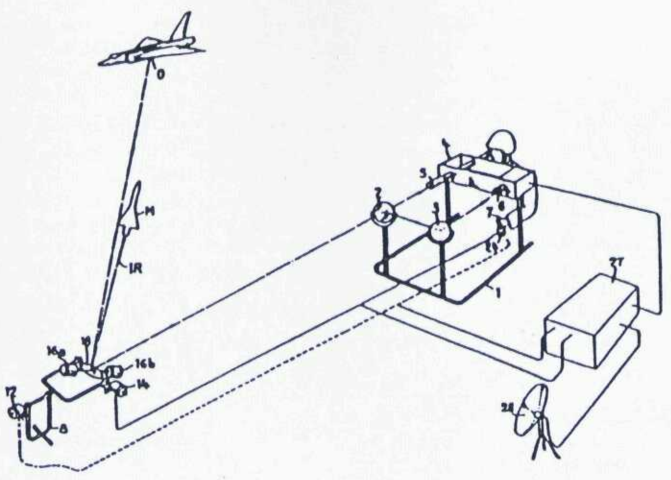


Figure 10.2.10

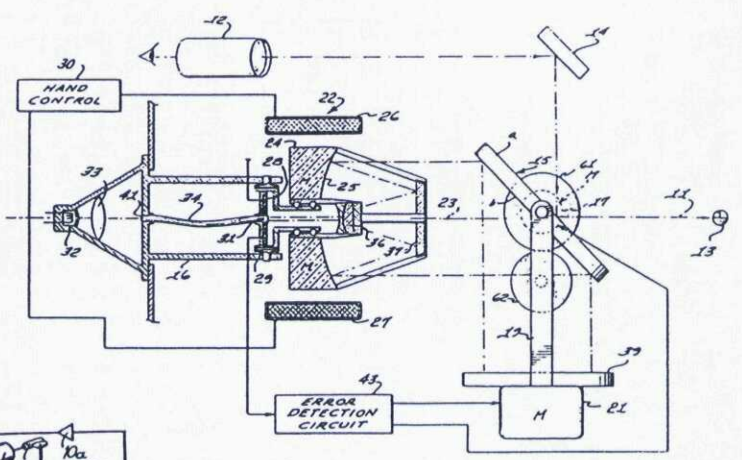


Figure 10.2.11

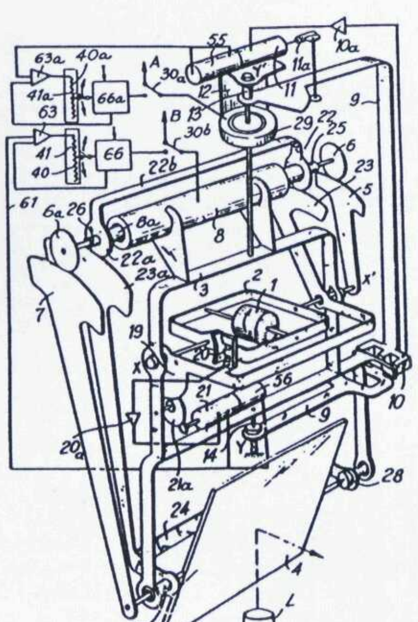


Figure 10.2.12

LIVERSIDGE alone GB 724900 (1955) under the aegis of METROPOLITAN-VICKERS ELECTRICAL COMPANY LTD. and by SPERRY GYROSCOPE INC. GB 754530 (1956).

HAMMOND US 2705371 (1955) disclosed a gyroscopically controlled sight line stabilizing device incorporating an optical compensating differential (Fig 10.2.9) for each of the two orthogonal axes of the gyroscope to stabilize the line of sight against turret lag. If we assume that the gyro is displaced to the position shown in the figure and some turret lag occurs which leaves pulley (70) lagging pulley (69) momentarily in a clockwise direction and angular distance of one degree or less, then the position of the upper part of belt 85 is fixed but the lower part thereof is slightly relaxed due to the displacement of pulley 70. Under the circumstances, no slack occurs in belt 85 because, due to the displacement of pulley 70, the upper part of belt 88 is wound up thereon and, since the opposite end of belt 88 is fixed to pin 86 in pulley 69, the cross arm 76, shaft 73 and mirror 35 are proportionately tilted, which tilt disappears smoothly as the servo mechanism restores the pick-off transformer 14 and its armature into register. When pulley 70 is displaced with respect to pulley 69 but in a counterclockwise direction, the parts of the differential function as described but in the opposite direction. By suitably proportioning the diameters of pulleys 69 and 70 with respect to the radii at which pivots 77 and 78 are offset from the axis of shaft 73 the desired proportional output deflection may be obtained. An approximate equation for the differential is

$$C \cong \left[\frac{R \sqrt{[r^2 + S^2] \left[1 - \dfrac{(R-m)^2}{r^2 + S^2}\right]}}{2rS \left[1 + \dfrac{(R-m)^2}{r^2+S^2}\right]} \right] [A-B]$$

for small values of C where R is the radius of pulleys 69 and 70, A is the angular displacement of pulley 69; B the angular displacement of pulley 70; C the angular displacement of output shaft 73; r is the radial distance from the axis of shaft 73 to the axes of the respective pulleys 79 and 80; m is the radius of pulleys 79 and 80; and S is the distance between the axes of shaft 73 and shafts 68 and 71. In the device as constructed, pulleys 79 and 80 are provided with jeweled bearings which turn on small pivots while pulleys 69 and 70 turn on antifriction bearings.

An integrated radar ranging sight was introduced by POWLEY and SYKES GB 815729 (1959) for Ferranti, it incorporated means for applying such rates of precession to the gyroscope rotor in response to signals indicative of the direction of a target object and derived from radar means as to maintain coincidence between the direction of the image and that of the target object.

NORD AVIATION SOCIETE NATIONALE DE CONSTRUCTIONS AERONAUTIQUES GB 1069266 (1967) GB 1236807 (1971) proposed a sighting device that improved the accuracy of taking aim. (See Fig 10.2.10) A missile (M) is directed toward a target (O) and an infra-red ray (IR) emitted from the missile to follow the optical path (18.2.3) and then to a goniometer (6) that measures the deviation (d) between the line of sight (5.2.18.0) where item 5 is a telescopic sight and items 2.18.3 are mirrors; mirror 18 being supported by a twin rotor gyroscope (16_a 16_b) actuated by torque motors 12.14 from a control stick in the hands of the gun operator. In the improvement of 1971 the rays (FIR) from the target incident upon the reflecting element are permanently parallel to the axis of the gyroscope rotor.

In the same year JOHNSON, CORONADEL, MAR and LUNDQUIST US 3326619 (1967) disclosed a gyro stabilized sight system with no precursors (Fig 10.2.11) that gives an auto- collimation technique wherein a beam of light (from source 32) is collimated by an optical path on a gyro (24) and projected by an auto-collimation mirror (39) from a back surface b and returned through the optical path system on the gyro to a detector (28) to provide a high resoltuion readout of the deviation of the line of sight (11) from the gyro spin axis (23).

We have referred above to the work of BEZU in respect of improvements in the stabilization of the telescope and we return now to his contribution to that of stabilized among sights.

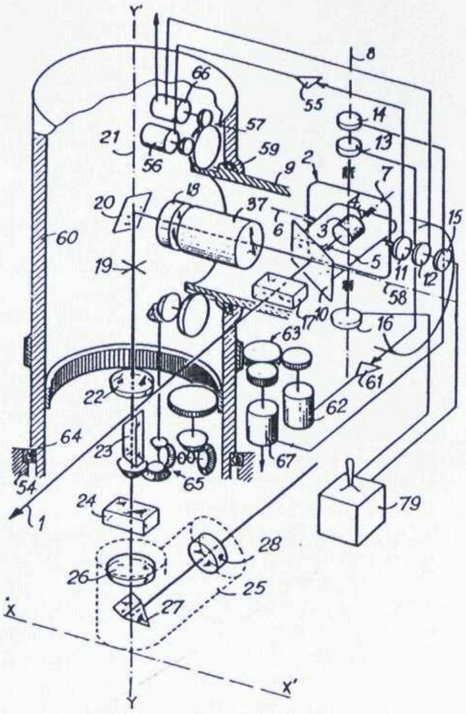


Figure 10.2.13

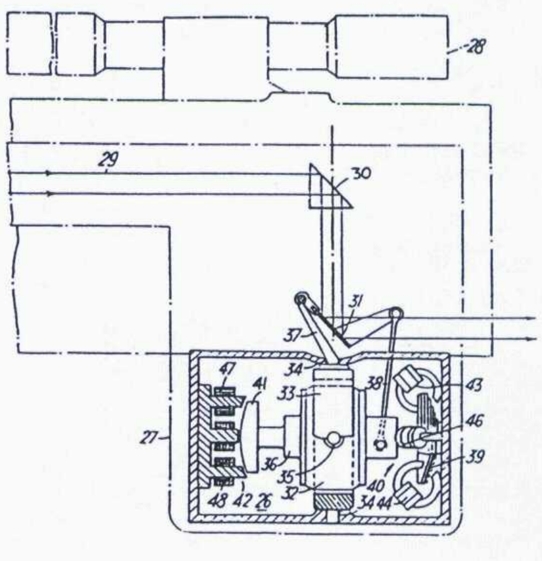


Fig A

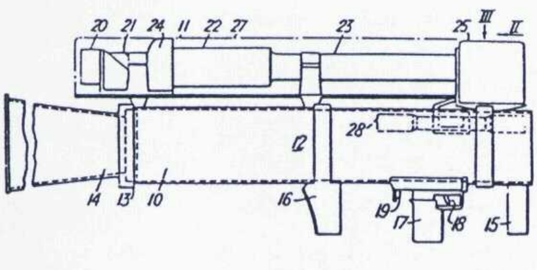


Fig B

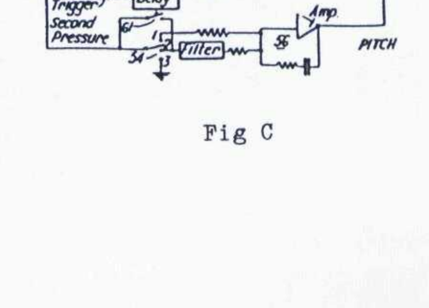


Fig C

Figure 10.2.14

BEZU GB 1114094 (1968) GB 1269817 (1972) GB 1340212 (1973) discloses a movable mirror ahead of a lens of his aiming sight (See Fig 10.2.12) coupled to a gyroscope, the mirror is movable about two orthogonal axes in azimuth and in elevation axes, in which the movement of the elevation axis (about XX) is but one half of that of the elevation gimbal (2) of the gyroscope but the azimuthal relation between mirror and gyro is direct (about YY). The one half relation is termed by BEZU 'demultiplication', it is achieved by the use of a sector cooperating with two pulleys. BEZU also specifies that the moments of inertia of the mirror (I) of the rotor of motor 8 (J_1) of the mechanical transmission including pulleys (6.6a) (J_2) are related to the demultiplication ratio n between the shaft of mirror (4) and the shaft of the pulleys (6.6a) is

$$\frac{I}{J_1 + J_2} = n\ (n+2)$$

BEZU (1972) discloses a not dissimilar sight using what he calls a Pechon prism* and an electromechanical device that upon actuation locks the gyroscope rotor and thus the optical beam against movement in elevation to give a high speed of traverse. Ideally a pair of such gyroscopic sights are deployed, one at the command of an observer and the other of a gunner.

BEZU (1973) assigned to SOCIETE D'ETUDES ET DE REALISATIONS ELECTRONIQUES (See Fig 10.2.13) his observation instrument having a fixed eye piece and a sighting axis that is stabilized by a gyroscope to give a panoramic vision. The sight deploys two mirrors (10.20) referenced to a single gyroscope double gimbal system. One mirror (10) is fixed to the inner gimbal and set at a fixed angle of incidence with its plane parallel to the rotational axis of the outer gimbal and inclined at 45 degrees of arc to the rotational axis of the inner gimbal. The other mirror (20) is slaved to the movement of the gyroscope about the axis of the outer gimbal. The device use two afocal anamorphic optical systems the magnification axes of which are on orthogonal axes to eliminate the anamorphosis of the image obtained ahead of the eye piece. The sight is to some extent anticipated by his own earlier work disclosed in BEZU GERMAN 1428733 (1969_1) FRENCH 1563217 (1969_2) and by the gyroscopic sight of RITCHIE US 3558212 (1971).

TONKIN GB 1161481 (1969) turns his attention to the more accurate firing of a beam-riding-projectile by introducing a gyroscopic control such that after gun recoil the sight and guidance beam are maintained onto the target and the guidance system corrects any trajectory errors by causing the projectile to steer into the centre line of the guidance beam. (Figs 10.2.14A, B and C) show a typical launcher for a one hundred millimetre missile.

In the example illustrated in Figs A and B the launcher consists of two main parts, a gun 10 and an optical projector assembly 11 mounted on a platform on top of the gun. The gun is of the "recoilless" type in which on firing, the forward momentum of the projectile is balanced by the rearward momentum of a proportion of the combustion gas which is exhausted rearwardly to atmosphere. The momentum finally imparted to the gun is small, but such as to cause sufficient gun movement to interrupt steady tracking of target. The gun consists of a barrel 12 and a breech 13 to which is attached a vent 14 through which some of the combustion gases are accelerated rearwardly. Attached to the underside of the barrel are a forward stock 15, a shoulder stock 16 and a pistol grip 17. The latter includes a trigger 18 and a cocking lever 19. The optical projector assembly 11 is arranged to project a guidance beam 29 (Fig A) along the axis of the launcher to a target and consists of a radiation source 20 (Fig B), such as a pulsed gallium arsenide laser, a projection lens system of known design housed within tubes 21, 22 and 23, and a pattern generator, mounted in housing 24, for dividing the beam cross-section into sectors and for rotating this sector pattern. To the forward end and to one side of the optical projector assembly there is attached to the gun barrel a housing 25 for a gyroscopically stabilised mirror system 26 (Fig A) to ensure beam stability both before, during, and after the projectile launch. The whole projector assembly is covered by a protective casing shown in broken outline at 27.

* I suspect he is referring to the well known PECHAN prism but his drawing shows a double DOVE prism. For accurate advice see LEVI L. Applied Optics 1968.

Below the optical projector assembly and to one side of the gun barrel is mounted a telescope sight 28 with which an operator locates and tracks a target.

Fig A illustrates the gyroscopically stabilised mirror system 26 in greater detail. The guidance beam 29 is projected on to a fixed prism 30, turned through 90° and directed on to a movable mirror 31. In the undisturbed position of this mirror the beam is reflected through a further 90° angle so that it emerges from the optical system parallel to its original course and to the longitudinal axis of the gun barrel. The gyroscopically stabilised mirror system consists of a gyroscope, the stator 32 of which is universally mounted relative to housing 25 by a gimbal ring 33. Gimbal ring 33 is pivotally supported in housing 25 by bearings 34, and the stator 32 is pivotally supported in gimbal ring 33 by bearings 35. The rotor of the gyroscope is not seen in the drawing, but one of the bosses which houses a bearing to support the rotor is seen at 36. In Fig A the stator 32 is shown in such an attitude that the rotor spin axis is parallel to the longitudinal axis of the gun barrel. An extension 37 to the gimbal ring 33 protrudes beyond the housing 25 and forms a support for the mirror 31 permitting pivoting movement of the mirror in yaw but not in pitch. In the pitch plane the mirror moves with the gyroscope rotor and because the light beam from the prism travels along the axis of this movement the direction of the beam emerging from the apparatus is always parallel, in the pitch plane, to the rotor axis, regardless of the gun-to-gyroscope angle in this plane. Angular movement of the launcher in yaw, however, results in a change in the angle at which the beam strikes the mirror 31. If the mirror were fixed, the yaw angle of the emerging beam would also change. To maintain the emerging beam substantially parallel to the spin axis of the rotor, we arrange for the mirror to undergo a compensating angular movement. Since the beam is being reflected by the mirror, this angular movement of the mirror in yaw is only one half of that of the launcher. To achieve this, a link 38, which is pivotally connected with the stator 32, provides a further support for the mirror 31. The position of the mirror 31 with respect to its support members 37 and 38 and to the gyroscope stator and gimbal is such that in the yaw plane the mirror 31 always moves through an angle substantially equal to one half that of the gun-to- gyroscope axis angle. The linkage shown achieves the correct relationship between gyroscope and mirror movement only at its central position, but it is approximately correct over the remainder of its angular sweep and is lighter and cheaper than systems of gearing, for example.

The guidance beam control system will now be described. The moving member 39 of a torque motor generally indicated at 40 is rigidly fixed to the gyroscope stator 32 by a support rod coaxial with the spin axis; and the moving member 41 of an inclination indicator or pick-off, generally shown at 42, is similarly fixed to stator 32 at the other end thereof by a support rod. The electrical connections to the gyroscope are shown in more detail in Fig C. The moving member 39 of the torque motor 40 consists essentially of a conducting coil wound upon a dome-shaped former and is arranged to move in the air gaps of four equally spaced electromagnets 43, 44, 45 and 46. Relative variation of the excitation of the four electromagnets will produce a torque to turn the gyroscope stator 32 in any required direction about its gimbal bearings 34 and 35. The moving member 41 of the inclination indicator 42 consists of a dome-faced iron armature, the movement of which causes e.m.f.'s generated in the coils 47, 48, 49 and 50 to become unequal. By connecting the coils in pairs in a suitable manner the resultant e.m.f.'s in the circuits of these pairs of coils indicate the direction and extent of deviation of the gyroscope spin axis from its undisturbed state. The output signals from the pairs of coils 47, 48 and 49, 50 are passed through phase-sensitive rectifiers 51 and 52 through three-way switches 53 and 54. The switches are mechanically interconnected to operate simultaneously. The terminals corresponding to switch positions 1 and 2 are each connected into a memory network indicated generally at 55 and 56, the outputs of which are fed into the electromagnets 43, 44 and 45, 46 respectively of the gyro torque motor 40. The terminals of switch position 3 are connected to earth. In this instance the gyroscope rotor is shown as gas driven, the gas supply is shown diagrammatically at 57.

In operation an operator will initially set the cocking lever 19 and in doing so will cock the firing pin of the launcher. The cocking lever will also release the gas supply to run up the gyroscope rotor and switch on the electronic circuit. The system is then in the condition described as mode 1 in which the switches 53 and 54 are in position 1. In this condition the gyroscope is tightly slaved to the launcher axis, i.e. it follows closely changes in direction of the launcher. In the system shown in Fig C the memory circuits 55 and 56 continuously store information from the pick-off 42 for a period of say one second previously. The information stored is, however, biased toward the more recent signals received, i.e. the influence of signals received by the memory circuits on their outputs decreases with the passage of time since the receipt of the signals in question. The operator can in this mode of operation quickly slew the launcher toward a target and then locate it accurately by means of his telescopic sight 28. The trigger 18 is then pressed, thereby moving the switches 53 and 54 to position 2. In this position the circuit is modified by the inclusion of a filter so that the beam direction follows changes in direction of the sight (or rather the launcher axis) but with a lower frequency response, ignoring rapid and temporary changes of direction. Random movements from the correct tracking path or aiming line are thus smoothed out. The trigger 18 is then pressed again thereby firing the projectile and simultaneously moving the switches 53 and 54 to position 3. The circuits between the pick off 42 and the torque motor 40 are broken so that the torque motor is provided only with signals corresponding to the information stored within the memory circuits 55 and 56. The tracking of the target is thus continued in a smooth uninterrupted fashion during the period of firing the projectile. Delay devices 58 and 59 actuated by the second of the said pressures applied to the trigger 18 and arranged to trip switches 60 and 61 respectively, ensure that after a brief period the system reverts to the second mode, for guidance until the target is reached. It will be seen that the memory circuit consists of an electronic amplifier with a capacitor in its feed-back circuit to cause it to operate as an integrator. When a target is being tracked steadily, the integrator capacitor builds up a charge such that no steady error signal is required at the input to the amplifier to maintain a steady output signal to drive the torque motor.

The French company SOCIETE D'APPLICATIONS GENERALES D'ELECTRICITE ET DE MECHANIQUES (SAGEM) GB 1183898 (1970) express an interest in a similar sight to that of TONKIN since they deploy what they call 'a propagatory phenomenon' that in one example is an optical radiation. They close to stabilize their mirror by the well known two gyroscope stabilized platform that is (See Arnold & Maunder (1961) p395) coupled to it through a linkage giving a reduction in movement of one half.

Aktiebolaget BOFORS the internationally famous Swedish company make two separate contributions to the art of stabilized gun sights.

In AB BOFORS GB 1352349 (1974) they disclose a stabilized sight having a gyro-stabilized line of sight using a conventional twin gyroscope stabilized platform coupled to a mirror disposed between a viewing window and a telescope objective. This gyro sight may be used in target simulators.

In AB BOFORS GB 1475112 (1977) there is disclosed apparatus, due to BLOMQVIST & STALFORS, US 4105174 for receiving or transmitting radiation in a gyro-stabilized direction. The principle is explained by reference to Fig 10.2.15. A positive lens (1) a flat mirror (2) pivoted about a point (3) and a detector (4) are arranged as shown. The distance between lens (1) and point (3) is equal to one half of the focal distance of the lens. It will be found that an incident beam at right angles to the mirror will be brought to a focus at point (4). The mirror in generally gyroscopically stabilized, indeed when used in a missile the mirror may be itself a gyroscope rotor. This device is to some extent anticipated by the work of ESTEY and VOGE US 2963973 and US 3434354. We noted above the work of DE LA CIERVA and he enters the present field of subject matter with his gun sight of 1975 (US 3871236). In this work there is disclosed a gun sight stabilized by a universally pivoted gyroscope rotor and contra-rotating reaction flywheel. This device is to some extent known from the earlier work of FIEUX GB 464315 (1937) and CLEVELAND US 2914945 (1959).

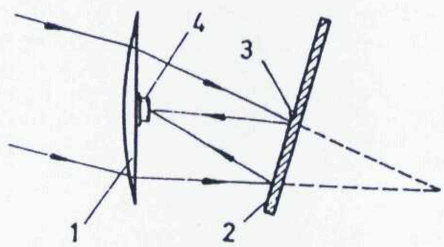
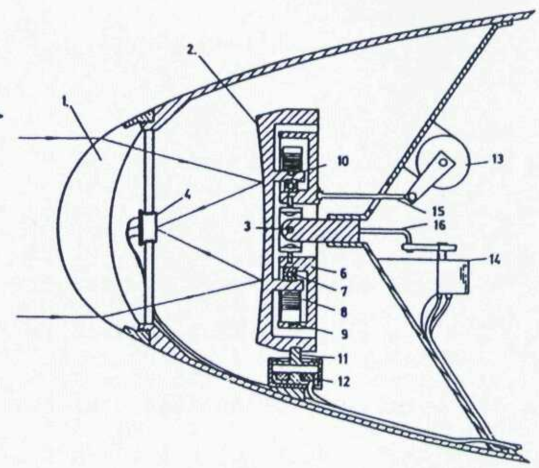


Figure 10.2.15



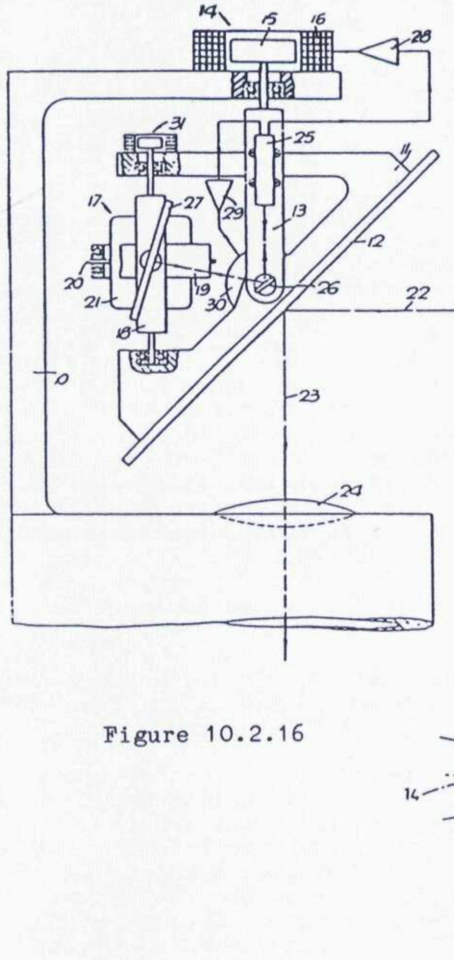


Figure 10.2.16

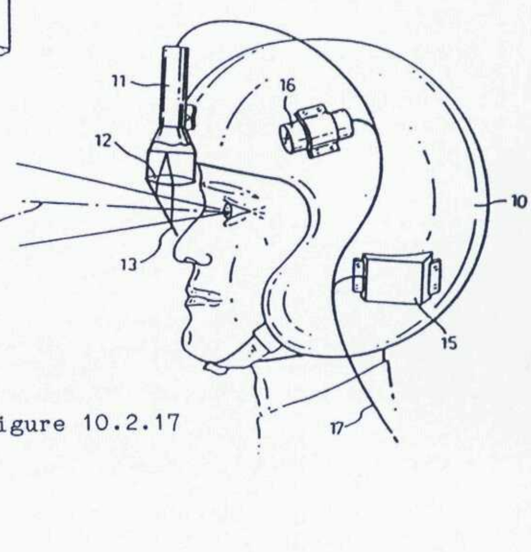


Figure 10.2.17

Figure 10.2.17

CANN and WHALLEY and CANN alone have assigned to FERRANTI LTD. GB 1491117 (1977) GB 1559218 (1980) a gyrostabilized sight for use primarily in helicopters.

The sight is shown in Fig 10.2.16, it comprises a main mirror (12) controlled by a two-degree of freedom gyro (17) and control means including a light source and a light sensitive detector in the form of a photoelectric auto-collimator (25) cooperating with a first small mirror (26) carried on a main mirror frame for movement about the pitch axis, and a second small mirror (27) carried on the inner gimbal of the gyro. When the sight line is stabilized the auto-collimator provides a balanced output and no error signals, but if unbalanced the error signals operate torque motors to restore the balance. The two small mirrors may be replaced by synchros that produce the error signals. In a modification due to CANN alone a balanced inertial mass in the form of a flywheel is attached to the main mirror so that mirror and flywheel are contra-rotating and the work load on the torque motor controlling the main mirror is small.

This stabilized Ferranti sight is said to remove the two to one mechanical linkages that are frequently used to give the necessary correct pitch stabilization and which linkages are referred to above as an essential part of the earlier gyro stabilized sights.

In a sight due to CANN & SOMERVILLE, FERRANTI GB 1512932 (1978) disclose a gyroscopic sight including a two-degree of freedom gyroscope of which the outer gimbal carries a mirror and the sight carries an indication means so that the observer has a warning of the proximity of the sight line when it is at the limits of its stabilization.

LLOYD has assigned to WESTINGHOUSE ELECTRIC CORPN. GB 1531871 (1978) what is in effect a gyroscopically stabilized sight called by them an inertially stabilized heliostat that includes elevation and azimuth gyros combined with a half angle drive that eliminates the need for counterweights.

A sophisticated piece of head gear for homo sapiens in aerial combat, in which there is for the first time an affinity with the diptera and strepsiptera, is disclosed by FERRANTI plc GB 2143948 for the inventor R.J. McFarlane. The Fig 10.2.17, would have delighted Giuseppe Arcimboldo.*

A sighting unit 11,12,13 carries a detector unit 15 which determines the movements of a helmet 10 without reference to apparatus external to the helmet. The detector is gyroscopic and the helmet also carries a boresight detector 16 which forms part of correction means operable to correct automatically for errors in the output of the gyroscopic detector unit from time to time as the head of homo sapiens moves through the reference direction. The sighting unit includes a cathode-ray tube 11 viewed superimposed on an outside scene by means of a semi-reflecting screen 13.

The detector unit 15 comprises an arrangement of gyroscopes arranged to detect movements of the helmet about the azimuth and elevation axes. It is possible to use either two single-axis gyros or a single two-axis gyro.

In use, the user is able to move his head, and hence the helmet, through a limited angle in both elevation and azimuth: Movements about the sight line axis 14, that is "roll" movements, are less likely. Any required display may be produced on the cathode-ray tube 11 for projection onto the screen 13 and this is usually focussed at infinity so that the user does not have to adjust the focus of his eyes when viewing both the display and the outside scene against which it is juxtaposed. If the display provides, say, a simple aiming mark or reticule, then movements of the helmet in azimuth and elevation will be required to cause opposite movements of the said reticule.

* The Arcimboldo Effect: Transformations of the Face from the Sixteenth to the Twentieth Century by Pontus Hulten et al. Abbeville Press, 402 pp.

NOTE Since writing the report I have been made aware of two most useful articles on the subject of gun sights.

(1) Callander. D.F.
Gunsights Through the Years
Ferranti Ltd. (1970). 15.p.

(2) Wallace-Clarke. R.
Drawing a Bead
Aeroplane Monthly.
(April-May 1983) pp. 200-204 pp. 279-283

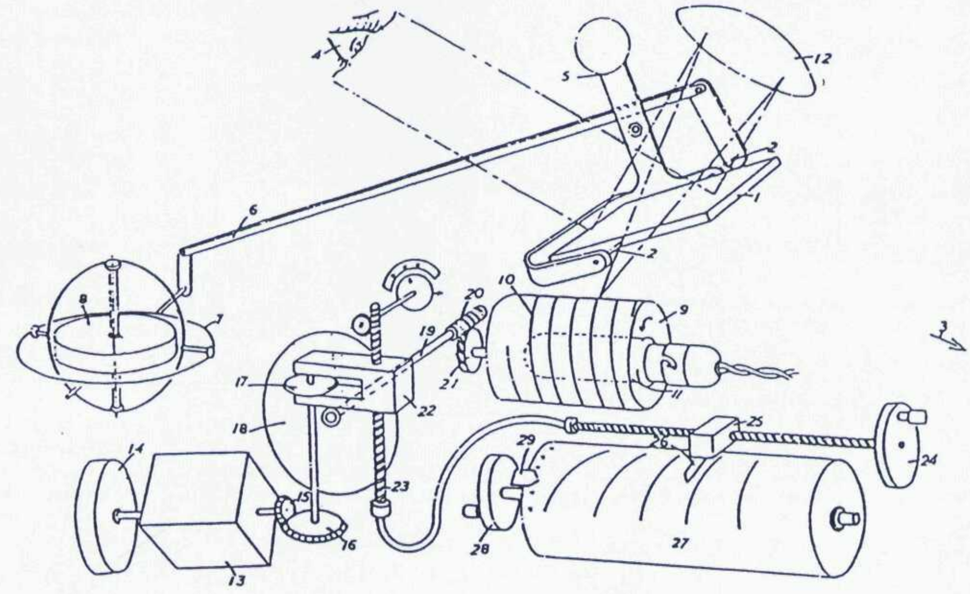


Figure 10.3.1

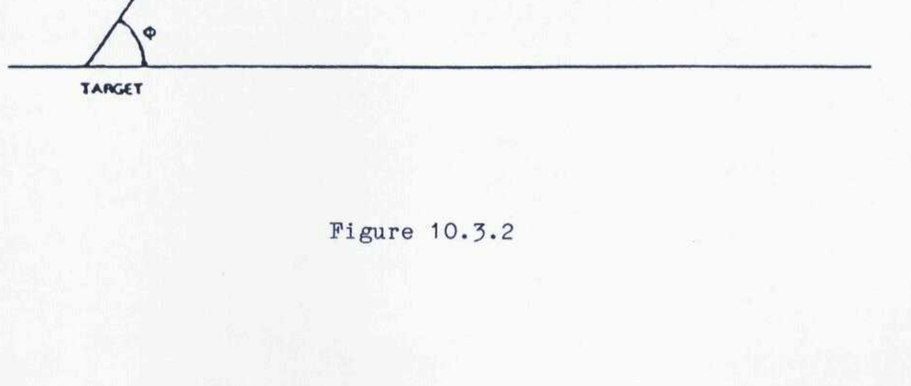


Figure 10.3.2

10.3 BOMBSIGHTS

OTTO KRELL US 940329 (1909) proposed a gyroscopic control for an optical sight to aid the navigation of an airship and assigned it to SIEMENS SCHUCKERTWERKE of Berlin. But the earliest uses of the gyroscope to steady a sight possibly for aerial bombing is that of GRAY and TAYLOR GB 127877 (1919) CARL ZEISS GERMAN 360390 (1922) and TITTERINGTON US 1645079 (1927).

BATES US 1783769 (1930) disclosed a bombsight that obviates the need to adjust the gyro stabilized reticule by introducing a mirror that is able to be moved to follow the target and assigned his rights in the device to Sperry Gyroscope Co. Inc of Brooklyn.

The major factor limiting effectiveness of a military flight mission is whether the aircraft or weapon reaches its geographical goal correctly. The air to ground acquisition of a physical target by man involves physiological problems of which accurate vision is the most critical, which desideratum is fully explored by HUDDLESTON et al (1972). The mathematics of the pure ballistics is well known and is given with clarity by WIMPERIS (1928) although it was made available as early as 1922 by ZEISS & KILLAT GB 186099 (1922).

JOHN W.R. TAYLOR (1974) reminds us that the US Army Air Force claimed that its highly secret NORDEN bombsight provided such accuracy that 'a bomb could be placed in a pickle barrel from 20,000 feet'.

The NORDEN sight is the subject of a disclosure by NORDEN & BARTH US 2428678 (1947) following a long period of secrecy from the spring of 1930.

Prior to 1947 a number of gyroscopic bombsights were known such as those communicated by SPERRY GYROSCOPE CO. INC to HILLIER GB 449238 (1936) said to be operational at altitudes of 20,000 feet and of the general type advanced by KILLAT G. to ZEISS in 1922 and referred to above, and by WATSON J.P. to VICKERS - ARMSTRONG LTD GB 413338 (1934).

SOCIETE ANONYME DE CONSTRUCTIONS AERONAUTIQUES (SACA) GB 473000 (1937) show how a mirror may be fixed directly to the gyroscope casing to provide a nose-dive aerial bombsight.

CHAFEE and VAN AUKEN GB 490811 (1938) and CHAFFE and MURTAGH US 2162698 (1939) use a gyro vertical to stabilize a reticule and give a general mathematical dissertation on the ballistics, which they advance further in the disclosure of CHAFEE and VAN AUKEN US 2371606 (1945).

Sir C.D. BURNEY et al GB 551880 (1943) give a basic dissertation on the use of a gyro- horizon for incorporation in a suitable bomb sight.

FEDDLE US 2350303 (1944) US 2432613 (1947) has proposed a sight using two gyroscopes based on the realization that a combination of two stabilized gyroscopes having the same kind of error law, but of unequal sensitivity provides an extremely accurate means for determining the departure of either gyroscope from its neutral or normal position.

AGA-BALTIC AKTIEBOLAG GB 563169 (1944) point to the errors associated with any attempt to follow the falling object since it does not in its descent conform with any known mathematical law. They deal with the problem empirically and they combine the gyro-sight with a fixed scale previously calibrated.

RICHARDS GB 579848 (1945) of the Royal Aircraft Establishment proposed a bomb sight stabilized in pitch by linking a transparent mirror through a 2 to 1 linkage to the pitch ring of a gyroscope. See Fig 10.3.1.

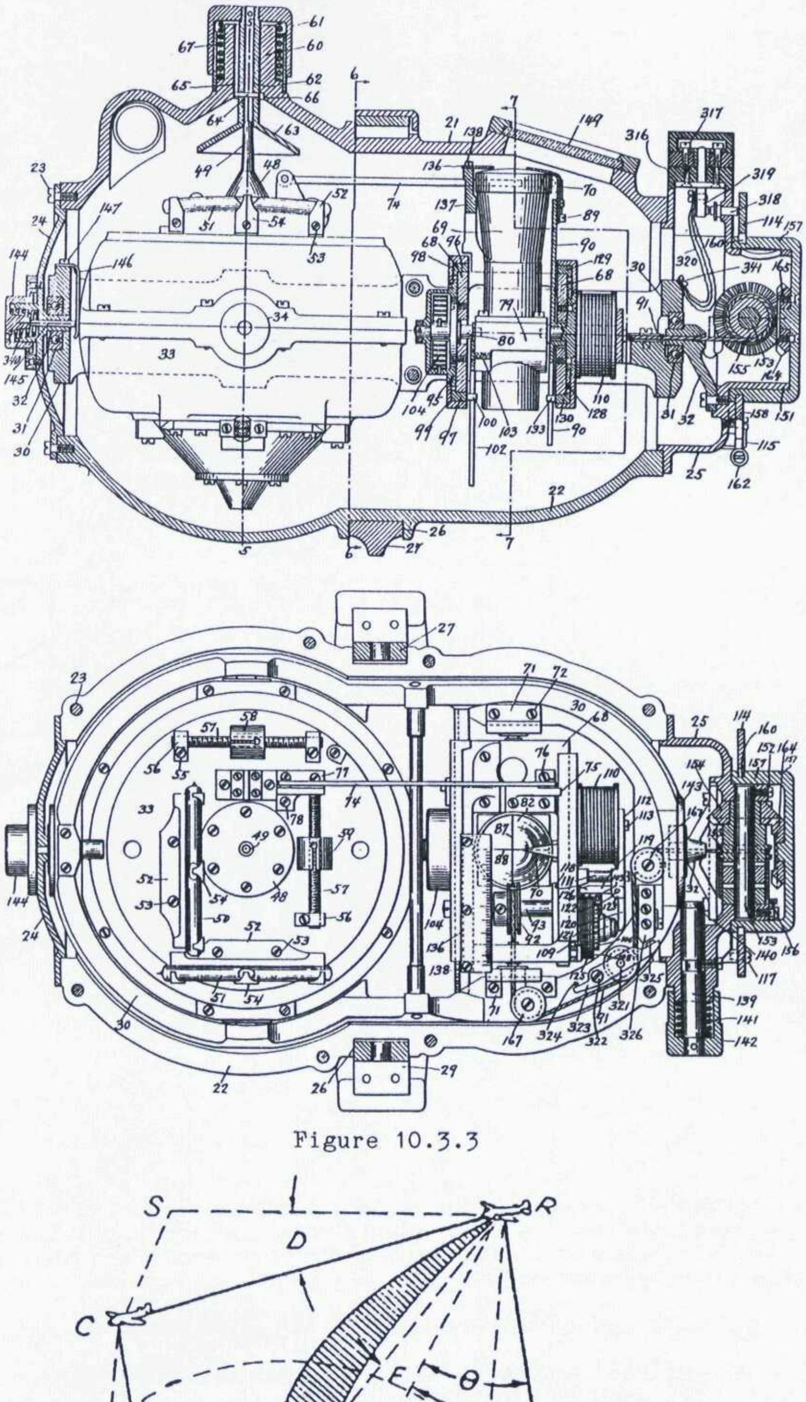


Figure 10.3.3

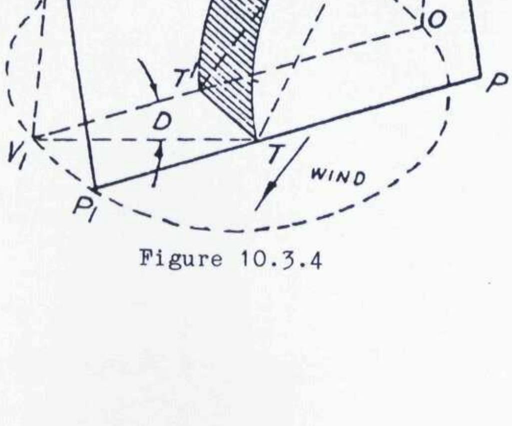


Figure 10.3.4

In its elementary form what is now called Bombs-mode requires that the sight line is depressed below the aircraft velocity vector by a gravity drop allowance inversely proportional to air speed so that release occurs at an approximately constant time-to-go to the target. Ferranti and others have explained that as time-to-go decreases, the 'bunt' increases (that is the downward rate of turn) and therefore the pitch rate of the airframe. Release instant occurs when the downward acceleration is a little over g/2. This method of lead angle prediction is known as KINEMATIC RANGING and is shown in Fig 10.3.2.

The aircraft is shown tracking a ground target with the sight line depressed below the velocity vector by a fixed gravity drop angle. At a given range R and dive angle θ, the required value of the gravity drop angle for the bomb to hit the target (neglecting drag) is

$$\theta req. = \frac{1/2g\ R\ \cos\phi}{V_f^2} \qquad (1)$$

or as θ is considered to be a fixed value, the required range R for the bomb to be released is

$$R_{req.} = \frac{V_f^2\theta}{1/2g\ \cos\phi} \qquad (2)$$

Now, the rate of turn of both the sight line and the velocity vector is

$$\dot{\phi} = \frac{V_f\dot{\phi}}{R} \qquad (3)$$

so that

$$R = \frac{V_f\theta}{\dot{\phi}} \qquad (4)$$

therefore, the aircraft is at the correct range for releasing the bomb when R = R req.

that is when

$$\frac{V_f\theta}{\dot{\phi}} = \frac{V_f^2\theta}{1/2g\ \cos\phi}$$

or

$$\dot{\phi} = \frac{1/2g \cdot \cos\phi}{V_f}$$

Since $\dot{\phi}$ is the rate of turn of the air frame, it can be measured by a rate gyro arranged to trigger the bomb release when $\dot{\phi}$ has the correct value for the chosen value of θ, the air speed and dive angle. The value of θ is chosen so that release occurs when time to collision $\frac{R}{V}$ is about six seconds, corresponding to a release range of 1400 metres at 400 knots.

WILLARD US 2408356 (1946) assigned to G.E. of Schenectady a highly sophisticated gyroscopic bomb sight and the mathematical dissertation he provides on the ballistic problem is rewarding for study, a not dissimilar line of reasoning inspired the work of BLACKETT* & BRADDICK GB 581970 (1946).

* Professor P.M.S. Blackett.
Baron Blackett of Chelsea 1897-1974.
See Sir Bernard Lovell's biographical essay.
Biographical Memoirs of Fellows of the Royal Society 21 Nov 1975.

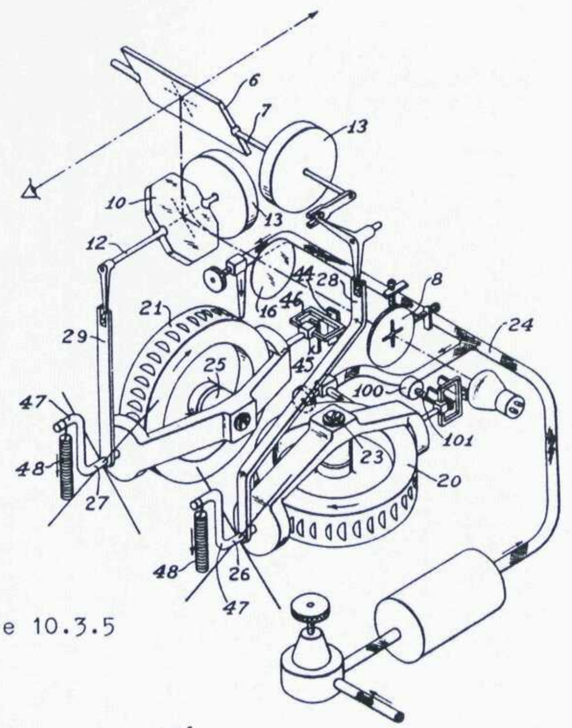


Figure 10.3.5

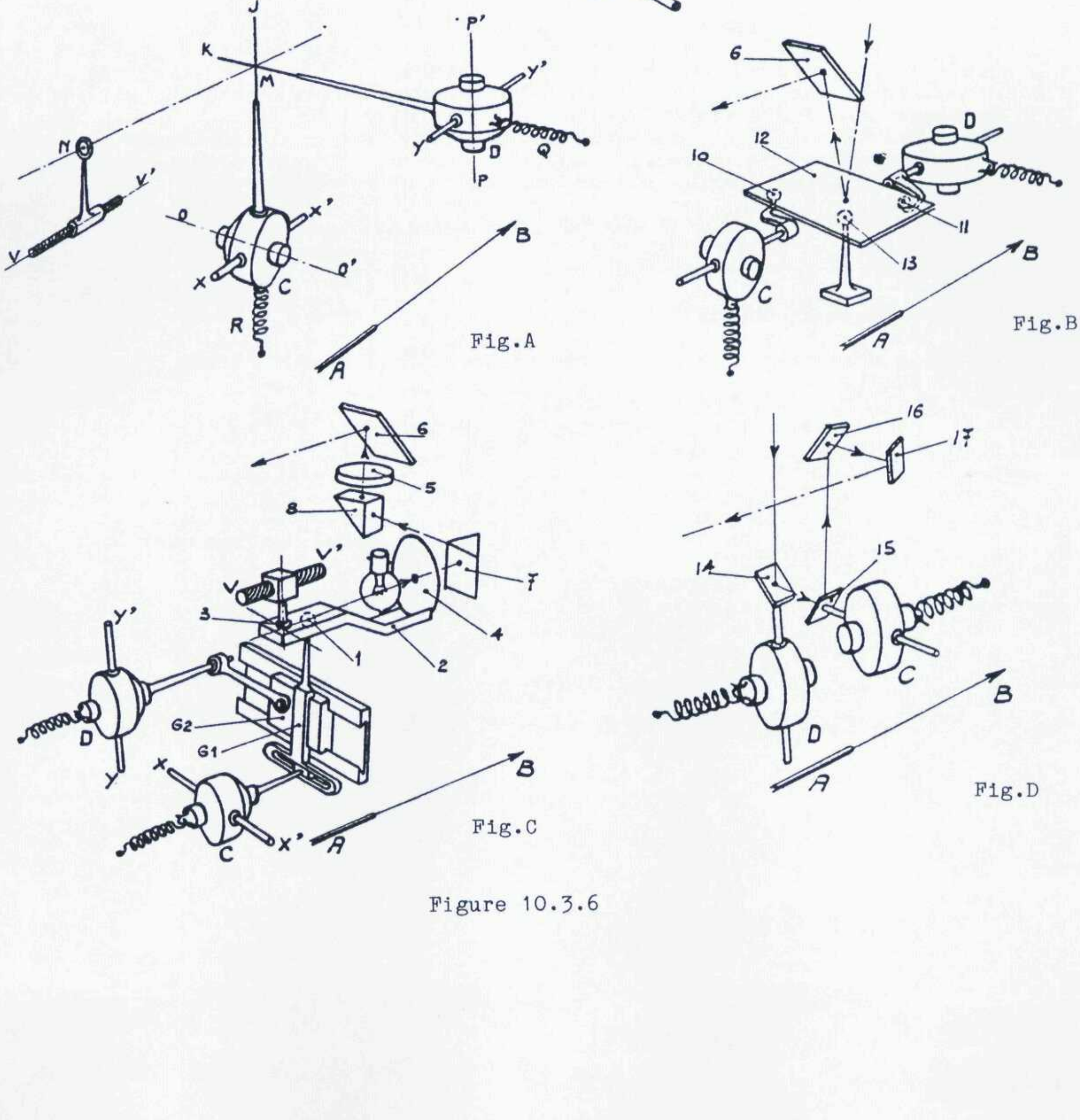


Fig.A

Fig.B

Fig.C

Fig.D

Figure 10.3.6

I turn now to the famous sight by NORDEN & BARTH US 2428678 (1947) to which I referred above. This sight widely used in World War II is the Norden bombsight of the US Air force. The sight was the subject of a U.S. Patent application dated May 27. 1930 and it was kept secret for seventeen years. The sight employs a pendulous gyroscope and this is shown at Figure 10.3.3.

Two gyroscopes are incorporated, a gyrovertical and a pilot director using a gyroscopic azimuthal stabilizer. A backsight is also included and this is combined with a clock that runs for a period corresponding to the time of fall of the bomb from a set altitude to give the correct dropping angle.

WYCKOFF US 3014280 (1961) uses the Norden sight with a time controlled mechanism for closing the lines of sight to the bomb and to the target, especially with a dirigible bomb that is the subject of his two U.S. Patent Specifications 2466528 and 2495304.

HENDERSON et al GB 586271/3 (1947) provide a stabilized sight for dive bombing and SPERRY CORPN GB 623721 (1949) provide a sight stabilized in azimuth that is independent of the yawing of the aircraft.

FAXEN and WILKENSON assigned to SVENSKA AEROPLAN AKTIEBOLAGET GB 624133 (1949) a gyroscopic sight suitable for the accurate release of the bomb during the pull-out from a dive or glide toward the target, their treatment of the problem is full and they do not fail to give a detailed mathematical analysis. In their disclosure of GB 665498 (1952) they show a gyroscopic wind drift computer in which the rotation axis of the gyro is placed parallel to the wind direction prevailing at the altitude above the target.

VAN AUKEN and ESHER assigned to SPERRY GYROSCOPE CO INC GB 624554/5 (1949) a gyroscopic bombsight incorporating a gyrovertical and a directional gyroscope. The ballistic problem solved by the sight is well set out in the document. (See Figure 10.3.4). In the figure the point R represents the position of a craft at the time a bomb is released therefrom, and the arrow represents the direction of the wind. The craft to proceed in the direction RC will therefore be headed up-wind somewhat as indicated in the vector diagram RSC in which RS is proportional to the air speed of the craft, SC to the wind velocity, and RC to the resultant ground speed. The angle between the longitudinal axis of the craft at R and the track (RC) of the craft, i.e., angle SRC (D), is termed the drift angle. When a bomb is released at R, its motion will be retarded by air resistance so that it will lag behind the craft backwards along the fore-and-aft axis of the craft. Thus at the time the craft has reached the point C the bomb will have reached its point of impact T, which, as shown, will be displaced from the point V, (which is vertically below the point C) downwind at an angle parallel to the longitudinal axis of the craft. The distance V_1T is termed the trail and the distance TT^1 (i.e., the lateral distance of the point of impact from the projection Y^1O of the ground speed (RC) of the plane is termed the cross trail).
From what has been said above therefore it will be realised that the bombsight must provide means for positioning the ground track of the craft on the upwind side of the target by the amount of the cross trail. This is accomplished, in the sight to be described, by tilting the optical system so that the vertical cross line of a sighting reticle will track along a line PP_1 parallel to the ground speed OV^1 at a distance down-wind from the ground speed equal to the cross trail T^1T. The angle through which the optical system must be tilted is termed the offset angle, and is shown at F. Also it will be realised that the range angle during the existence of cross wind will be the angle TRP(θ). The reticle of the optical system is stabilised in all planes by being mounted on the top of a suitable form of gyro-vertical which, in turn, is mounted on a platform stabilised in azimuth by remote control from a directional gyro.

BURLEY US 2583815 (1952) discloses a twin gyroscopic sight allied with a computer for use by a plurality of observers.

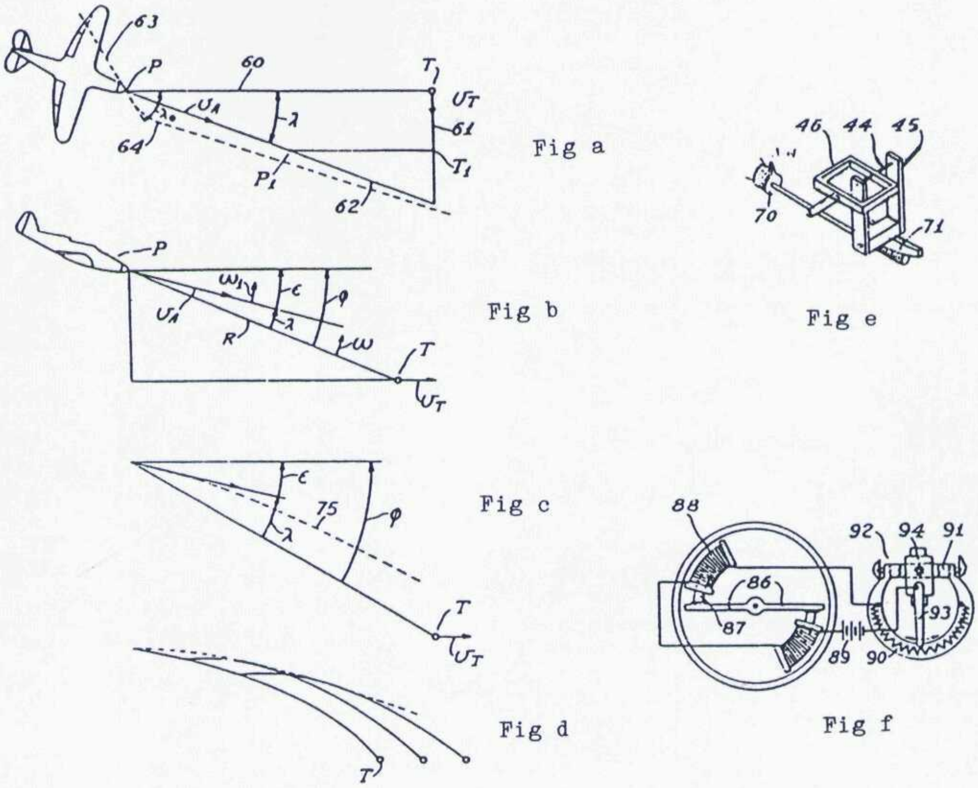


Figure 10.3.7

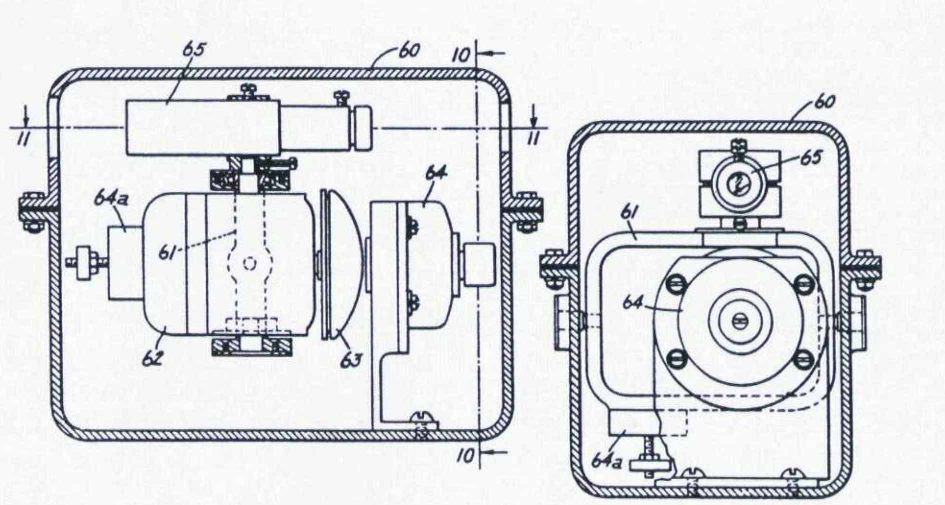


Figure 10.3.8

BENTLEY & DRAPER, assigned to MASSACHUSETTS INSTITUTE OF TECHNOLOGY US 2609606 (1954) a bombsight kept secret from 1943. As shown in Fig 10.3.5 two air spun gyroscopes are orthogonally placed and via a mirror system and a reticle all significant motions are resolved into component motions of the two gyroscopes whereby any motion of the aircraft in space as observed by the pilot through the image of the reticle on the main mirror (6). The gyroscopes are elastically unrestrained and used to control a line of sight that is offset with respect to the flight path of the aircraft; a system that gives effective freedom from most of the disturbing influences such as range wind and target motion. This system was postulated in part by ALKAN FRENCH 749767 (1933) and shown in the delightful drawings of Fig 10.3.6.

The mathematical analysis of the ballistic considerations is well given by Bentley and Draper and I reproduce them since they have been ignored in the general discussion above. (See Fig 10.3.7). What Bentley and Draper offer in effect is a null indicating steering device capable of maintaining constant angular leads comprising means for indicating a line of sight, and an angular rate gyro capable of precession through a small angle the precession being substantially free of elastic restraint, connections for operating the indicating means in accord with the precession to displace the line of sight by an angle λ with respect to the path of motion of a steered body, and thereby introducing an instability-producing torque proportional to $\frac{d\lambda}{dt}$, and stabilizing means to neutralize any tendency toward instability for all values of $\frac{d\lambda}{dt}$.

Fig a shows the condition for a target T moving across the line of sight. When the aircraft is at P the line of sight between P and T is 60, the target path is 61, and the "collision course" of the airplane is 62. The velocities of the airplane and target are represented by the vectors ^{V}A and ^{V}T. There is a constant lead angle λ between the line of flight and the line of sight. There is only one constant angle λ that will dictate a straight course of the airplane and also maintain the reticle fixed on the target throughout the path. For example, when the airplane reaches P_1 and the target reaches T_1, there will be the same constant lead angle between the line of sight and the collision course. The foregoing illustrates what may be termed the "steady-state" condition; it assumes that the correct lead angle has been introduced and that the pilot has been flying the collision course 62 for a sufficient time to wipe out all transient effects. It will now be shown that the pilot, in flying the plane in on any course, will necessarily bring the airplane on to a collision course, if he so flies the airplane that the reticle bears continuously on the target. Suppose that the pilot is flying the course indicated by the dotted line 63, and that at a point P the reticle bears on the target with an original lead angle λ_o. For purposes of theoretical discussion this initial angle λ_o may be of any value, although in practice, it would be natural for the pilot to come in on an estimated collision course and uncage the gyro in such a way as to indicate an approximately correct lead angle. However, assuming a fairly large initial lead angle, as shown by Fig. a, the pilot will find that although the reticle is on the target momentarily at point P, it cannot be held on the target without turning to the left as indicated by 64. The exact course taken from there, on the assumption that the pilot flies in such a way as to hold the reticle continuously on the target, can be determined by the transient solution of a differential equation, to which reference will be made later. The path 64 approaches a straight line, which is a collision course differing from the course 62 and intersecting the path of the target at a different angle. The curved portion of the path 64 represents a "transient" which dies out rapidly.

The effect of cross-wind is also illustrated by Fig. a. Assume that the target is stationary and that a cross-wind is blowing from the right. To keep the reticle on the target it is necessary to maintain a heading at an angle λ to the line of sight. The flight path with respect to ground will correspond in this case with the line of sight 60. This line of flight is therefore a collision course with respect to the target. If the bomb is released at any point of the path, it will continue at the same velocity and direction as the airplane. This neglects the sidewise acceleration of the bomb under the action of the cross-wind, but the deviation of a heavy bomb under this acceleration is negligible.

The conditions as viewed in the vertical plane are similar. Neglecting gravity for the sake of simplicity in preliminary explanation, Fig. b shows how the elevation gyro indicates the proper lead angle, whereby the airplane flies at a certain angle ε from the horizontal for a collision course with a moving target. Since the bomb is released at the same velocity as the aircraft and is here assumed to continue at that velocity, it may be released at any point in the path. The effect of head or tail wind is merely to change the ground speed of the airplane and thus to change the relative velocity whereby the lead angle is automatically brought to the correct value. As in the case of the crosswind above described, the wind accelerates the bomb after release, but the deviation due to this cause is usually negligible. Since any motion may be resolved into components handled independently by the two gyros, the proper flight path is determined simply by holding the reticle on the target.

Before passing to a consideration of the effect of gravity, we shall now discuss the theory of operation under the idealized conditions assumed above. Gravity will later be accounted by compensating means. For satisfactory operation, certain relationships must be maintained between the physical constants. These relationships will be developed by dynamic analysis for one gyro only, namely the elevation gyro, since the same principles apply to the other gyro. In Fig. b, let:

ε = angle of flight path as measured from the flight path to some reference line, here horizontal

ϕ = angle from the line of sight to the same reference line

λ = indicated lead angle $= \phi - \varepsilon$

θ = angle of precession of the gyro from neutral position

Si = $\frac{\lambda}{\theta}$ = sensitivity of the optical system

c = coefficient of damping about the axis of precession

k = net elastic coefficient due to the gyro suspension and springs

H = angular momentum of the gyro rotor about its spin axis

ω_1 = angular velocity of the airplane in its flight path

ω = angular velocity of the line of sight.

According to the conventions herein adopted as shown in Fig. b,

$$\omega_1 = \omega + \frac{d\lambda}{dt}$$

Then, from gyro theory, the precessional torque is

$$H\omega_1 = H\left(\omega + \frac{d\lambda}{dt}\right) \tag{1}$$

This torque is resisted by the inertial torque, the damping torque and the elastic restoring torque. The inertial torque is small and is here neglected for simplicity, since it does not materially affect the conclusions. Hence

$$C\frac{d\theta}{dt} + k\theta = H\left(\omega + \frac{d\lambda}{dt}\right) \tag{2}$$

By virtue of the optical system the indicated lead angle λ is proportional to θ. The angle θ is held to small values, preferably not more than 2° from the zero position, and a sensitive linkage is relied on to indicate sufficiently large values of λ.

Since $\theta = \frac{\lambda}{Si}$,

$$\left(\frac{C}{HSi} - 1 \right) \frac{d\lambda}{dt} + \frac{k}{HSi} \lambda = \omega \tag{3}$$

The quantities c, H and Si are characteristics of the instrument, hence the coefficient of $\frac{d\lambda}{dt}$ may be taken as a design constant. Let

$$\frac{C}{HSi} - 1 = \sigma$$

Then the differential equation becomes

$$\sigma \frac{d\lambda}{dt} + \frac{k}{HSi} \lambda = \omega \tag{4}$$

The invention contemplates that the net elastic coefficient should be zero, or nearly so. Hence the equation reduces simply to

$$\sigma \frac{d}{dt} - \omega = 0 \tag{5}$$

It can be shown from the geometry of Fig. b that in any case

$$\omega = \frac{v_A}{R} \left[\frac{v_T}{v_A} \sin \phi - \sin\lambda \right] \tag{6}$$

where v_A is the velocity of the aircraft, v_T the component of target velocity in the plane illustrated by Fig. b, and R is the range.

Then (5) becomes

$$\sigma \frac{d\lambda}{dt} + \frac{v_A}{R} \left[\sin\lambda - \frac{v_T}{v_A} \sin \phi \right] = 0 \tag{7}$$

with a "steady-state" solution of

$$\mathrm{Sin}\, \lambda = \frac{v_T}{v_A} \sin \phi \tag{8}$$

Thus, when the gyro has settled down, so that $\omega = 0$ and $\frac{d\lambda}{dt}$ likewise equals 0, the device indicates a constant lead angle determined by (8). The value of λ given by this equation is the proper lead angle for predicting the collision course. To maintain this constant lead angle, absence of restoring torque is necessary and this accounts for the requirement that $k = 0$.

The transient effects are best determined by going back to (5). Since $\omega = - \frac{d\phi}{dt}$, (5) becomes

$$\sigma d\lambda + d\phi = 0$$

Then

$$\sigma \lambda + \phi = C,$$

the constant being determined by the fact that at some initial point when the reticle is first brought to bear on the target,

$$\phi = \phi_o \text{ and } \lambda = \lambda_o$$

Therefore

$$\sigma (\lambda - \lambda_o) + (\phi - \phi_o) = 0 \tag{9}$$

This is itself a differential equation since λ is the angle between the line of sight and a tangent to the flight path. The solution shows that the transient error dies out approximately in proportion to $\frac{R}{R_0}\frac{1}{\sigma}$ where R_0 is the distance to the target from the initial point at which the reticle is brought to bear on the target. For example, with $\sigma = 0.2$, the error reduces to 1/32 of its initial value in traversing half the distance from the initial point to the point of ultimate collision. Thus in Fig. a, the initial "error" is the angle between lines 62 and 64. After traversing half the distance to the target, the line 64 will differ from a true collision course by about 1/32 of that initial error. In any actual case the pilot will be able to initiate the action with an error of only a few degrees, hence the error will be insignificant after travelling a relatively short distance. This shows that σ should be small. It should be noted that σ is a net damping coefficient. The quantity $H \frac{d\lambda}{dt}$ is a torque which depends on the fact that the information conveyed to the pilot is based entirely on the line of sight, while the only control at the disposal of the pilot is his ability to manoeuvre the airplane. This quantity $H \frac{d\lambda}{dt}$ is a torque which inherently tends to produce instability. This tendency to produce instability is removed by the dissipative torque $\frac{c}{Si}\frac{d\lambda}{dt}$. The dissipative torque must be greater than the torque that tends to produce instability, that is, σ must be greater than zero. If σ were made exactly zero, there would theoretically be no transient. It must be remembered, however, that the foregoing analysis accounts for gyro characteristics only and does not account for the dynamic characteristics of the aircraft. A condition of $\sigma = 0$ would be physically significant only if it were possible to bring the airplane suddently on to the correct path, for which an infinite acceleration would be required. If σ were negative, it would be impossible for the pilot, by any natural responses, to hold the line of sight on the target. In such a case the system of pilot, airplane and instrument would be "unstable". Therefore, σ must always be positive, and must be sufficiently greater than zero to avoid the chance of its accidentally becoming negative through some slight variation in the gyro constants. Practically, σ may be of any value from about 0.05 to 0.5.

From the foregoing it will be observed that the gyros are used in somewhat different manner from conventional gyro operation. Gyros are usually used in one of two ways, namely, to indicate a fixed position in space, as in a flight indicator; and to measure a rate of turn, as in a turn indicator. The present invention is more properly viewed as a "null indicator", since it gives an indication of zero angular velocity of the line of sight. While the gyro axes remain in constant position after the gyros have settled down, that constant position is not related to any earthbound coordinates, but is determined only by the condition of zero angular velocity when the proper flight path is determined with relation to the objective.

The foregoing indicates the remarkable independence of the present invention of such usually disturbing factors as target speed range and wind. Only two other factors need to be considered, namely, gravity and air resistance. These may be combined into one, as will be shown later, but for the present, let us consider the effect of gravity alone.

It may be noted that the expression for σ includes a term Si, the optical sensitivity. For a desired value of σ, the greater the sensitivity, the greater must be the damping coefficient.

To compensate for gravity, it is necessary to fly with some "superelevation", as indicated by Fig. C, whereby the bomb, after release, will follow a curved trajectory 75 to the target. Superelevation is introduced into the elevation gyro by applying a bias torque. This may be conveniently accomplished by a manual setting knob 70 operating to tilt the bracket through a worm drive 71, as shown in Fig. C. Let the bias torque be represented by T. Then equation (7) becomes

$$\sigma \frac{d\lambda}{dt} + \frac{vA}{R}\left[\sin\lambda - \frac{v_T}{vA}\sin\phi\right] = \frac{T}{H} \qquad (10)$$

an approximate steady state solution for which is

$$\sin\lambda = \frac{v_T}{v_A}\sin\Phi + \frac{R}{v_A}\frac{T}{H} \qquad (11)$$

The term $\frac{v_T}{v_A}\sin\Phi$ represents, as in (8), that part of the lead angle which predicts the collision course. It can be shown that the superelevation required to correct for gravity is approximately

$$\frac{R}{2v_A^2}\cos\varepsilon \qquad (12)$$

wherefore the proper superelevation will be introduced if T is proportional to $\cos\varepsilon$ and inversely proportional to v_A. The bias torque is independent of range, but it must be adjusted for angle of the flight path from the horizontal and also for airplane velocity. If the velocity is kept nearly constant the only necessary adjustment is for the angle of the flight path.
Since the aircraft will ordinarily be used for but one type of bombing, a set adjustment will usually suffice. Thus, for very low level bombing, $\cos\varepsilon$ may be taken as 1. For dive bombing at high angles, the superelevation is so small that a constant value for ε may be assumed, and no appreciable error will be introduced by variations of ε from the assumed value.
If the airplane is to be used for various types of service, with considerable variation of ε, the pilot may be relieved of the necessity of adjusting the bias torque, by providing a connection with the flight indicator or gyro vertical of the airplane, as diagrammatically shown in Fig. f. The pointer of the gyro vertical moves with relation to a rheostat to control the current to bias-control solenoids, the armatures of which are connected to the precession axis of the elevation gyro. One trunnion of the elevation gyro 20 is provided with radial arms 86 carrying arc-shaped armatures 87 entering solenoids 88. The solenoids are variably excited by a connection through a battery 89 and a rheostat 90. The rheostat is mounted on the gimbal ring 91 of the flight indicator 92. A movable rheostat arm 93 is mounted for movement with the gyro element 94. Since the axis of the element 94 maintains a substantially vertical position in space at all times, the position of the arm 93 with respect to the rheostat 90, and hence the current in the solenoids 88, is a measure of the angle of the longitudinal axis of the aircraft from horizontal. The rheostat is "tapered" to apply a bias torque proportional to $\cos\varepsilon$.

The preferred method of introducing superelevation torque is by means of a weight 100 applied to an arm 101 attached to the supporting wire of the elevation gyro (Fig. 70.1). Let M be the mass of the weight and b its distance from the axis. Then the bias torque due to gravity is $Mbg\cos\varepsilon$. However, the weight is subject to other forces due to the curvature of the path. The torque due to these forces is

$$Mb\,v_A\,\omega_1 = Mb\,v_A\left(\omega + \frac{d\lambda}{dt}\right)$$

The differential equation (10) modified to take account of these several torques is

$$\sigma 1\frac{d\lambda}{dt} + \frac{v_A}{R}\left(\sin\lambda - \frac{v_T}{v_A}\sin\Phi\right) = \frac{\frac{Mbg}{H}}{1 + \frac{Mb\,v_A}{H}}\cos\varepsilon \qquad (13)$$

where

$$\sigma 1 = \frac{\frac{c}{HS1} - 1 - \frac{Mb\,v_A}{H}}{1 + \frac{Mb\,v_A}{H}} \qquad (14)$$

An approximate "steady state" solution of (13), neglecting the dynamic error due to changes in λ from following a curved path is

$$\sin\lambda = \frac{v_T}{v_A}\sin\phi + \frac{R}{v_A}\,\frac{Mbg\cos\epsilon}{H\left(1+\frac{Mb\,v_A}{H}\right)} \tag{15}$$

It will be observed that (15) involves a target-speed term and a gravity-correction term. The target speed term accords with that previously developed for the collision course. This leaves only the gravity-correction term to consider. This term is proportional to range and to cosine of the angle ϵ. If the airplane velocity is assumed constant, and if the weight is chosen so that

$$\frac{Mb\,v_A}{H} = 1, \tag{16}$$

the lead angle corresponds to (12) and is correct (within the limits of approximation of the superelation formula) for all values of ϵ and for all ranges.

In examining (13) for transient solutions, is noted that the coefficient of $\frac{d\lambda}{dt}$ now includes a term involving the speed of the airplane, that is

$$\sigma^1 = \frac{\frac{c}{HSi} - 2}{2}, \text{ if (16) is satisfied.}$$

σ^1 should be between 0.05 and 0.5, and this merely requires more damping than in the gyro without the bias weight. Some variation of σ^1 with changes in speed will occur, and for this reason, it is best not to use too small a normal value of σ^1, since an increase in speed might bring its value too close to zero. Referring to (14) it will benoted that if vA changes, a compensating change may be made in b to keep σ^1 constant. This may be done manually by setting the weight 100 at different positions on the arm 101.

With increase in technology WHEELER and CARBARINI assigned to SPERRY RAND CORPN GB 804700 (1958) a bombing navigation computer incorporating a stabilization unit having three gyroscopes within it. The specification is a veritable text book of information on 56 pages with 110 figures and a detailed mathematical dissertation yet subject to anticipatory documents from about 1933 to 1958 that can be found in some ten earlier United States Specifications*. A dive bomb gyro sight using a gyrovertical is the subject of a further disclosure by the same authors, see WHEELER & CARBARINI US 2955356 (1960). GRIMSHAW US 2859526 (1958) discloses a sight that is of a type established by JOHNSON US 2467831 (1942) US 285655 (1958) (Fig 10.3.8 on Page 62) in which the sighting mechanism controls the line of fire of the gun so as to give the gun the correct 'lead angle' with relation to the line of sight of the target as required by the speed of the target. A support is utilized on which is maintained a gyroscope free to move about a predetermined point of suspension and to be precessed to follow the sight as the sight tracks the target. GRIMSAW introduces further corrections and refinements into the system for ballistics connections for the effect of wind blowing the projectile back along the line of flight and the effect of the dropping of the projectile due to gravity over the range distance of the target. This he achieves by means *inter alia* of a resolver bar (65) cooperating with an eddy current disc.

The famous Hookes joint gunsight of CUNNINGHAM et al GB 578958 (1945) reappears in the work of HARPER & SHIPPLEY & SMERDON assigned to FERRANTI LTD. GB 950634 (1964) GB 1072926 (1967).

* See for example US Patent Specifications 19191; 2116508; 2408356; 2428770; 2480208; 2507567; 2577313; 2593902; 2823585; 2825055.

In the first disclosure the sighting lead is substantially identical with that of Cunningham et al but the pilot is constrained to fly a course that maintains his line of sight to the target at a fixed angle below the flight line, thereby causing both lines to turn at an accelerated rate. A critical value of the pitch rate inversely dependent upon airspeed is eventually reached such that a bomb released at that instant will hit the target.

In the second disclosure due to SMERDON a gyroscopic sight is proposed for use in a low level attack with retarded bombs. The gyro is an eddy current device not wholly dissimilar to that of CUNNINGHAM et al and it is constrained to lower the aiming mark in accord with the equation (Fig 10.3.9)

$$\omega_V = (S/D_V)\ \mathrm{Sin}\ 2\theta_V$$

where ω_V is the angular velocity of the sight line,
S is the ground speed
D_V is the pass distance and
θ_V is the depressed angle between the velocity vector PA and the sight line PQ.

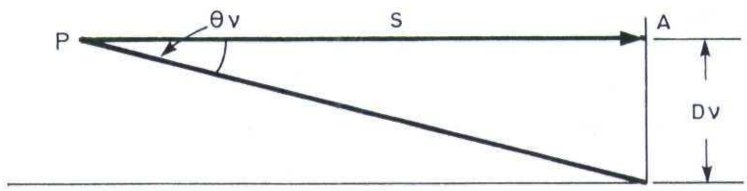


Figure 10.3.9

From this, a critical value of the angle θ_V may be computed using analogue techniques to indicate the instant of bomb release from the craft to hit the target.

A kinematic bombsight by TYE US 3427437 (1969) is in effect a bombsight computer having a reticle for tracking the target at a predetermined angle of deviation and means for displaying indicia representing the range D, from the sight to the target, means for measuring the rate θ at which the aircraft-borne sight rotates, means for measuring the aircraft velocity $\underline{V}$ and computing means responsive to the means for V and θ for driving said indicia according to the relation

$$-\dot{D}_1 = V-K\ (V\gamma-D_1\theta)$$

where K is a constant and $\dot{D}_1$ is the time derivative of the range D_1. The all important pitch rate θ being sensed by a rate gyro.

An advanced projectile delivery system is proposed by COX and FERRY US 3880043 (1975). It is related to canned delivery bombing techniques in which the pilot attempts to release the bomb at a predetermined set of values corresponding to the dive angle of the aircraft, airspeed, target depression angle and the altitude that has been precalculated to result in a target hit; while permitting the aircraft to recover from a dive with a specified ground clearance. The aiming reticle must be kept vertical, for success, in inertial coordinates by gyroscopic means and the desired position of release is determined in a processor solving the two following equations:-

$$\lambda_r = \lambda_{r_o} + K_3\ (\theta - \theta_o) + K_4\ (V - V_o) + K_5\ (\alpha - \alpha_o)$$

$$\lambda_T = \alpha_o + \cot^{-1}\ [K_1(H-H_T)-K_2]$$

where
θ = Pitch angle
θ_o = Preplanned pitch angle
α = Angle of Attack
α = Preplanned Angle of Attack
V = Airspeed
V_o = Preplanned Airspeed

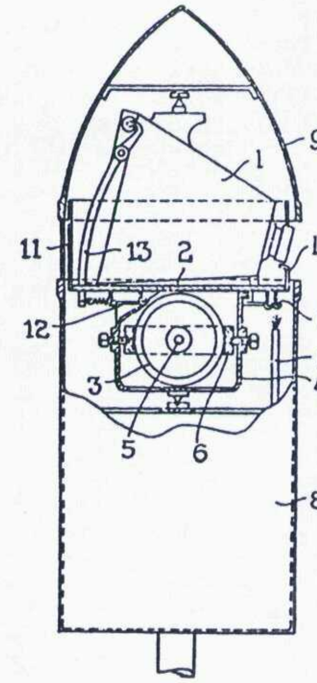


Figure 10.4.1

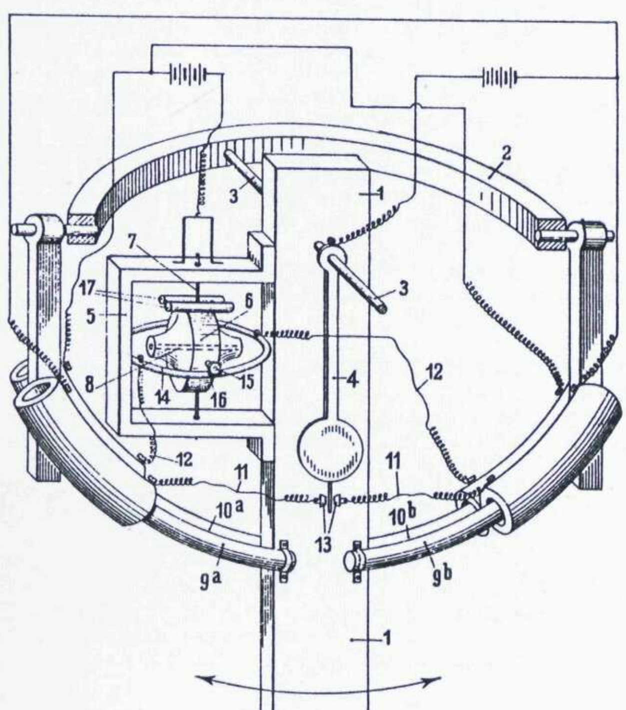
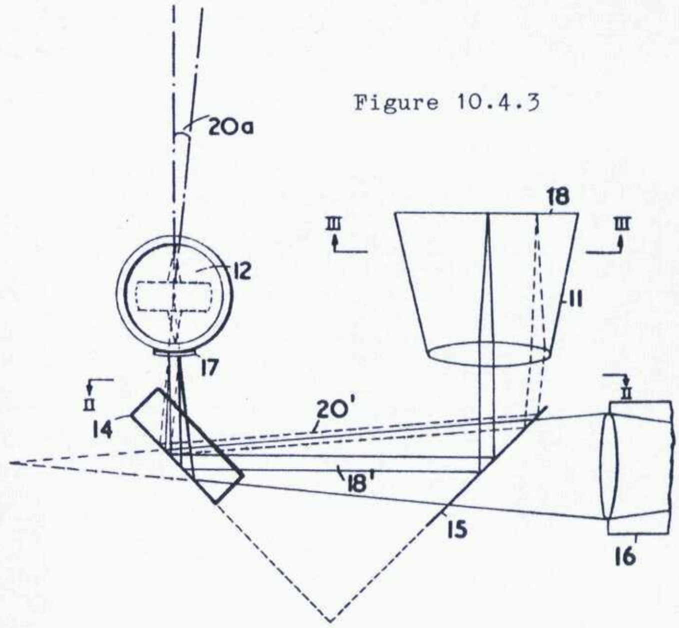


Figure 10.4.2

Figure 10.4.3



H_T = Target Altitude above sea level
H = Aircraft Altitude above sea level
λ_r = Release Cue Depression Angle
λ_{r_o} = Preplanned Release Cue Depression Angle

λ_T = Reticle Depression Angle
$K_1 - K_5$ Precomputed Ballistic constants as a function of Preplanned Release Parameters V_o, θ_o, α_o, λ_{r_o}

It can be observed that the solution to the two equations will generate λ_r and λT which are the only parameters necessary to establish inter alia the compensating release cue.

10.4 CAMERAS

The camera is too well known to call for a description here; it grew out of the camera obscura or dark room and was adapted to the making of a permanent image from the work inter alia of NIEPCE AND DAGUERRE in France c.1820. To achieve any success it has to be held steady and BOULT GB 10757/1906 acting as communicatee to MAUL of Dresden disclosed gyroscopic means for holding an aerial photographic camera steady in a set position. The construction is well shown in Fig 10.4.1. The gyroscopes vertical flywheel is suspended by means of a universal joint in a rocket case, an advanced conception for the early year's of this century.

I have referred earlier to the work of HENDERSON in respect of the stabilization of the telescope and I should remind the reader that this work is said to apply with equal validity to that of the camera.

OPTISCHE ANSTALT C.P. GOERZ Akt Ges. GB 150995 (1920) proposed using for the stabilization of aircraft cameras a gyroscope and a pendulum combined (see Fig 10.4.2).

COOKE US 1586071 (1925) assigned to AERO SURVEY CORPN of America his aerial gyroscopic camera. These early devices tried to stabilize the whole camera and it was JACKSON GB 218415 (1926) acting as communicatee to the SPERRY GYROSCOPE COMPANY of USA who showed that a gyro-controlled mirror may be placed in front of the camera lens to maintain the line of sight vertical for the camera.

In the same year HOLEKA US 1573343 (1926) suspended a gyroscope from beneath a simple camera on a tripod.

TITTERINGTON US 1645079 (1927) and SCHUELLER GB 275649 (1929) each deploy twin gyroscopes to achieve stability. Titterington place the rotational axes of the gyroscopic rotors in radial planes at right angles to each other, but Schueller places the axes in orthogonal planes, each axis coupled by a flexible shaft to a camera in a gimbal support.

BOWEN GB 402890 (1934) discloses the use of a gyro-vertical combined with an aerial camera to give a photographic record of camera tilt so that in the assessment of the photographic image any distortion may be corrected for. WESTINGHOUSE ELECTRIC INTERNATIONAL CO. GB 590682 (1947) similarly appeal to the use of a gyro-vertical for their more complex camera mounting.

Again the gyro-vertical is appealed to for recording a gyroscopically defined 'plumb' point in aerial photography by SPEAK and WALTERS, who assigned their work to The MINISTER OF SUPPLY GB 650826 (1951). This optical arrangement of Speak and Walters is shown at Fig 10.4.3. An image of a graticule is made to be central of a circle when the sight line of the camera is plumb. The construction comprises a camera 11 projector 16 for the graticule, a right angled roof-top mirror 14, a flat mirror 15 and a gyrovertical 12 with a spherical mirror 17.

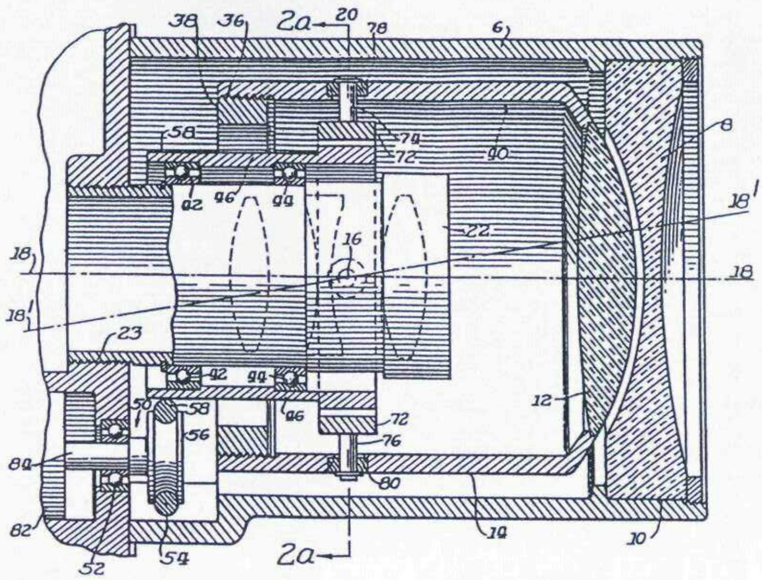


Figure 10.4.4

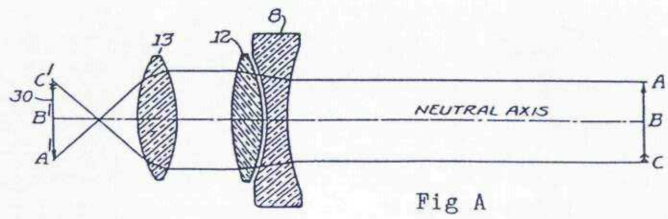


Fig A

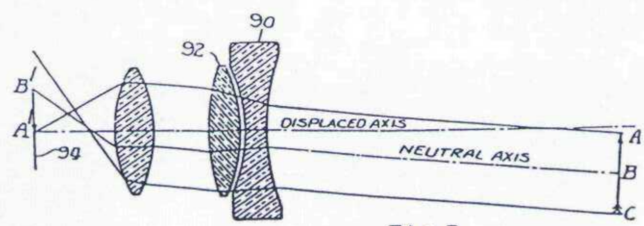


Fig B

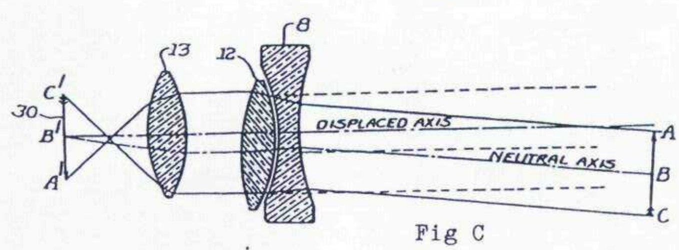


Fig C

Figure 10.4.5

WATTS, THOMAS & COULTHARD assigned to MINISTER OF SUPPLY GB 808829 (1959) a stabilizing apparatus in which two aerial cameras are mounted on a carrier that is supported from a pillar by a Hooke's joint, the carrier having on it two diametrically displaced gyroscopes the spin axes of the rotors of which are orthogonally placed.

DUBISSON assigned to SOCIETE FRANCAISE D'OPTIQUE ET DE MECHANIQUE GB 892453 (1962) what is in effect an elaboration of the gyroscopic device of BOWEN by providing an accurate scale for giving the orientation of an aircraft in space onto cartesian coordinates on a film.

I have referred earlier to the work of de la CIERVA and to that of RICHARDS & SHIN that was assigned to DYNASCIENCES CORPN GB 1056527/8 (1967) GB 1339397 (1973) in respect of the DYNALENS and to the work of ALVAREZ to BELL & HOWELL COMPANY GB 1099026 (1968) that has important applications to the use of aerial satellites and space vehicle photography.

CALL US 3409350 (1968) proposed a stabilized lens assembly for a movie camera, in which the lens per se is part of a gyroscopic rotor in a gimbal suspension. The motor and the lens being spun about a common axis at the same speed as the gimbal suspension. A mating lens is mounted in front of the rotating lens to form a Boscovitch* wedge described more fully by ORT US 2180017 (1939) and utilized effectively in pendulous mode by RANTSCH and even earlier by GRAY GB 127877 (1919).

In the following year CALL US 3424522 (1969) explains the use of a rotor having a substantially spherical surface that is in CALL US 3459473 (1969) modified to include a friction element to inhibit undesirable precessional motions.

KOPPENSTEINER US 3424521 (1969) introduces into the system of CALL a refined rotor drive and BRANIGAN US 3424523 (1969) introduces means to change the precessive force in accordance with changes in the operating conditions, a gyroscopic trick known to both KNUTSON US 2709922 (1955) and LAHDE US 2951377 (1960). The BELL and HOWELL COMPANY is responsible through the researches of CALL, KOPPENSTEINER, KOBER & BRANIGAN, referred to above, for seven disclosures in England, all in the same year (1969).

One arrangement for stabilising a lens is shown in Figure 10.4.4 it comprises a rotor having the lens mounted thereon, a gimbal suspension system, pivotal connecting means for connecting the rotor to the gimbal suspension system, and means to rotate the gimbal suspension system so that the rotor and the lens are spun abount an axis at the same speed as the gimbal suspension system, but pivotal with respect thereto.

In Figs 10.4.5A, B and C let arrow ABC represent an object that it is desired to photograph at a time when the lens stabilisation system is in its neutral position, that is when the axis of the focusing lens housing (22) and the spin axis of the rotor are superposed. At this time, light rays coming from the right hand side of the figure pass through the lens elements 8 and 12 that form a Boscovitch type wedge, then through the focusing lenses (say 13) from which an inverted image of the object $A^1B^1C^1$ is focused at the focal plane 30. Fig B shows corresponding lens elements 90.92 of a camera that does not have a stabilised optical system. Only the AB portion of the object has an image A^1B^1 formed at the focal plane due to disturbance. Fig C shows a similar disturbance to that of Fig B with the stabilised system. The lens elements 12 maintain stability about the neutral axis, the lens element (8) and the camera focusing lens (13) are displaced with the camera housing. With respect to the camera housing therefore lens (8) is fixed while lens (12) is relatively movable although stationary in space. The entire image $A^1B^1C^1$ of the object ABC is placed onto the focal plane.

* Boskovic R.J. 1711-1787.
Croatian, Jesuit polymath astronomer and optician see his Dissertationes quinque ad dioptrician pertinentes VIENNA 1767.
See Dictionary of Scientific Biograph SCRIBNER 2 (1970) p.326-332.

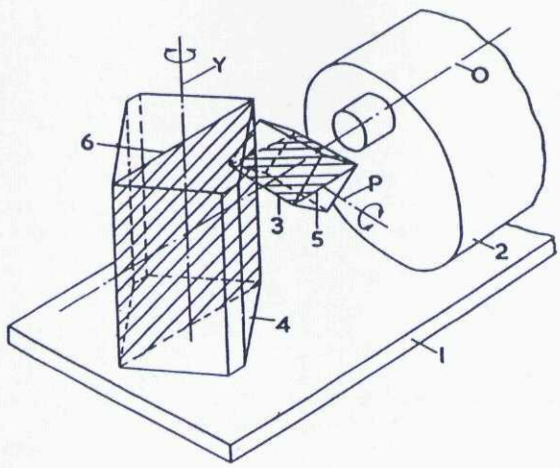


Fig A

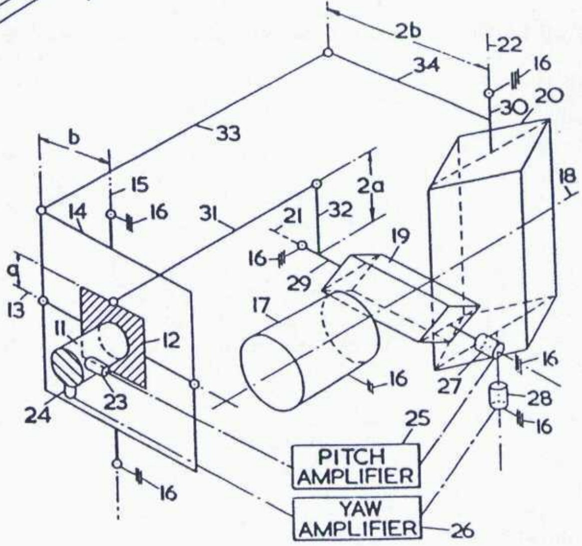


Fig B

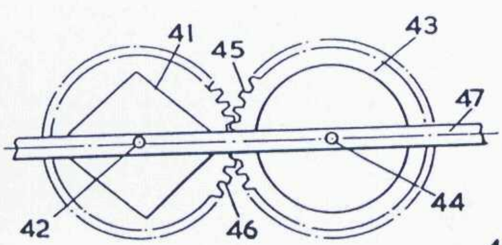


Fig C

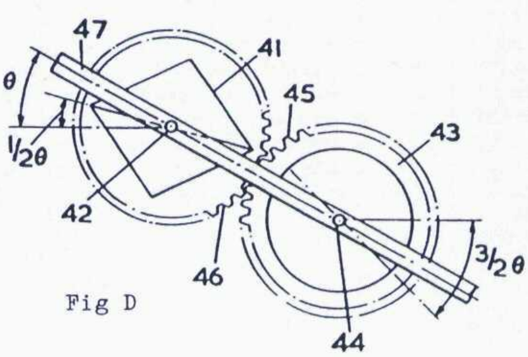


Fig D

Figure 10.4.6

I close with but a short reference to the work of HUMPHREY in respect of camera lens stability, having noted earlier his skills in respect of stabilising inter alia the telescope. In HUMPHREY the same principles are applied mutatis mutandis see Figures 10.1.21/22.

There is also the work of WILLS disclosed by SECRETARY OF STATE FOR DEFENCE, LONDON GB 1450027 (1976) using double compensatory Dove prisms and integrated gyroscopic means for the stabilisation of cameras. (See Fig 10.4.6A and B). In Fig B the camera is at (17) and the reference gyroscope of the Hooke's joint type described in U.K. Patent Specification 578958 is at 11. The two orthogonal double dove prisms are respectively at 19 and 20. The reference gyroscope is secured to an inner gimbal 12 for angular movement about an inner gimbal pitch axis 13 in an outer gimbal 14 supported about an outer gimbal yaw axis 15.

Two double Dove prisms 19, 20 are supported on the frame 16 for angular motion about support axes 21, 22 respectively which axes are mutually normal and normal to the axis 18 and are respectively parallel with the gimbal axes 13 and 15. Pick off means 23, 24 afford control signals responsive to angular motions of the gyroscope rotor with respect to the inner gimbal 12 and are connected through corresponding pitch and yaw amplifiers 25, 26 to pitch and yaw torque motors 27, 28 respectively. The torque motors 27, 28 are operative to drive the prisms 19, 20, through drive shafts 29, 30, respectively as will later be described. Pivoted linkages having arms 31, 32, and 33, 34 connect the inner gimbal 12 to drive shaft 29 and the outer gimbal 14 to the drive shaft 30, respectively. The lengths of the linkage arms 32 and 34 shown as 2a and 2b, are twice that of the arms of the gimbals 12, 14 (shown as a and b), respectively. Small errors will be present in the linkage systems of approximately 1 minute of arc for angular motions of the camera of 10° about each axis and 30 minutes of arc for 30° deflections. The resulting position errors of the prisms which are approximately proportional to the third power of the deflection angle can, if desired, be eliminated by measuring the gimbal or prism deflection angles and feeding a corresponding correction signal into the control loop to the torque motors.
In operation the camera 17 is aimed at an object, and provides an optical image thereof. If now the apparatus is rotated with respect to the pitch and/or yaw axes 13, 15 the picture would normally move correspondingly on the viewing presentation. In this case, however, signals proportional to the angular displacement of the gyroscope rotor with respect to its casing (which is secured to the inner gimbal 12) are fed from the pick off means 23, 24 through the amplifiers 25, 26 to the torque motors 27, 28. At the same time relative movements of the inner gimbal and outer gimbal 12, 14 with respect to the frame 16 are operative through the linkages 31, 32 and 33, 34 to rotate the prisms 19, 20 through substantially half the corresponding angular motion of the inner gimbal 12 and outer gimbal 14, respectively. This double control movement of the prisms has the optical effect of compensating for image movement which would otherwise take place as a result of movement of the camera.

At least one of the dove prisms is reduced in its inertial effects as shown in Figs C and D. The prism 41 is supported for angular movement about a longitudinal 42 corresponding to the axes 21 or 22 of Fig B. A free inertial mass 43 is supported for rotation about an axis 47 parallel to axis 42 and is coupled through gear teeth 45, 46 to the Dove prism 41. The prism 41 and the mass 43 are pivotally supported in a link 47 corresponding to the frame 16 of Fig B. The gear ratio between 41 and 43 is unity and the inertia J of mass 43 is arranged to be equal to one third of the inertia I of the prism 41. In general, when the gear ratio between the prism 41 and the free inertial mass 43 is N the two inertias are related by the expression

$$J = \frac{N^2}{(2N + 1)} I$$

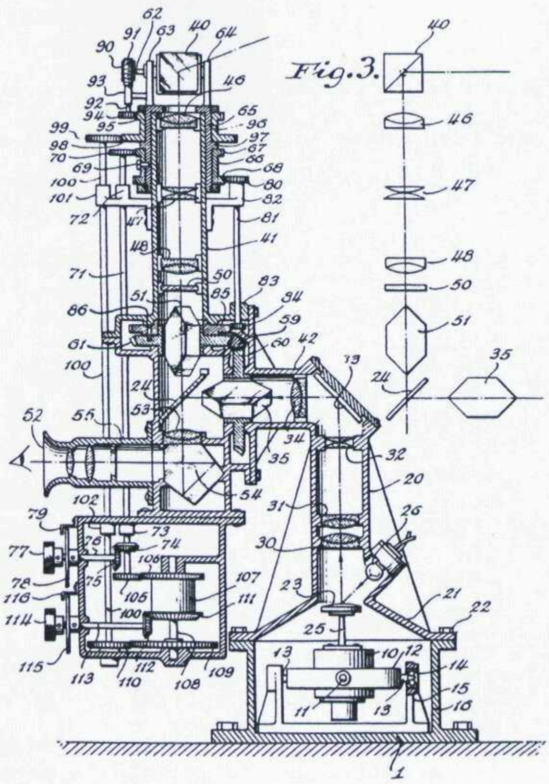


Figure 10.5.1

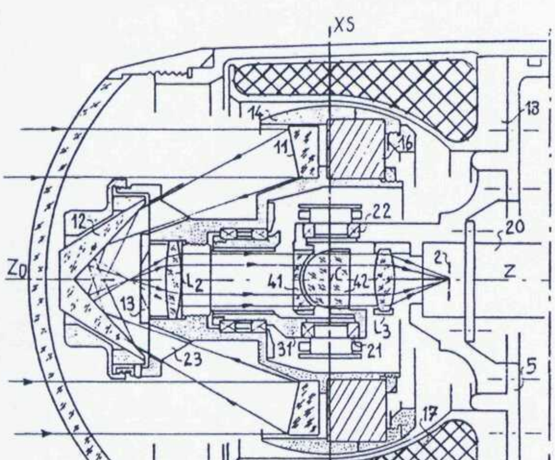


Fig A

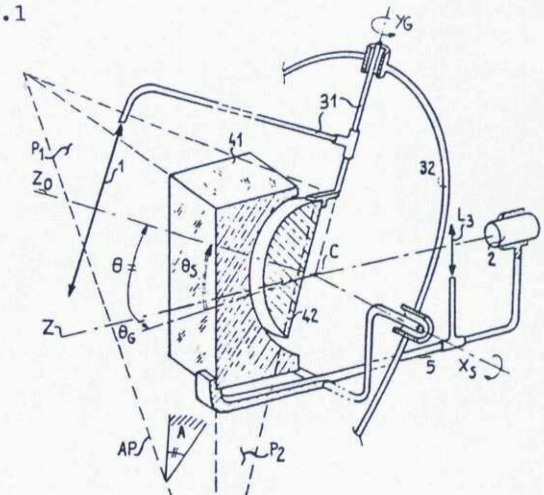


Fig B

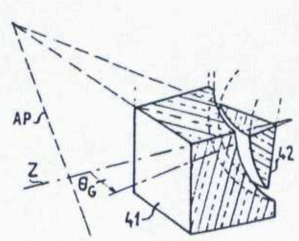


Figure 10.6.1

In the case of Figs C and D if the link 47 moves through angle θ the prism 41 moves through an angle $\frac{\theta}{2}$. At the same time the free inertia mass 43 moves through an angle $\frac{3\theta}{2}$ in the opposite sense to that of the prism and the inertias cancel each other and the torque to move the prisms is effectively reduced.

For completeness the reader should be referred to DORNIER SYSTEM GmbH GERMAN 2731134 (1977) and ANGELROTH G. BELGIAN 851740 (1977).

The first discloses a vertical sensor for the stabilization of a camera and the second uses two flywheels in contra rotation and with appropriate masses that annul the gyroscopic reaction of a single wheel.

10.5 PANORAMIC SEXTANTS

The proposal for a gyroscopically stabilized sextant is to be found from the work of BURKA & CRANE US 2220884 (1940). They disclose a gyroscopic means for the stabilization of an artificial horizon or reference mark against vibration or short period oscillations as experienced in an aeroplane in flight.

CRANE, THURLOW & BURKA US 2266741 (1941) also disclose a panoramic sextant for aircraft using a gyroscopically controlled reticle and a not dissimilar piece of apparatus is put forward by ESVAL et al in which a gyrovertical keeps an housing of an illuminated reticle stabilized as a reference onto a viewing mirror.

Yet a further panoramic sextant is due to WRIGLEY US 2505819 (1950). The instrument is shown in Fig 10.5.1. A reticle 23 is supported by a spindle 25 mounted on a gyroscope casing 10 in alignment with the spin axis of a gyroscope. The geometry of the optical system is seen to include an entrance prism 40 rotatable about intersecting horizontal and vertical axes. The light rays from a celestial body are reflected by the prism to reflector 24. Two double Dove prisms are deployed in the optical train 35, 51. Gearing is used to couple the Dove prisms with the entrance prism at one half the rate of rotation of the entrance prism in azimuth to give erect images in the eyepiece.

10.6 PANORAMIC FILM VIEWER

BENSON and CAMPBELL have assigned to CINERAMA CAMERA CORPN GB 1010615 (1965) a gyroscopically stabilized panoramic film viewer, which allows the viewer to swing to the right or left and bring progressively different segments of the entire panorama into his view. It was shown by LUMIERE US 705771 (1902) and by HATTU US 888236 (1908) that panoramic views can be obtained in a simple hand held device and it may be argued that to combine such a device with a gyro-attitude indicating such as that of DRAPER US 2515200 (1950) is an obvious extension of an old art.

An advanced extended vision system for helicopters having an inertially stabilized vision turret able in use to maintain a substantially constant orientation with respect to the terrain regardless of vehicle maneouvering is the subject of disclosures by HUGHES AIRCRAFT CO. GB 1307548 (1973) GB 1317772 (1973).

A further advanced video stabilized system including a Cassegrainian optical system with the gyro at point c. (Fig 10.6.1) is that due to Pepin and Emmanueill for THOMSON-C.S.F. E.P. 0130869 published 9 Jan 1985.

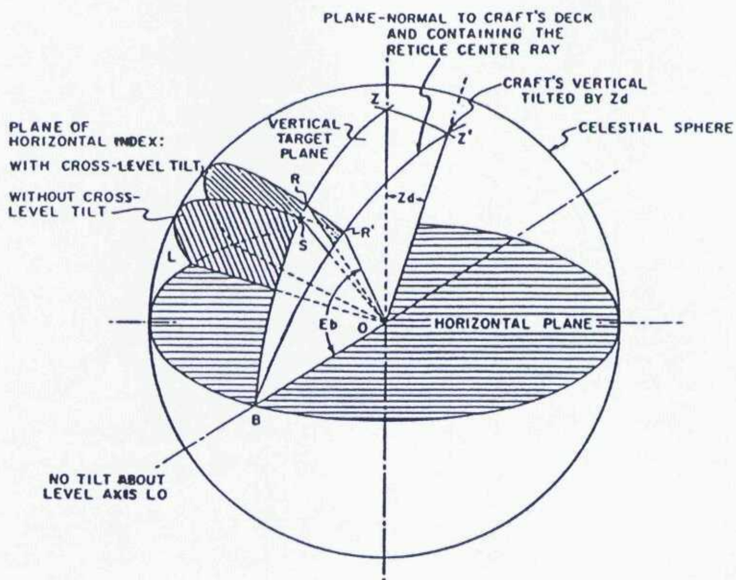


Figure 10.7.1

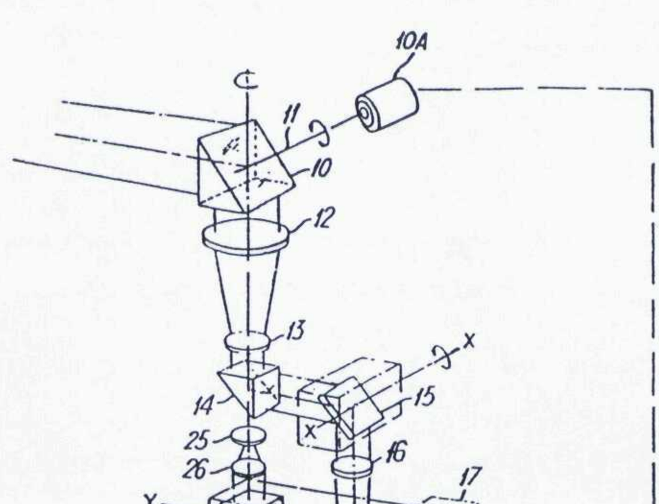


Figure 10.7.2

10.7 PERISCOPES

The word periscope as originally used in the English tongue means a 'look round', a general or comprehensive view, in 1822 Dr FERGUSON was said to be taking a medical periscope of the island of Antigua. Not until 1899 did the Westminster Gazette record 'that various experiments are being carried out in order to provide these boats (presumably submarine boats) with eyes and notably with an apparatus known as the periscope which is based on the principle of the dark room in photography, and which by means of a tube, can be raised to the surface of the water'. The word is now used almost exclusively for an optical instrument used in land and sea warfare to enable an observer to see his surroundings while remaining under cover behind armour or submerged. One of the most authoritative articles on the instrument is by SMITH (1923). The first abridgement volume of United Kingdom Patents for invention to use the word 'periscope' (See Camera obscuras, polenoscopes etc telescopes etc) was the edition of 1901 to 1904 (Class 97).

The first periscope proper is due to COMMON A.A. GB 5990/1901 to be followed by one to Sir H. GRUBB GB 10373/1901; both for use in submarine boats.

The first gyroscopic periscope is due to SCHNEIDER ET CIE GB 226163 (1926). An early gyroscopic periscope gun sight is disclosed by HAWKER AIRCRAFT LTD GB 637358 (1950) utilizing the work of HOLLYHOCK. This instrument enabled accurate fire to be deployed from a gunner behind the pilot's compartment.

McCARTNEY & BRADDON US 2986966 (1961) concern themselves with a periscope enabling a submerged boat to obtain a correct measure of the angle of elevation or altitude of a celestial body, such as a planet or a star. The optical device includes a gyrovertical and a corrective output in the form of an electrical signal of a value proportional to

$$\frac{Zd^2}{4} \times \sin 2E_b$$

See Fig 10.7.1 where Zd is the tilt between the vertical target plane and the plane $Z^1R^1B^0$ and

E_b is the elevation angle.

TEN, BOSCH and LANG US 3035477 (1962) deploy two gyroscopes as sensing elements of a control system to be coupled to an aircraft periscope, one gyroscope having a vertical spin axis to give a signal as the aircraft maneouvres about the roll axis and the other an horizontal spin axis to give a signal as the aircraft maneouvres about the vertical or yaw axis.

A more advanced periscope system with a stabilized line of sight is due to RITCHIE US 3558212 (1971) assignee to BARR & STROUD LTD. The component parts are well shown in Fig 10.7.2, a prism (10) is placed above a Galilean telescope (lens 12.13) providing a collimated output reflected by a prism 14 and a gyro-stabilized mirror 15 stabilized about the transverse pitch axis XX. Disposed beneath a lens 16 is a carriage 17 supporting a gyro-stabilized mirror 18 stabilized against roll about the YY axis and vertically movable by means of a rack and pinion. The observer's eyepiece system (not shown) includes a rotatable dove prism*.

We have noticed above some references to gyro gunsights that may be termed periscopic gyro-gunsights such as for example, Fig 10.7.3.

We should include here the famous FERRANTI gyroscopic periscopic sights AF 120 and AF 530. The first is a gyrostabilized periscopic telescope with binocular vision and two viewing magnifications of X2.5 and X10. The instrument has an all-up weight of about 81.25 lb. (36.85 kg). Fig 10.7.3. The second instrument is a monocular instrument of about 20.6 kg. Fig 10.7.4.

* See KINGLAKE. Applied Optics and Optical ENGINEERING 1965, Vol. 3, p.297.

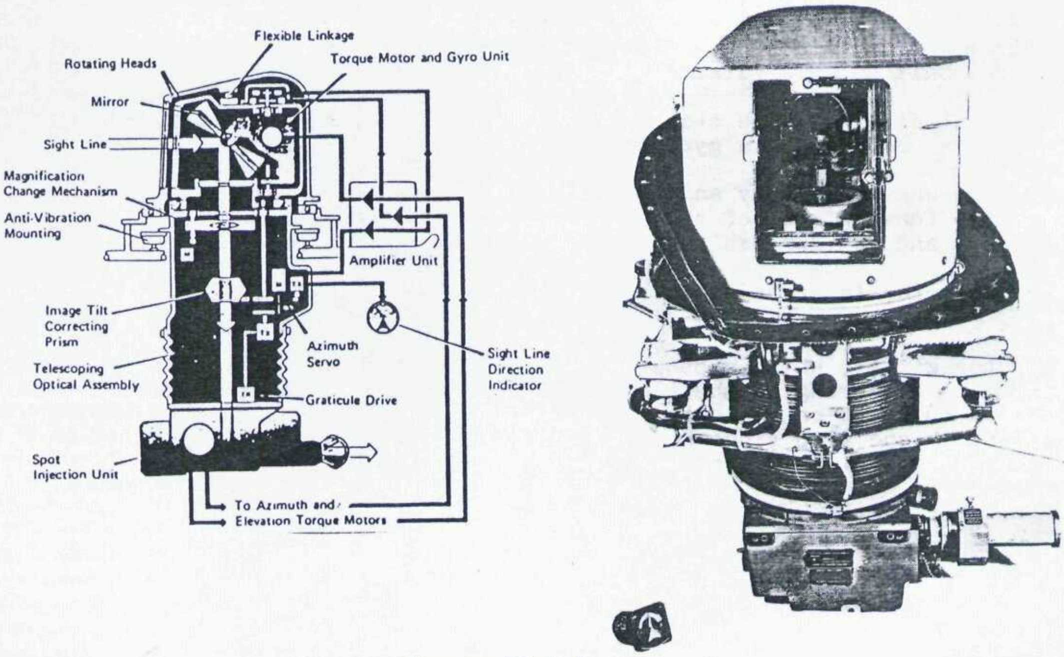


Figure 10.7.3

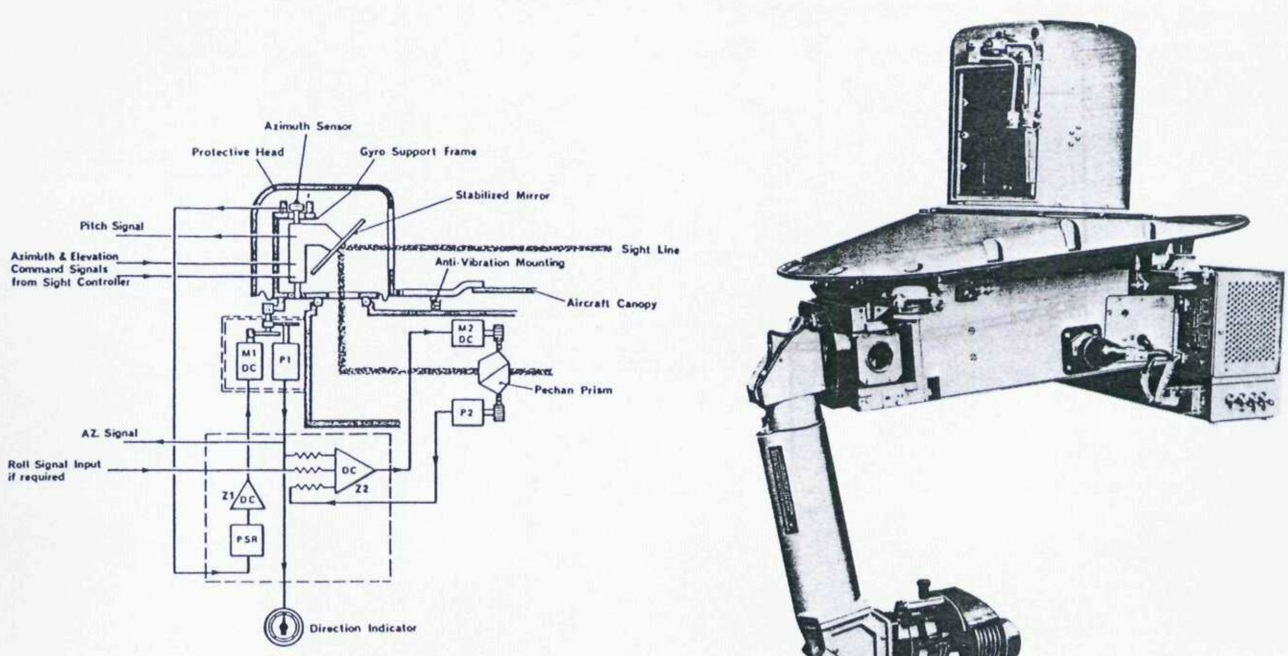


Figure 10.7.4

Photographs by courtesy of Ferranti Ltd.

An aerial periscope is made by Ferranti. Fig 10.7.5 providing an airborne surveillance system including a gyro-stabilized television camera all contained within a 15 inch dia sphere (380 mm) for suspension above the Earth by a tethered balloon or a kite.

The essential sight-line stabilization, which eliminates the effects of angular motion, is provided by a gyroscope based upon a brushless d.c. motor.

Both gyro wheel assembly and television camera are mounted on a plate which forms the inner gimbal of the two degrees of freedom gyroscope, allowing 10° elevation and 80° depression of the television camera from the horizontal.

A torque motor is arranged to react on the support shaft of the inner gimbal plate.

The outer gimbal, which supports the horizontally pivoting inner gimbal, is carried on a bearing at the top of the gyro assembly. Another torque motor is arranged to react on the outer gimbal. Electrical connections are made by a brush-block and slip rings between the outer gimbal and its support frame, allowing unrestricted rotation of the camera in the azimuth plane.

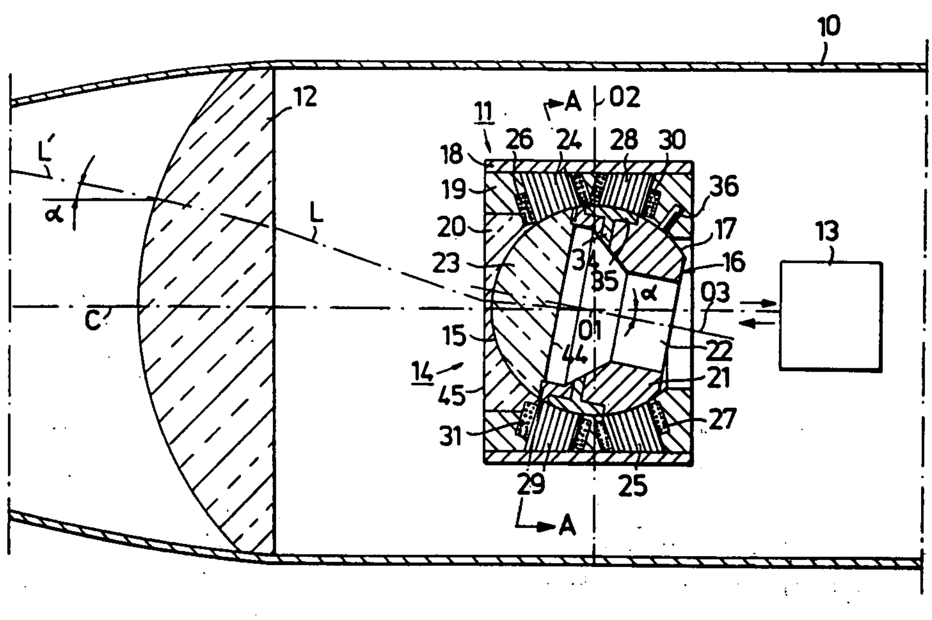


Figure 10.7.5

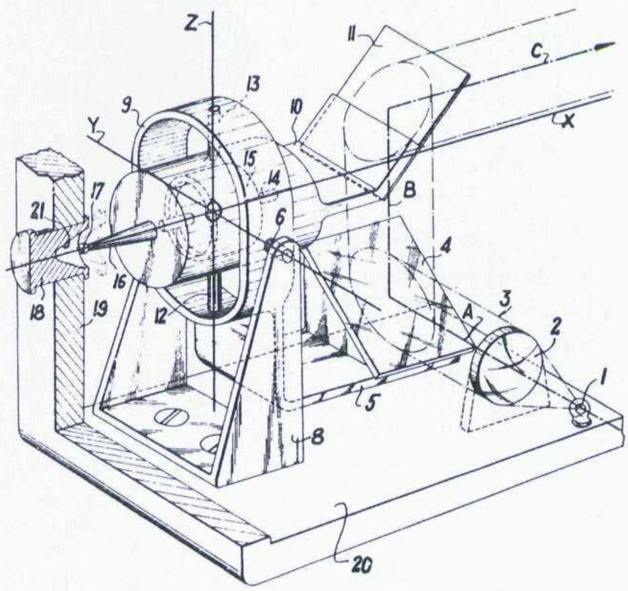


Figure 10.10.1

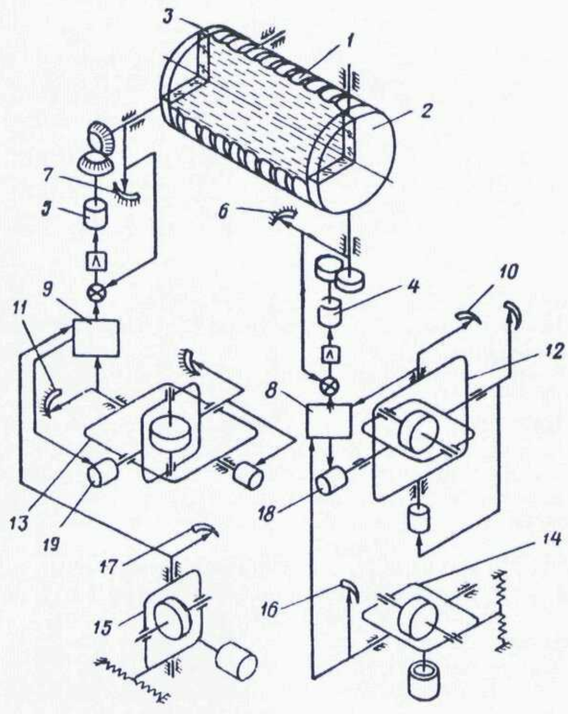


Figure 10.10.2

10.8 NAVIGATIONAL AIDS

ZEISS and KÖNIG GB 284882 and GB 286514 (1929) are responsible for a gyroscopic sighting device for the transformation of terrestrial coordinates into ship coordinates an exercise dependent essentially upon a vertical line that is established by a gyrovertical.

SCHULTZ, SCOTT & WING have disclosed through The SPERRY CORPN GB 749987/8/9 (1956) a navigation system, in which a primary axis is set into a direction parallel to the Earth's polar axis and the position of said axis is gyroscopically stabilized by three gyroscopes having their spin axes mutually perpendicular. The system includes an optical star follower.

SPERRY RAND CORPN GB 805535 (1958) draw attention to the work of GARBARINI, WENDT and RAWITZ directed to a gyroscopic directional reference system using an astronomical computer that is dependent upon a gyro stabilized optical sight.

Of historical interest only is the selfrecording rolling indicator of GIPPS GB27508/1907. His proposal is directed to a gyroscopic optical device that records the motion of a ship due to pitching rolling and yawing. A concave mirror is fixed gyroscopically in space and light from it gives a trace, as the ship moves, on a suitable screen that is provided with cartesian coordinates.

10.9 RANGEFINDERS

We are indebted to Professor J.B. HENDERSON GB 162677 (1923) for a solitary disclosure, of a single observer gyroscopic rangefinder for use on shipboard.

10.10 STABILIZATION OF IMAGE

McGEE GB 750889 (1956) proposed a stabilization of a television image used in a guided missile by means of a gyroscopically variable angle prism such as explained in detail by de la CIERVA to DYNA SCIENCES CORPN (1967) treated earlier. The gyroscopic control operates to stabilize the course of the missile and an opposite motion is imparted to the prism.

SMITH US 3293360 (1966) is also concerned with the stabilization of a television image in a missile, but he uses the gyroscope to control in space a second light image produced by an incident first light image, so that the second image remains substantially unaffected by changes of position.

POWLEY has assigned to FERRANTI LTD GB 1015916 (1966) a not dissimilar stabilization means for a television camera in the nose of a missile using a gyro-controlled mirror that brings an image onto a television screen. The length of the optical path between the principal point of a lens system and the mirror being approximately half the focal length of the lens system.

PHILCO-FORD CORPN GB 1073446 (1967) in a system devised by Johnson & Lundquist use two gyroscopes to provide an auto collimation technique wherein a beam of light is collimated by the optical path system on the gyro and projected onto an autocollimation gyrostabilized mirror from the back surface of a sight line deflecting mirror and returned through the optical path system on the gyro to a detector to provide a high resolution readout of the deviation of the sight line.

CROVELLA US 3371161 (1968) electronically stabilizes the image using the gyroscope to provide deviation voltages for pitch and yaw.

GIRAVIONS DORAND GB 1136054 (1968) have proposed a gyroscopic device for holding an optical beam parallel to a predetermined direction especially useful in the firing of guided missiles. The device which is a neat exponent of gyroscopic stabilisation in its essential parts is shown at Fig 10.10.1. The component parts comprise a point source of light 1 placed at the focus of a convergent lens 2, two plane mirrors 4.11 are set at 45 degrees of arc. Mirror 4 is on a stirrup 15 with pivots on axis YY_1 and mirror 11 on a frame 10 pivoted on axis ZZ_1. A gyroscope has its spin axis parallel to XX_1 and is centred at 0, the meeting point of the three orthogonal axes XX_1. YY_1. ZZ_1. On an extension of the gyro axis is a setting pin 16 cooperating with a plunger 18. It is a simple matter to incorporate such a device with a firing simulator so that a marksman may follow the stabilized light spot, the original orientation of which is maintained.

A sight of not dissimilar form is that disclosed by HUMPHREY US 3468595 (1969) and ALLARD US 4027540 (1977).

BENTLEY and DEMAINE have assigned to The RANK ORGANISATION LTD GB 1374765/6 (1974) a Schmidt type system in which the image plane input end incorporates a coherent fibre optical bundle that is gyroscopically stabilized, the output end being fixed.

A motion picture filming apparatus using gyro stabilizers reminiscent of the Dynalens is due to TOROCHKOV and KALABIN of the MOSKOVSKII INSTITUT GEODEZII AEROFOTOSIEMKI I. KARTOGRAFII RUSSIAN 458710 (1975). In their apparatus a liquid prism is distorted by transparent end plates driven by two gyrostabilizers controlled by gyroscopic angular velocity gauges (See Fig 10.10.2).

JOHNSON & RICHARDSON have assigned to The MARCONI COMPANY LTD and ELLIOT BROTHERS (LONDON) LTD GB 1520845 (1978) a rotational balance for a revolving mirror incorporating a gyroscope that detects rotational movement whereby rotation of the mirror and its support in one direction causes rotation of a balance member in the opposite direction. The inertia of said balance member tending to compensate for the inertia of the mirror.

11. SPECIAL APPLICATIONS OF THE GYROSCOPE

11.1 THE GYROSCOPE AS A MEANS OF TESTING GENERAL RELATIVITY*

Anyone wishing to enter this recondite subject should first peruse the two widely separated contributions of EVE. A.S. (1926) and SCHIFF. L.I. (1961). The latter has pointed to the motion of the spin axis of a gyroscope that is either at rest in an earth-bound laboratory, or in a free-fall orbit about the Earth. In either case, the Newtonian theory predicts no precession of the spin axis if the gyroscope is spherically symmetric. General relativity theory, however, predicts both the geodetic precession arising from motion through the Earth's gravitational field, and the Lense-Thirring precession[x] that represents the difference between the gravitational field of the rotating and the nonrotating Earth. If the gyroscope is at rest with respect to the Earth, it is carried around the Earth once a day by the rotation of the Earth, and its weight must also be supported by a nongravitational force; the latter gives rise to an additional Thomas (special-relativistic) precession. In this case, all three terms are of the same order of magnitude, and the total precession is about 0.4" of arc per year. If the gyroscope is in a satellite at moderate altitude, the geodetic precession is about 7" per year, the Lense-Thirring precession is about 0.1" per year, and the Thomas precession is zero.

One proposed gyroscope consists of a superconducting sphere supported stably on a static magnetic field. The difference between the local acceleration of gravity g and the true acceleration of the satellite arises mainly from atmospheric gas and should be of the order of 10^{-7} g at moderate altitudes; this greatly simplifies the problem of supporting the spinning sphere. Ambient electric and magnetic fields can be greatly reduced by using a superconducting shield, and the low temperature required also decreases thermal distortion since all coefficients of thermal expansion are very small. A temperature of around 4°K can be maintained for a year by sublimation of a hundred pounds of solid hydrogen, and an additional five liters of liquid helium would keep the temperature below 1°K. The orientation of the spin axis would be observed by putting a spot of a suitable radioactive material on the sphere, and using the Mossbauer effect to align the spin axis of a synchronously rotating absorber with that of the sphere. Experiments are under way on all aspects of this system.

A different kind of extreme-precision gyroscope consists of a conducting sphere which is supported by an electric field with the help of a feedback loop.

Its drift rates can be held to less than 3×10^{-8} rad/sec when it is supported against normal earth gravity; it is expected that this can be reduced by a factor 30. (Note that 1" of arc per year is equal to 1.5×10^{-13} rad/sec.) Satellite operation of 10^{-7} g would certainly lower the drift rate by several orders of magnitude. Reading out the orientation of the spin axis is accomplished by an optical method; this can now be done with an accuracy of 0.2".

A simplified orbiting experiment is proposed by PALAMARA. R.D. (1966).

* Note these comments are to the General Theory.
The Special Theory is, though widely acclaimed, open to severe criticism, see
DINGLE. H. (1972)
ESSEN. L. (1971-1972)
MARINOV. S. (1982)

x Regarding the Lense-Thirring precession see
LENSE J. THIRRING H. (1918)

It is now technically feasible according to ENSLEY. D.L. (1970) to perform another test of Einstein's Theory of General Relativity in an Earth laboratory. This is possible as a result of a new inertial mass support technology, in which a gyroscope rotor is resonantly supported within a coherent radio-frequency soundwave field in superfluid helium II at 1°K.

With this technology it should be possible to detect a predicted 1/2 arc sec/yr relative precession of the spin axes of two similar gyroscopes located 39° latitude apart on the earth's surface (at Livermore, California and in Greenland). In no way would this test depend on astronomical observations. For the first time a significant test of gravitational theory, involving the effects of both the static field and the spin of the earth, can be done in which all experimental variables will be under the direct control of the experimenter.

The long history of this elaborate project has not been free from trouble, but William Fairbank, professor of physics at Stanford, now thinks the doubters have been won over. NASA has promised space for a proving flight on the shuttle in 1988, and there is a prospect that the satellite may be launched (into an accurately polar orbit) in 1991.

Fairbank says that the outcome may not be just another test of general relativity but perhaps a pointer to the physics that will have to be understood before gravitation can be quantized. If it works, it should help further to make general relativity part of ordinary physics. (See MADDOX. J. (1984))

The task of the experimentalists is now known to lie in the production of a sphere of fused quartz accurate to one part in ten million. There is, however, a quadrupole moment that affects the drift rate. BARKER. B.M. and O'CONNELL. R.F. (1975) have shown that if the gyroscope's spin axis is one degree out of either the orbit plane or perpendicular to the orbit plane, then the drift rate will exceed one millisecond of arc per year.

Again the Earth's quadrupole moment will contribute an amount of four milliseconds of arc per year, and the contribution from the Sun gives rise to a geodetic precession of the gyroscope of the value of 19.2 milliseconds of arc per year. The need to test the goedetic and motional precessions is met by the suggestion by SCULLY. N.O. ZUBAIRY. M.S. HAUGAN. M.P. (1981) that a ring laser interferometer may be used on Earth to effect a more accurate measurement, whereas BRAGINSKY. V.B. POLNAREV. A.G. THORNE. K.S. (1984) suggest measuring the Earth's gravitomagnetic field by monitoring its effect on the plane of swing of a Foucault pendulum at the Earth's south pole. Finally, the data provided by TAYLOR. J.H. and WEISBERG. J.M. (1982) suggest to BARKER. B.M. and O'CONNELL. R.F. based on their calculations of (1975) that the pulsar PSR.1913 +16 discovered by HULSE. R.A. and TAYLOR. J.H. (1975) may provide a test of the spin orbit precession rate presently in doubt.

Since writing parts I and II of this report I have been made aware of the Stanford Gravity Probe B experiment described by Cheung.S.et al.* which is said to be an experiment aimed at testing general relativistic precessions of superconducting gyroscopes orbiting the Earth[x]. The goal is to measure the gyro effects (7.9 arcseconds/year for the geodetic precession and 0.040 arcsecond/ year for the motional precession) to a precision of one milliarcsecond/year. The spin signal of the gyroscope, which is a quartz sphere about 1.5 in. (38 mm) in diameter and coated with a superconductor, manifests itself as the London magnetic moment # which can be detected by a readout scheme using multiturn and micron-sized superconducting loops placed near the equator of the gyroscope and coupled to a SQUID magnetometer.

* Cheung. S., Bardas. D., Gill. D., MacGirvin. M., Saldinger. T. <u>Superconducting thin film gyroscope readout for gravity-probe-B.</u> 1986. Conference on Precision Electromagnetic Measurements CPEM.86 Digest p.10.

x See J.T. Anderson, et.al., Proceedings of the Second Marcel Grossmann Meeting on General Relativity, ed R. Ruffini (Amsterdam, North Holland, 1982), Part B, pp. 939-957.

\# Re London magnetic moment field, given as $H = 10^7$ gauss

See London.F.London.H.
<u>The Electromagnetic Equations of the Supraconductor.</u>

Proceedings of the Royal Society of London.
<u>A149</u> (1935) pp.71-88.

London. F. London. H.
<u>Supraleitung und Diamagnetismus.</u>
Physica <u>2</u> (1935) pp.341-354.

Fairbank W.M.) <u>Superconductivity.</u>
Fetter. A.L.) Chapter 12. pp.5.166-5.177 in Handbook of Physics.
2nd Ed. 1967. Condon & Odishaw.

11.2 THE USE OF GYROSCOPES TO PROVIDE STABILITY

The desire for stability in dynamical structures has produced a large number of investigations especially in relation to the role of the gyroscope in bringing the desideratum about. LEIMANIS, E. (1965) in one of the more erudite of the general investigations into the ramifications of the gyrosocpe makes but a single reference in his preface to the broad question of gyroscopic stabilization; he refers his readers to the work of I.I. METELICYN (1952). A closer inspection reveals that METELICYN'S paper directed to on gyroscopic stabilization is in Russian and that Leimanis has merely translated the Russian title. I give below a translation[x] from the Russian into the English tongue.

Metelicyn. attracts different spellings in English and German. Magnus. uses METELIZYN. some prefer METELITSYN. the spelling adopted by Leimanis is unusual, the Russian title and a transliteration from the Cryllic is given below:-

И. И. МЕТЕЛИЦЫН

К ВОПРОСУ О ГИРОСКОПИЧЕСКОЙ СТАБИЛИЗАЦИИ

Доклады Академии Наук СССР

from

1952. Том LXXXVI, № 1

I.I. Metelitsyn. K voprosu o giroskopicheskoi stabilizatsii.
Doklady Akademii Nauk SSSR. Mekhanika
1952 Tom. LXXXVI, No 1

Gyroscopic Stabilisation (translation from Dokl Akad Nauk. SSSR. 86 (1952) p31-34)

1. The criteria for the stability of movement of mechanical systems include points which the forces applied must satisfy in order that the roots of the characteristic equation will have a negative real part.

The theorems of Lagrange, Kelvin, Routh and Zhukovsky, which express the conditions for stability in such a form, are well known; these theorems also reveal the effect of dissipative and gyroscopic forces on the stability of movement of conservative systems.*

At present these conditions are almost unused in practical problems, since the mechanical and electromechanical systems the stability of movement of which we have to examine are not conservative. It is no accident that the stability criteria of Hurwitz, Mikhailov and Nyquist have driven out the classical criteria, since they are applicable to non-conservative systems and are convenient in practical calculations.

However, these latter criteria do not let us answer general questions, such as what effect dissipative or gyroscopic or non-conservative forces have on the stability of movement, whether stability of movement can be enhanced or even guaranteed by the addition of dissipative forces to the forces that are present if the system is non-conservative, etc.

x The translator wishes to remain anonymous.

* according to SNEDDON I.N. (1976) p.154

CONSERVATIVE FORCE OR FIELD:- If the total energy of a body moving in a field of force is constant (or conserved) the force (or field of force) is said to be conservative. In such a field the work done on the body in moving between any two points is independent of the path taken. For each point in a conservative field it is possible to define a scalar quantity V, known as the potential energy, such that the work done against the force F in moving along paths from A to B is

$$\int_A^B F\, ds = V_A - V_B$$

If we take into account that in contemporary machine-building technology there are methods for the artificial realisation of different categories of force, it is useful to predict in advance what effect could be obtained by adding forces of one category or another to the forces which act on the system in its natural state.

Below are set out certain theorems which allow us to answer these questions; the theorems are not exhaustive, but they do allow us to orientate among these questions before going on to the detailed investigation of stability by the well known methods.

2. Let $q_1, \ldots, q_n$ be the generalised coordinates of the system under examination. The equations of movement of the system (linear) have the appearance

$$\Sigma(a_{ks}\ddot{q}_s + b_{ks}\dot{q}_s + c_{ks}q_s) = 0 (k = 1,2,\ldots n)$$

Here the coefficients satisfy the conditions $a_{ks} = a_{sk}$ as coefficients of the square form of the live force of the system. The remaining coefficients can be presented in another form, putting:

$$\beta_{ik} = \frac{b_{ik} + b_{ki}}{2} = \beta_{ki} \quad : \quad \gamma_{ik} = \frac{b_{ik} + b_{ki}}{2} = \gamma_{ki}$$

$$\delta_{ik} = \frac{c_{ik} + c_{ki}}{2} = \delta_{ki} \quad : \quad \varepsilon_{ik} = \frac{c_{ik} + c_{ki}}{2} = \varepsilon_{ki}$$

Using this method we usually separate the generalised forces depending on the generalised speeds into dissipative forces $\beta_{ik}q_k$ and gyroscopic forces $\gamma_{ik}q_i$; similarly, the generalised forces depending on the generalised coordinates can be separated into conservative forces $\delta_{ik}q_i$ and "inherently non-conservative" forces $\varepsilon_{ik}q_i$.

We can now put the equations of movement as follows

$$\Sigma(a_{ks}\ddot{q}_s + \;_{ks}\dot{q}_s + \;_{ks}\dot{q}_s + \delta_{ks}q_s + \varepsilon_{ks}q_s) = 0 \quad (K = 1,2,\ldots n) \tag{1}$$

We introduce the dissipative function D and potential energy V

$$D = 1/2 \sum_k \sum_s \beta_{ks}\dot{q}_k\dot{q}_s \tag{2}$$

$$V = 1/2 \sum_k \sum_s \delta_{ks}q_s q_k \tag{3}$$

Putting in equations (1)

$$q_s = A_s e^{\mu t}$$

We find equations for determining the constants A_s and the characteristic equation

$$\Sigma\left[a_{ks}\mu^2 + (\beta_{ks} + \gamma_{ks})\mu + (\delta_{ks} + \varepsilon_{ks})\right] A_s = 0 \quad (k = 1,2\ldots n) \tag{4}$$

$$\Delta(\mu) = a_{ks}\mu^2 + (\beta_{ks} + \gamma_{ks})\mu + (\delta_{ks} + \varepsilon_{ks}) = 0 \tag{5}$$

We assume that the characteristic equation (5) permits complex conjugated roots μ and μ^1 for each of which there is its own system of conjugated values of constants

$$A_s = M_s + N_s i \qquad A_e' = M_s - N_s i \tag{6}$$

Multiplying equations (4) in turn by A_s and summing, we find the following relationship

$$T(AA^1)\mu^2+[D(A.A^1) + i\Gamma\,(A.A^1)]\mu + V(A.A^1) + iE(A.A^1) = 0 \qquad (7)$$

The coefficients for μ^2, μ and the free term have the following values

$$T(A.A^1) = T(M) + T(N) \qquad D(A.A^1) = D(M) + D(N)$$
$$\Gamma(A.A^1) = i\,\Sigma\gamma_{ks}(M_kN_s - M_sN_k) \qquad iE(A.A^1)\ 0\ i\Sigma\,\varepsilon_{ks}(M_kN_s - M_sN_k) \qquad (8)$$

Without dwelling on the derivation of these well known relationships, we note only that T, V, D, Γ and E are real quantities and that in addition T is a definite positive quadratic form. We suppose that D also is a definite positive quadratic form, i.e., we shall examine systems in which the dissipative forces have the general character of resistance forces. The approach that we used above can lead to a dissipative function of another character.

We take equation (7), which after suitable transformations can be brought to a characteristic equation which does not contain arbitrary constants. It differs from the characteristic equation only in the grouping of the terms, because of which it has the appearance of a quadratic equation with respect to μ.

Solving equation (7) for μ, we obtain

$$\mu = \frac{-D+i\Gamma\,) \pm\sqrt{(D + i\,\Gamma\,)^2 - 4T(V + iE)}}{2T} = \frac{-(D\mp X) - i\Gamma(1\mp\frac{D}{X})}{2T} \qquad (9)$$

where

$$X2 = \frac{D^2 - \Gamma^2 - 4VT}{2} + 1/2\sqrt{(\Gamma^2 + 4VT - D^2)^2 + 4\,(D\Gamma - 4TE)^2} \qquad (10)$$

Since the quadratic form D is positive definite the movement will be stable if the inequality $x^2 < D^2$, is satisfied,

or $$4TE^2-2D\Gamma E < D^2V \qquad (11)$$

It is this last inequality which expresses the condition of stability of non-conservative systems.

We will note the theorems arising from the inequality (11).

Theorem 1. If a conservative system is statically unstable, the addition of inherently non-conservative forces (without dissipative and gyroscopic forces) cannot make the system stable.

If D=0 and Γ=0, the condition $4TE^2<0$ cannot be satisfied.

Theorem 2. If a conservative system is statically stable, the addition of inherently non-conservative forces (without dissipative and gyroscopic forces) can make the system unstable.

Theorem 3. An inherently non-conservative system (V=0) can be made stable only if gyroscopic and dissipative forces are simultaneously added to the forces that are acting.

If V=0 the conditions for stability (11) can be satisfied only if $4TE^2<4E\Gamma D$, i.e., Γ and D are not equal to nought.

Theorem 4. A statically unstable system can be made stable if to the forces that are applied are simultaneously added dissipative, gyroscopic and inherently non-conservative forces.

Since in the case examined $V < 0$, then $4TE^2 - 2D\Gamma E < VD^2 < 0$. From this it follows firstly that $4TE^2 < 2D\Gamma E$, i.e., $\Gamma E > 0$, and secondly that the dissipative and gyroscopic forces are sufficiently strong.

<u>Theorem 5</u>. If the conditions for stability (11) are satisfied and if gyroscopic forces dominate over the others, the frequencies of vibration of the system diverge, i.e., some become very small and others very large.

From equation (10) we find

$$x \simeq D - \frac{4TE}{\Gamma} - D\,\frac{4VT-D^2}{2T^2};$$

From here it follows that x differs little from D, and therefore of the two quantities $v_1 = \frac{\Gamma}{2T}\left(1-\frac{D}{x}\right)$, and $v_2 = \frac{\Gamma}{2T}\left(1+\frac{D}{x}\right)$ the second will be many times greater than the first: $v_2 \gg v_1$.

<u>Theorem 6</u>. If the conditions of stability are satisfied and if the gyroscopic forces are dominant, the fading of higher frequency vibrations is more intense than that of slower vibrations.

For vibration of frequency $\frac{\Gamma}{2T}\left(1 - \frac{D}{x}\right)$

the coefficient is $\frac{D-x}{2T}$

and for vibrations of frequency $\frac{\Gamma}{2T}\left(1+\frac{D}{x}\right)$ the coefficient is $\frac{D+x}{2T}$

Since $x>0$ and with large gyroscopic forces x differs little from D, the slow vibrations fade slowly and fast vibrations fade faster.

This property of the vibrations of gyroscopic systems simplifies investigations, since practical calculations can be limited to slow precession vibrations.

Discarding from equation (7) terms with coefficients T and D, we get the "shortened" equation

$$i\Gamma\mu + V + iE = 0, \tag{12}$$

from which

$$\mu = -\frac{E}{\Gamma} + \frac{V}{\Gamma}i.$$

If $E/\Gamma > 0$, slow precession vibrations fade, and strictly non-conservative forces act as dissipative forces. One could incorrectly conclude from this that a gyroscopic system could be stabilised by adding just non-conservative forces. It follows from the foregoing that the dissipative function D must not equal nought, and in any gyroscopic system the source of the dissipative forces must be found, if there are no special devices for damping vibrations.

The above theorems can be easily illustrated in well known gyroscopic machines.

In complete contrast to the erudite mathematical exposition of Metelicyn, I now give two practical examples of the use of the gyroscope to provide stability. See Fig 11.2.1 and Fig 11.2.2.

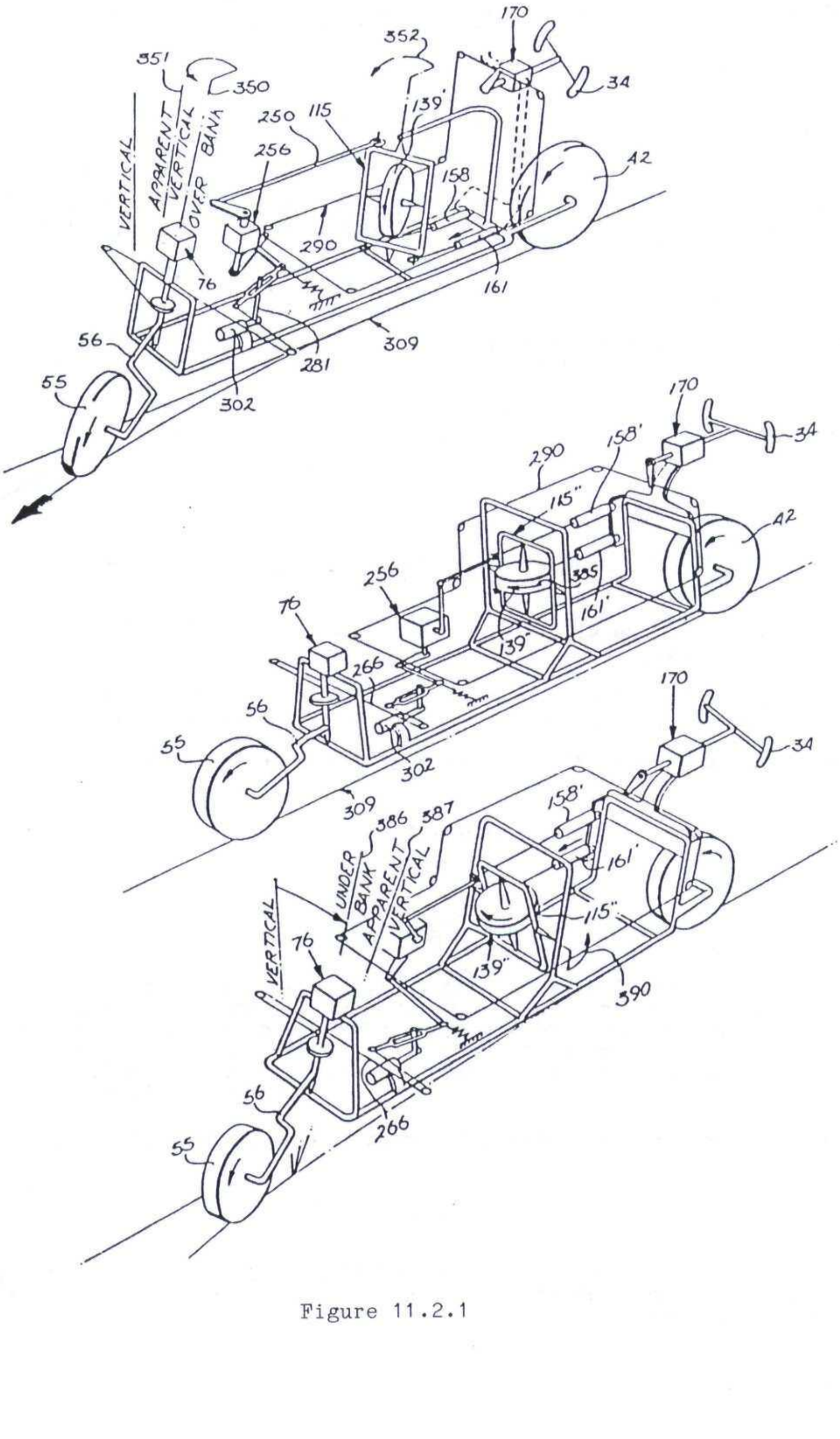

VERTICAL
APPARENT VERTICAL OVER BANK
351
350
352
170
34
250
115
139'
256
42
158
290
76
161
56
281
309
55
302
170
34
290
158'
115''
42
256
385
161'
76
139'
266
56
55
302
309
170
34
386
387
UNDER BANK
APPARENT VERTICAL
158'
161'
VERTICAL
115''
76
139''
390
56
266
55

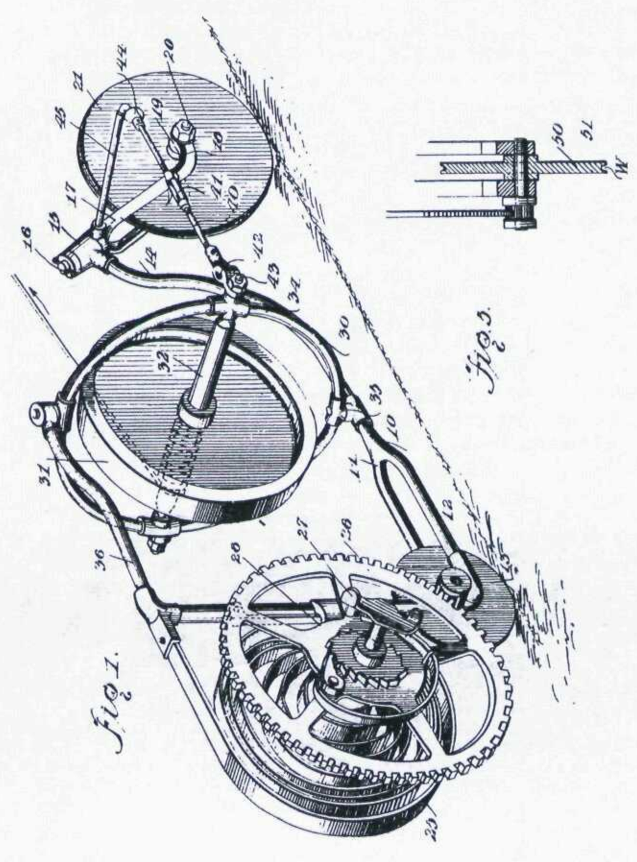
Figure 11.2.1

Oct. 28, 1924. 1,513,143

E. F. WELCH ET AL

GYROSCOPIC CONTROLLED WHEELED TOY

Filed Dec. 6, 1922 2 Sheets-Sheet 1



INVENTORS
Edward F. Welch
Peter E. Frarow
John F. Phillips
BY Munn & Co.
ATTORNEYS

Figure 11.2.2

11.3 GYROSCOPIC INERTIAL DRIVES

In 1891 Nikola Tesla in his lecture[x] to the members of the A.I.E.E. in Columbia New York made the following euphoric visionary statement:-

> "But there is a possibility of obtaining energy not only in the form of light, but motive power, and energy of any other form, in some more direct way from the medium. The time will be when this will be accomplished, and the time has come when one may utter such words before an enlightened audience without being considered a visionary. We are whirling through endless space with an inconceivable speed, all around us everything is spinning, everything is moving, everywhere is energy. There must be some way of availing ourselves of this energy more directly. Then, with the light obtained from the medium, with the power derived from it, with every form of energy obtained without effort, from the store forever inexhaustible, humanity will advance with giant strides. The mere contemplation of these magnificent possibilities expands our minds, strengthens our hopes and fills our hearts with supreme delight."

It is this essential human asperation that lies in the minds of the inventors of the machines that will form the subject of this note on the complex subject of gyroscopic interial drives.

The machines all make an appeal to the forces that are of themselves not easily resolved by the use of the orthodox laws of physics and the orthodox equations of mathematics. We need but turn to a homely example to show that the feline ability to make a "retournement spontane" in falling to land on its feet is an apparent contradition of the principle of the conservation of angular momentum in a closed system (see DARIUS J. 1894).[xx] The chronophotographic picture by MAREY E.J. (1894) does not resolve the problem as shown by the contribution from GUYOU E. (1894) the famous French mathematician whose contributions to the cause of science and gyroscopic phenomena are considerable. In his opening remarks he states that "ce retournement spontane de l'animal parait impossible". (Figure 11.3.1.)+

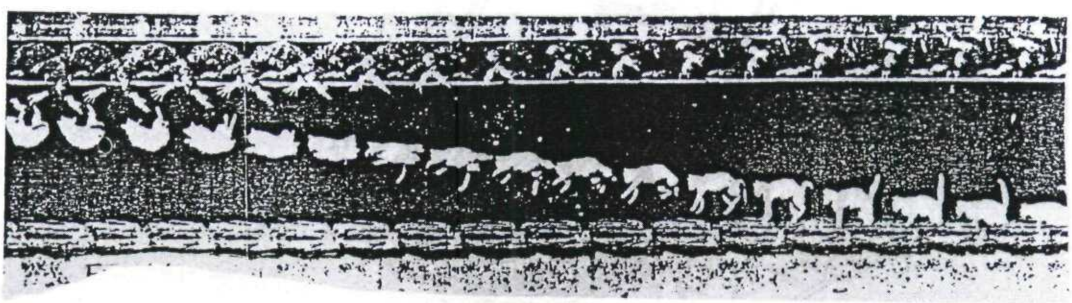


Figure 11.3.1
Animal Mechanism - Note concerning the Presentation of Mr. Marey; by Mr. Guyou translated from the French original.

At first sight, this spontaneous turning of the animal appears impossible. It is seen, in effect, by comparing the initial and final positions, that each part of the body has rotated 180° about a longitudinal axis, and this result seems incompatible with the forces theory. But this incompatibility does not exist. The total sum of forces does not only depend, as does that of angular rotations, upon the initial and final positions; it depends also upon the intermedial phases of the movement. And, in the case under consideration, this sum remains always nil, although the algebraic sum of rotations is positive.

x Experiments with alternate currents of very high frequency and their application to methods of artificial illumination (20 May 1891).

xx See also Nature 308 (1984) p.109.

\+ Comptes Rendus 119 (1894) pp714-718.

When, in effect, the animal, by a contraction of muscles, gives a twisting movement to its body, it also gives, by extension of its limbs, a large moment of inertia to the part which rotates in the negative direction. It results, therefore, from the area theory that the negative rotations have an angular value less than the positive rotations.

Let us suppose, to make it clear, that the respective moments of inertia of the front and rear parts of the body have value I when the limbs are extended and i when they are brought in line with the body. When the animal, having folded up its front paws, gives to the corresponding part of the body at rotation Ω, the rear part turns by a negative quantity ω such that one has

$$\frac{\Omega}{\omega} = \frac{I}{i}$$

When, subsequently, reversing the moments of inertia by folding up its rear paws and extending the front ones, it gives its body a twist in the opposite direction, the rear part turns in the positive direction by quantity Ω, and the front part in negative direction by quantity ω. It follows that the algebraic sum of rotations of the two parts equals $\Omega - \omega$. The body has thus reached a position such that all parts have turned by this amount in the positive direction. The total rotation can thus be achieved by successive differential movements.*

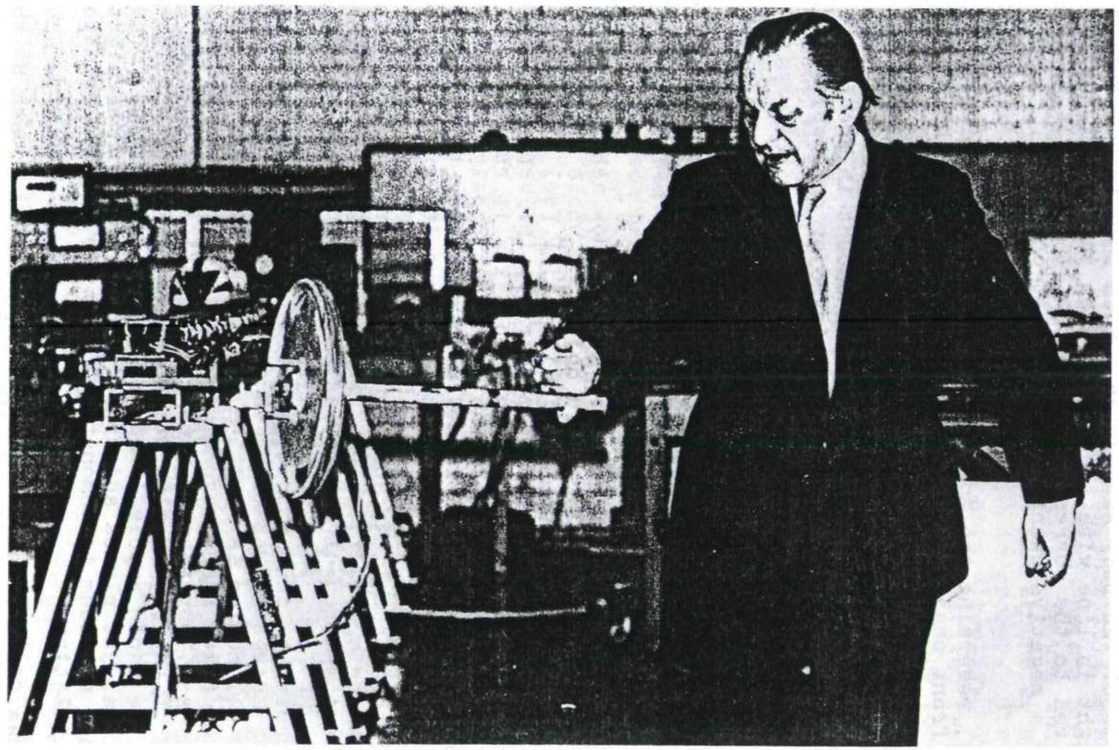
If now we turn to scientific instruments we can excite our interest further by restating the case of the Foucault pendulum as presented by Arthur Koestler in his well known book The Roots of Coincidence (1972). In the chapter entitled The Country of the Blind, Koestler makes what he calls 'one last excursion into physics' -

> "On the shadow desk in front of me there is a shadow ashtray. For ordinary purposes it is quite a sensible, solid object, a whole in itself, with no quantum nonsense about it. But when I lift it, I feel its weight, which means that it is subject to a rather mysterious influence which we call the earth's gravitational field. And when I push it, it resists. This is partly due to the friction against the desk, but partly to the massive ashtray's inertia. Now inertia is defined, according to Newton's First Law of Motion, as the tendency of a body to preserve its state of rest or uniform motion in a given direction. But if I were to suspend my ashtray on a fine thread from the celing, and turn it into a replica of Foucault's pendulum in the Paris Invalides, the plane of its oscillations would not remain fixed in its given direction, as the principle of inertia requires, but would slowly rotate, completing a turn in twenty-four hours. We explain that this is caused by the earth's rotation, and that my ashtray pendulum did preserve its direction relative to the fixed stars, so all is well. However, since all motion is relative, we are entitled to regard the earth at rest, and the fixed stars revolving around it - as the ancients did; and if this is the case, why should my ashtray's motions be governed by the stars, instead of the earth below it? The same argument applies to the flattening of the earth's poles, and the so-called Coriolus force which deflects missiles, jet planes and trade winds from their straight inertial direction. They all seem to demonstrate that the earth's rotation is absolute, not relative.
>
> This paradox was first pointed out by Bishop Berkeley, then by the German physicist Ernst Mach (after whom the units of supersonic speed are named). Mach's answer was that we are indeed entitled to regard the earth as at rest, and to explain the phenomena which we ascribed to its rotation as somehow caused by the fixed stars and galaxies - that is, by the mass of the universe around us. According to this theory, known as Mach's Principle, it is the universe around us which determines the direction of Foucault's pendulum, and governs the inertial forces on earth responsible for the flattening of the poles. Einstein took over Mach's Principle and postulated that the inertia of earthly bodies is merely another manifestation of gravity, not due to the stars as such, but rather to their rotation. This is the accepted theory today. How the rotation of the stars produces the inertia of my ashtray is anybody's guess.

* See also: DIAMOND J.M. Animal Behaviour: Why cats have nine lives Nature 332 (1988) p.586-587.

Photograph by courtesy of Professor Eric Laithwaite

Figure 11.3.2



The wheel is rotating at 2400 RPM it weights 24.lb (10886.g) it is 13 inches in diameter (330mm) and the steel shaft has a length of 30 inches (762mm).

> Inertia is the most tangible, down-to-earth phenomenon in our daily existence: you are up against it whenever you shift a piece of furniture. And yet it has now been discovered that its resistance to being shifted is due to the circumstance that it is surrounded by the rotating mass of the universe. In 1927, Bertrand Russell, though subscribing to Einsteinian Relativity, nevertheless felt impelled to protest:
>
> It is urged that for "absolute rotation" we may subsitute "rotation relative to the fixed stars". This is formally correct, but the influence attributed to the fixed stars savours of astrology, and is scientifically incredible."

If the Foucault pendulum can ignore the blandishments of its 'close-bosom friend' the EARTH, what of the free moving gyroscope?

That extra-terrestrial forces may be involved in sustaining the precession of a gyroscope in accordance with Mach's principle is shown in the fine photograph of Professor Eric Laithwaite holding with consumate ease, on the little finger of one hand, a heavy overhanging fast rotating wheel that is moving in a precessional path (Fig 11.3.2).

A number of vibrational machines have been proposed that seek to convert angular momentum in a closed system into a linear propulsive force; but these are not gyroscopic devices. One of the most famous is that of Norman L. DEAN US 2886976 (1959). His machine provides a system for converting rotary motion into a unidirectional motion and this is achieved from the oscillatory movement of a freely suspended inertial mass. The machine has three precursors, that of GODITIABOIS and PIRLOT Italian 379821 (1939). NOWLIN A.C. US 2350248 (1944) and GEYER H.M. US 2700542 (1955).

A later machine that may wrongly have been placed in this vibrational group is that of DIBELLA A. US 3404854 (1968). We are indebted to LAITHWAITE E.R. iii (1975) for an exposition of the gyroscopic nature of the machine.

DiBella states that in January 1962 he proposed to initiate a study on the rotatory movement of a mass in space, to see if the dynamic actions produced by it could make way for possible applications in the field of propulsion. He decided to begin by considering the rotatory motion of a mass around a point.

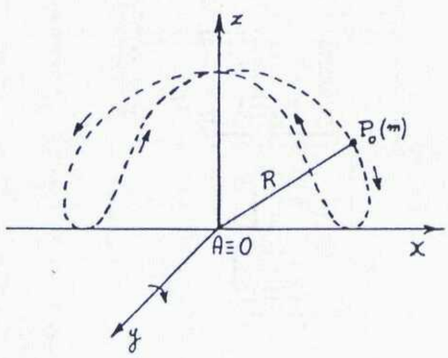


Figure 11.3.3
Rotatory motion of a mass around a point.

The device indicated in Fig 11.3.3 immediately appeared useful to his study. It executes the motion of a point on a hemisphere. With simple mechanisms, it was possible to have an arm AP = R, rotating around a point 0, having the extremity A coincide with 0, and the extremity P free to move on the hemisphere. A mass m was concentrated in P.

As a matter of interest, he recalls that the trajectory described by P belongs to the hypopedes family,° studied in astronomy by Eudoxus, a contemporary of Plato. More precisely, the trajectory represents the window of Viviani, a pupil of Galilei, who posed the problem of tracing four windows of maximum area on a hemisphere. (the solution of the problem, given by Gauss, requires that each window have as its contour the trajectory described by P, which is also the intersection of a hemisphere with a cylinder of circular section, having the ray of the sphere as its diameter.)

The device was tested extensively on the ground and on the surface of the water with satisfying results on the whole.

The DiBella machine is shown in Fig 11.3.4. DiBella has stated, not withstanding his high abilities, that he is unable fully to explain the modus operandi of the device. No one present at the 7th Symposium on Naval Hydrodynamics in Rome, where DiBella presented his views, and exhibited his machine was able to make any constructuve comment; indeed the solitary comment in the discussion was from Professor M. POREH who made a reference to the 'polarized acceleration' exhibited by the Mexican jumping bean.*

On a rotatable shaft 9 is fixedly mounted an arm 12 to the free end of which is fixedly attached a weight 13. The arm 12 and the weight 13 together define a mass m. A frame 4 is mounted on base plate 2 and is provided with bores 7 and 8 through which the shaft 9 passes and which act as bearings for the rotation of the shaft 9.

A motor 1, which may be battery-powered for example, rotates a shaft 3 upon which is mounted, for rotation with the shaft, the frame 4. The shaft 3, which continues on the other side of frame 4, is rotatably mounted on bearings 5 and 6, bearings 5 and 6 being mounted on the base plate 2.

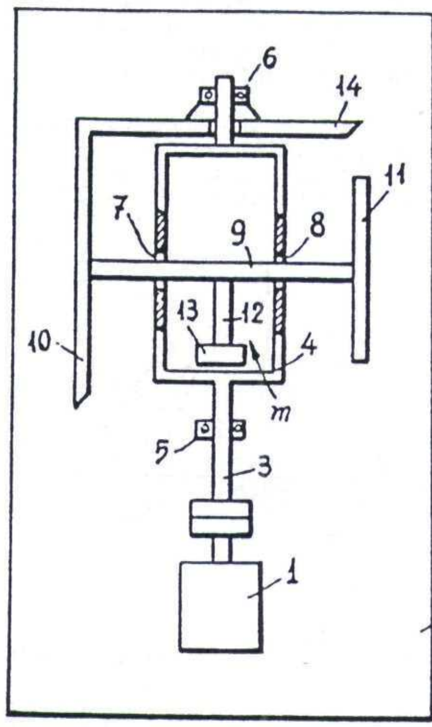


Figure 11.3.4

° Hypopedes. Hippopede (horse fetter) eight-shaped curve of Eudoxus, see Neugebauer O. (1957) 2nd Ed. p182-183.

* Mexican jumping bean; the seed of interalia the shrub Sebastiania that contains larvae of the moth Carpocaspa salitans.

Aug. 31, 1965 H. D. KELLOGG, JR **3,203,644**

GYROSCOPIC INERTIAL SPACE DRIVE

Filed Jan. 5, 1961

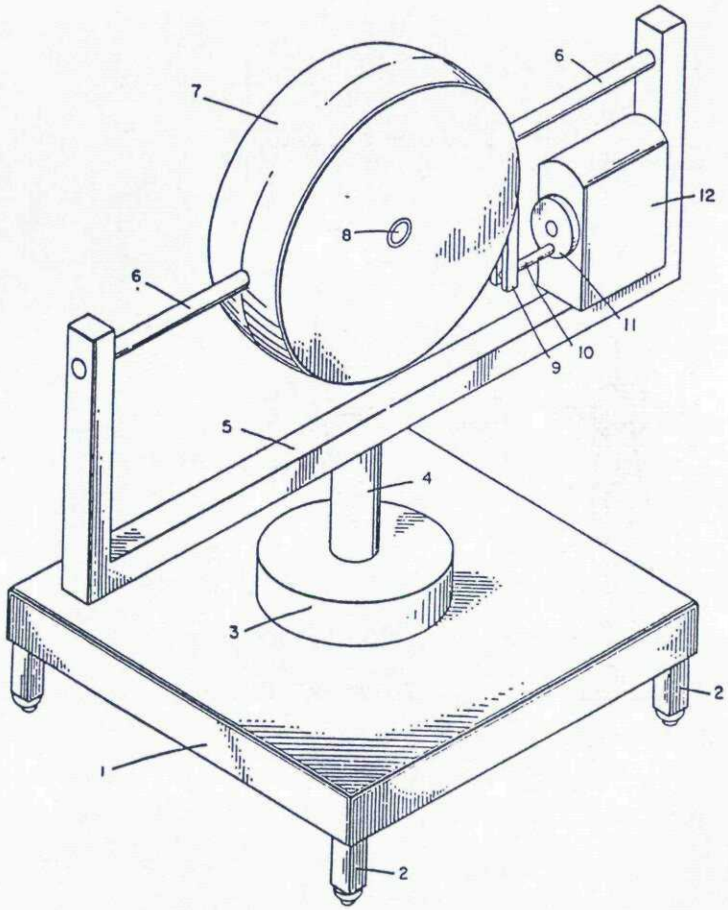


Figure 11.3.5

GB 2 090 404 A

Inventors
Geoffrey Colin Russell

United Kingdom (GB)
Application published
7 Jul 1982

A gyroscopic propulsion system

(57) A gyroscopic propulsion system comprises a series of hollow spheres (e, c, f, g) rotatably mounted one within the other. A series of three equi-angularly, spaced gyroscopes (g1) are located in a vertical plane around the inner surface of inner sphere (e) which is rotated about a horizontal axis. The gimbals of the gyroscope (g1) can be locked in sequence to produce gyroscopic precessional torque impulses for driving a craft, in which the system is located, in horizontal directions. A further series of gyroscopes (e2) are located at equi-angularly spaced locations in an annular chamber fixed externally to sphere (e). These gyroscopes are also mounted in gimbals which can be locked to produce further torque impulses for moving the craft in vertical directions.

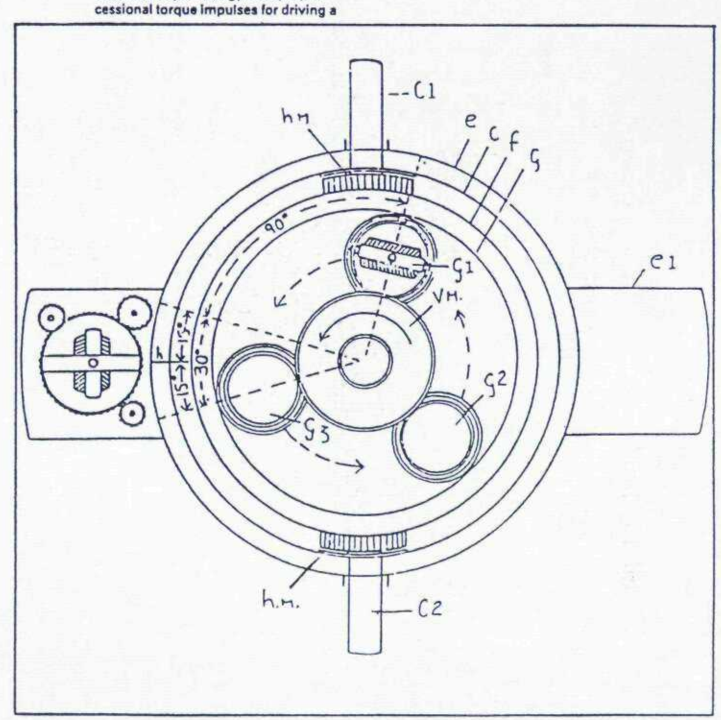


Figure 11.3.6

Jan. 19, 1971 H. W. YOUNG, JR 3,555,915

DIRECTIONAL FORCE GENERATOR

Filed Dec. 11, 1967

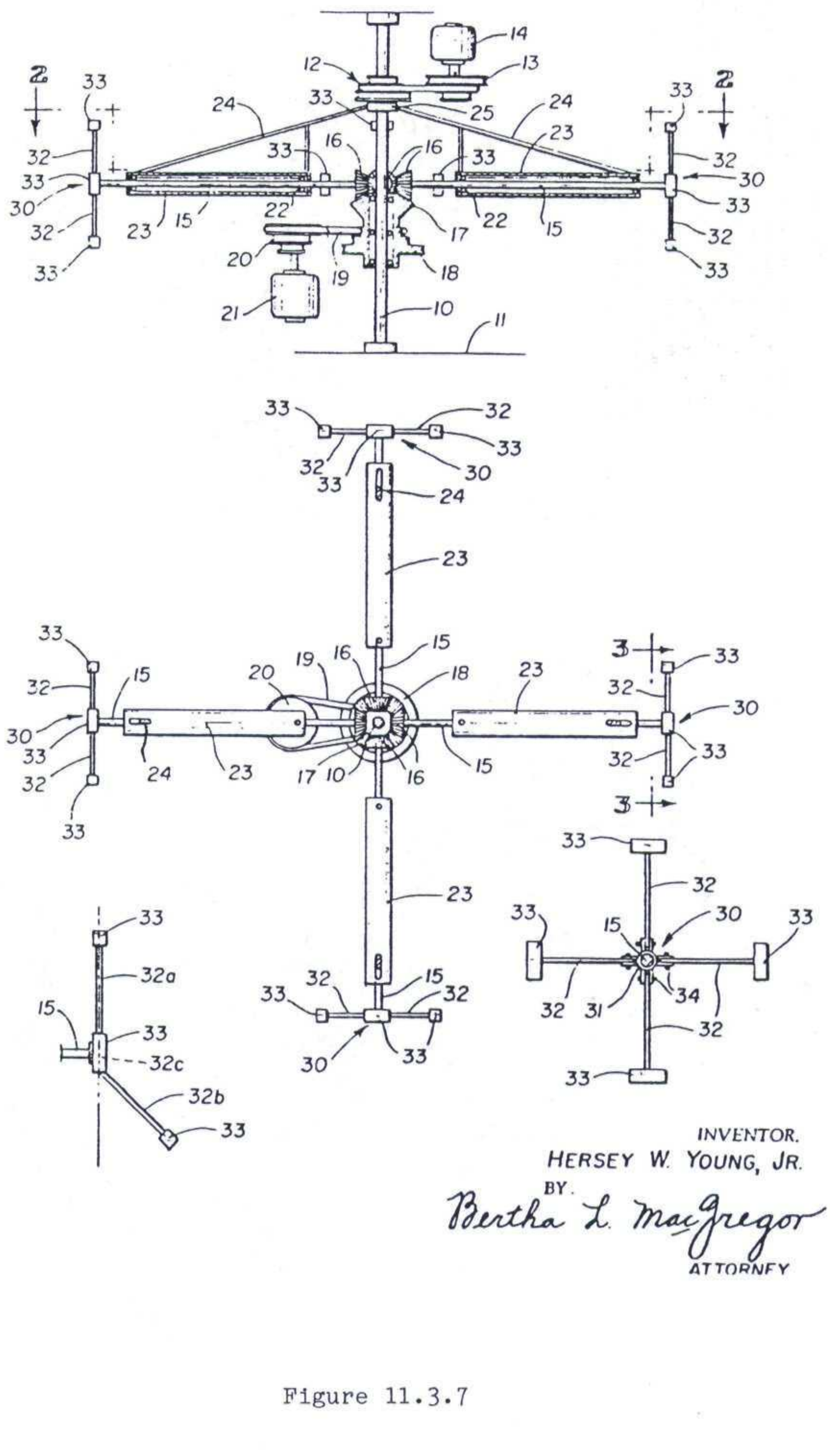


Figure 11.3.7

At one end of the shaft 9 is mounted a gear 10 and at the other end of the shaft 9 is mounted a balance weight 11, in the configuration of a disc, to balance the gear 10. On the base plate 2 near the free end of the shaft 3 is fixedly mounted the gear 14, which messes with the gear 10.

When the motor 1 rotates the shaft 3, the frame 4, being attached to the shaft 3, rotates with the shaft 3. the frame 4 carries with it in rotation about the axis of the shaft 3 the shaft 9. The meshing of the gear 10 with the gear 14 causes the shaft 9 to rotate about its own axis as it rotates about the axis of the shaft 3. Thereby, as the shaft 9 is rotated about the axis of the shaft 3, the mass m is rotated about the axis of the shaft 9 at a fixed distance from the axis of the shaft 9 since it is fixedly mounted on the shaft 9.

The only precursor of the DiBella machine known to the U.S. Patent Office is that of MME. PREVOT FRENCH 1063784 (1954) but as far as I am aware it is a paper proposal only and is a purely vibrational machine. A number of similar proposals appealing to gyroscopic forces span the years from 1928 to 1981 and these are shown in Figs 11.3.5 to 11.3.7.

An early gyroscopic inertial drive made by Laithwaite et al is shown in Fig 11.3.8. The latter was exhibited at the Royal Institution in 1974 and was the subject of much discussion. A full disclosure of the machine per se by Mr. Peter Green on behalf of Professor E.R. Laithwaite was given in the pages of Research Disclosure. I was responsible for writing the disclosure and I give it below in full:-

Unidirectional force generating device.

This disclosure relates to a unidirectional force generating device that makes use of a spinning rotor that is also able to precess.

It is known that a freely suspended spinning rotor will, when subjected to a torque, precess in a path in a precessional plane at right angles to the plane containing the couple that produced said torque and that the rate of precession which is usually expressed as an angular velocity in radians/sec is equal to the said torque divided by the moment of inertia of the rotor times the angular velocity of spin of the rotor - which rate of precession is hereinafter termed the natural rate of precession of the rotor.

As is known the natural rate of precession for any spinning rotor is determined solely by its moment of inertia, its angular velocity of spin hereinafter termed its rotation and the torque applied to it.

If now the couple producing the natural rate of precession is allowed to act about a suitably placed member, such as for example a pivoted arm carrying the rotating rotor so that it is overhung, then the centre of gravity of the rotor will be revolved about the pivot of the arm in an orbital plane hereinafter called the natural precessional plane.

According to the present disclosure a unidirectional force is generated by driving an overhung rotor, that is rotating on its axis and that is precessing at its natural rate of precession such that for a part of the precessional movement the rotor moves out of its natural precessional plane and said rotor is then forcefully moved in a direction back toward its natural precessional plane.

The drive may be such that the angular velocity of the revolution of the rotor is in excess of its natural precessional rate, or a torque may be applied to the rotor to cause it to move out of its natural precessional plane.

In the Figure, two overhung rotors A1.A2 are shown each rotating (arrow r_1, r_2) about axis X_1X_2. Consider now the conditions that would obtain within the system if a synchronous motor (not shown) on shaft S_1 were to be de-energised and its electric rotor able to free-wheel; then by virtue of the rotation of the rotors and the force of gravity (arrow G) downward, a precessional force would revolve the rotors in the direction of arrows R_1, R_2 and the rotors, if friction is ignored, would persist in a steady precessional motion in the natural precessional plane Cp.

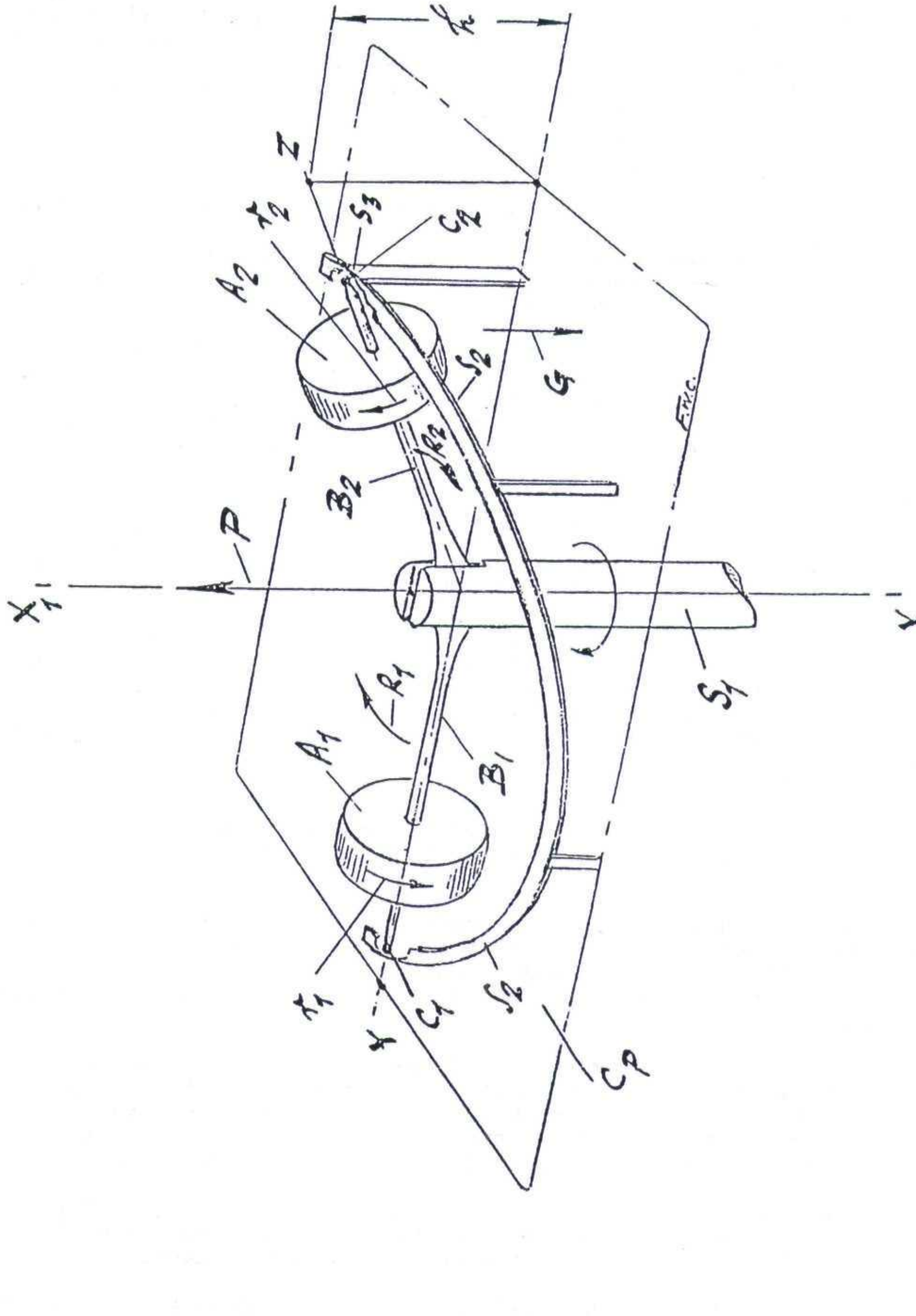


Figure 11.3.8

If now the synchronous motor is energised and its electrical rotor on shaft S1 made to revolve the radial arms B1, B2 of rotors A1, A2 at a speed in excess of the natural precessional speed of revolution, then the arms of the rotors will rise. Let the cam C_2 now contact a reaction member or surface S2 at a point S3. If now the cam C_2 is made to take a path on the inner surface of the reaction member such that the centre of the gravity of the rotor A_2 for example is made to descend along a preferably sinusoidal or similar arc to give ideally a constant or sinusoidal acceleration of descent from point Z to Y, then the rotor A2 will descend from a height h at Z to zero height at Y and the reaction surface S2 will move with the pivot S1 attached to it in the opposite direction, that is to say in the direction of arrow P, against the force of gravity G. At the same time the cam C1 of rotor A1 moves out of contact with the surface S2 and being freed from any constraint rises to allow C_1 to meet the surface S2 at S3, while the rotor A_2 now leaves the surface S2 at Y. Each time a rotor leaves the natural precessional plane Cp it is forced back into that plane and a continuous thrust is thereby produced in the direction of arrow P.

The rotors need not be journalled in the arm as shown in the figure but may be placed in other planes, or indeed have their spin axes skewed to the axis of revolution so that they lead or lag on the arm.

Clearly more than two rotors may be deployed and phased to give a mass rate of descent that is an optimum.

Such a force generating device is one which, if placed in a vehicle freely suspended in a field of force, such as a gravitational field of force would counteract said force. Clearly, if said counteracting force was of sufficient magnitude it would propel the vehicle continuously in a straight line in opposition to said field of force and would constitute an anti-gravity device. Surprisingly the device relies for its modus operandi, on reaction to dynamic forces generated solely by a plurality of simultaneously rotating and revolving rotors, an energy release arrangement that is unusually powerful as is exhibited in the bursting of a flywheel.

It is to be made clear that such a force generating device is able to produce a force in any direction and that the force of gravity can be produced, where none exists, by the use of inter alia springs or torque motors. Such a force generating device may be used to lift and propel a vehicle such as a rail car, or propel a boat wholly from within the hull thus making no contact with the water, it may also operate to propel a space vehicle.

A gyroscopic propulsive device that I know to have worked is that of RICKMAN. E.J.C. (G.B. Patent Specification 1479450).

As shown in Fig 11.3.9 it comprises two inclined discs or rotors, both rotating in the same direction and the whole structure is also rotated in the same direction as that of the rotors. The principle of operation is said to depend upon the differential movement in space of the relative outer part and inner part of the periphery of each disc 4 and 7. The outer edge of each disc is moving faster than the inner edge of the disc at all times when the disc is itself being rotated in the same direction as it is being turned with axle 1 about the axis of axle 1. When the discs 4 and 7 and the axle 1 are rotated in the same direction and at the same angular velocity, any point on the periphery of a disc describes a hyperbola. The speed of this point, relative to a point which is stationary with respect to both the discs and the axle 1, varies from substantially zero at the point closest to the axis of rotation of the axle 1 to a maximum at the outermost point furthest from the axis of axle 1.

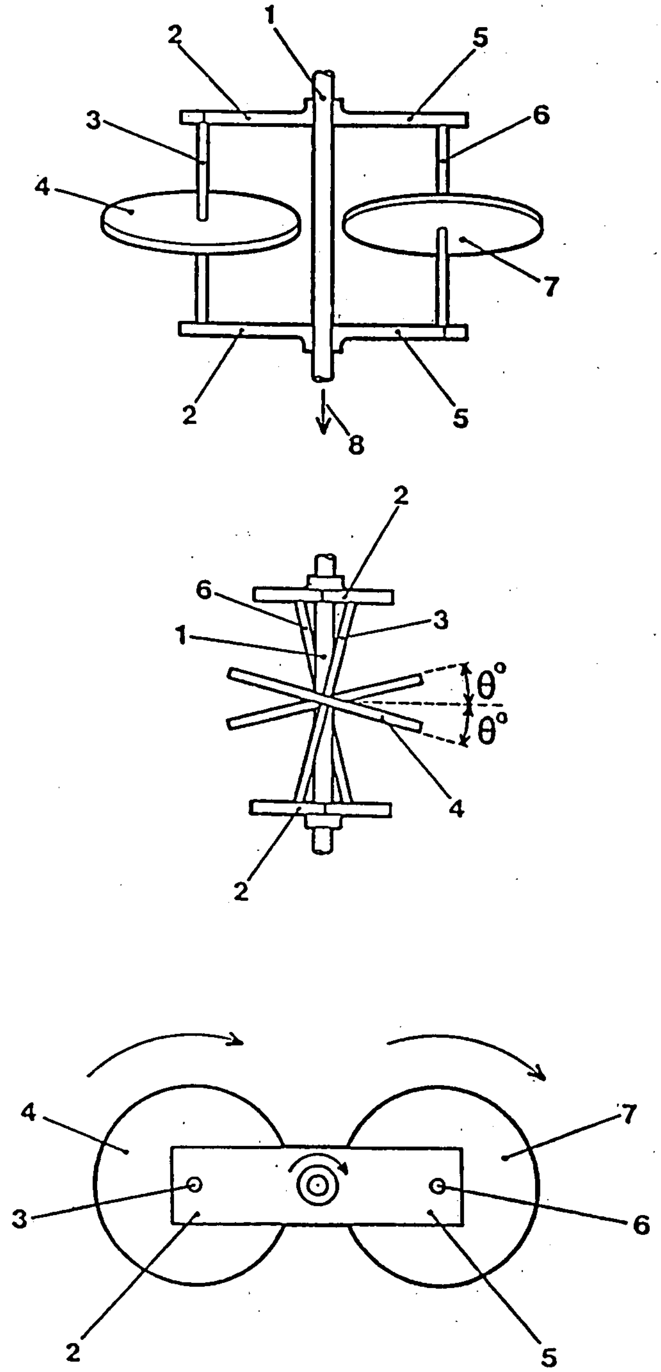


Figure 11.3.9

With the discs inclined as already described, the outer edge of each disc, when travelling at maximum speed, moves not in a plane perpendicular to the common axis, but in a direction crossing this plane at the angle 0. The reaction from the motion of the fastest moving point of each disc imparts a thrust aligned with the axis of rotation of the axle 1 and acting in a direction opposite to that of the fastest moving part of the inclined disc, i.e. in the direction of arrow 8 with the example described.

Greater thrust may be achieved by disposing more discs around the axle 1 each crossing during rotation the same plane at right angles to the axis of rotation of the axle 1 and also by stacking further pluralities of rotatable discs along the length of the axle 1.

To control the thrust produced, the discs may be rotated at a predetermined angular velocity with the axle 1 stationary, and the axle 1 accelerated as thrust is required to be developed.

Two machines were made by RICKMAN, a small prototype machine and a larger machine with rotor discs of about 10 inches (254 mm) diameter. The larger machine behaved with considerable violence and destroyed a part of Mr. RICKMAN'S bungalow at Langham in Colchester.

Now follows a brief discussion of two machines that have taken-up much of my time[xx] over the past fourteen years. The machine of JONES A.C. (1975) and the machine of Laithwaite E.R. (1986).

The Jones' machine is well illustrated in Fig 11.3.10. this machine was shown to Professor Eric Laithwaite in 1973 and the professor was good enough to swear an affidavit to the machines' performance before a Notary Public on 20th Oct. 1973 in my presence. The affidavit was subsequently presented to the German Patent Office since the German patent application had fallen into the hands of German examiners who regarded it under the broad heading of a perpetual motion device and therefore not open to patent protection per se.°

The Jones' patent application, however, went much further than the machine of Fig 11.3.10, it showed a machine as illustrated in Fig 11.3.11 where the rotor rotational axes R_1R_2 met at point 0. on the line of linear propulsion P_1P_2.

xx I wrote the original English specification that formed the basis of Offenlegungsschrift 2341245 in 1973 and I exhibited the JONES' machine before the German Patent Office in Munich with Mr. Jones and Her. Jurgen Weisse in late April 1974. I wrote the International Patent Specification WO86/05852 for Professor Laithwaite in 1985.

° The German law contained a provision similar to the British Patent Act of 1949-1961 which under section 10(1)a, excluded perpetual motion machines; now under the new British act of 1977 and the European Patent Convention Article 52 all inventions have to be 'susceptible (capable) of industrial application'. The locus classicus on the subject of Perpetuum Mobile is to be found in the work of DIRCKS.H. (1861).

An unchanged reprint of the original edition was published in 1968 (2 vols) by N.V. Boekhandel and Antiquaraat. B.M. Israel, Amsterdam. PERPETUUM MOBILE or search for self-motive power.

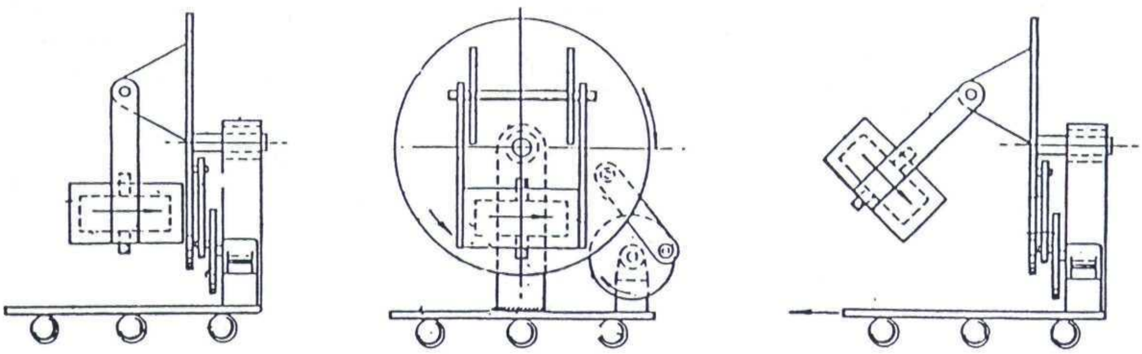


Figure 11.3.10

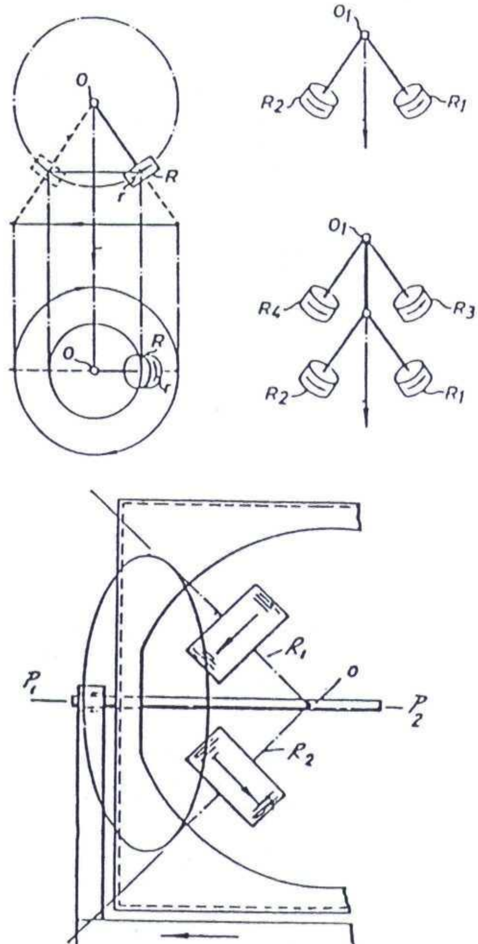


Figure 11.3.11

It was this feature essentially that was taken-up by the Space Division[x] of British Aerospace (as they now are) when they examined one of the Jones' machines. Their mathematician pointing out that the choice of axis system is of fundamental importance. The system he found, most suitable from his understanding of Jones' hypotheses being shown in Fig 11.3.12.

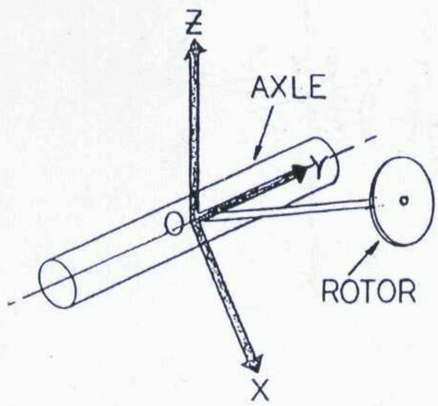


Figure 11.3.12

The axes OXYZ move with the body, such that OZ is always vertical, OY is along the axle, and OX completes a right-handed set, and is normal to the axle.

OX, OY are always in the horizontal plane.

The axes are free to translate in the horizontal plane, and are able to rotate about OZ.

The conclusions reached from the mathematical investigations were stated as follows, and went against the acceptance of the Jones' machines:-

> A mathematical analysis has been made of the above mechanism designed by Mr. A. Jones to covert angular momentum into linear momentum. The applications of such a mechanism would be numerous, particularly in a space environment.
>
> Intuitively, the laws of classical mechanics deny the possibility of such a mechanism working. This paper presents the results of an analysis using classical mechanics and shows that theoretically the mechanism does not produce a net change in linear momentum.
>
> As is typical of this type of mathematical problem, it is one step in the solution to generate a series of differential equations to describe the system, but a separate, more complex, step to obtain a solution giving the position and orientation of the body as explicit functions of time. An attempt was made to obtain the solution using a Runge-Kutta Numerical Integration Routine* in order to compare the motion with traces obtained during tests with the 'space drive'. Difficulties with the stability of the integration preclude the possibility of including theoretical traces with this paper.
>
> The mathematics of the problem have been arranged such that it can be seen that there is no movement of the centre of mass of the mechanism during the motion.
>
> If a similar mechanism was manufactured and tested under more stringent specifications than the prototype already seen by the Company, and this demonstrated a net change in linear momentum, recourse would have to be made outside accepted theories of classical mechanics to account for the motion.

x Hawker Siddeley Dynamics

* The method of Runge-Kutta - see Sneddon. I.N. (1976) p.182.

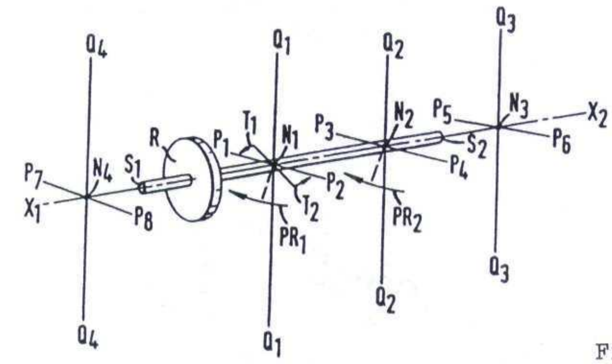


Fig.A

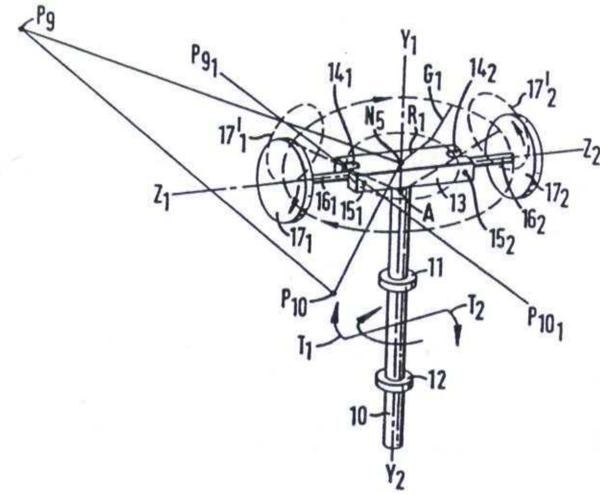


Fig.B

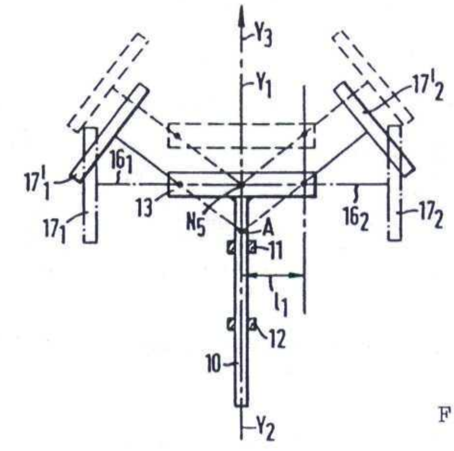


Fig.C

Figure 11.3.13

The machine of Laithwaite to my mind resolves the difficulty and the essential ingredient for success is clearly stated - it is to arrange the rotating mass such that it is denied its true precessional point and the mass is thereby constrained to translate the said point to a position where the precession can occur. The Laithwaite machine is fully described below (See Fig 11.3.13):-

Fig A is a schematic representation of a rotatable disc mounted to allow precession;

Fig B is a perspective view of a thrust-producing device in accordance with the invention;

Fig C is a schematic view of the device of Fig B;

Referring initially to Fig A a rotor disc R has a centre mounted for spinning about axis X_1X_2 on a shaft S_1S_2. Let us first consider the shaft S_1S_2 as being capable of pivoting about vertical axis Q_1Q_1 and horizontal axis P_1P_2. The disc R is spun and the shaft S_1S_2 carrying it rotates, i.e. precesses, in the direction PR_1 about the axis Q_1Q_1, this precession being initiated by a torque. When a torque T_1T_2 is applied to the shaft in the rotational sense of precession about axis Q_1Q_1, the shaft rotates about P_1P_2. Precession is taking place about N_1 i.e. about the intersection of the axis of rotation of the shaft, the axis about which the torque is applied, and the axis about which the shaft can precess.

If, therefore, the torque axis is transferred to the point N_2 on the intersection of Q_2Q_2 and P_3P_4, and X_1X_2 and the torque applied as before about Q_2Q_2 then the shaft will precess about N_2 pivoting about P_3P_4.

This principle holds good even if torque is applied about points N_3 and N_4 spaced from the axle.

From the above it can be perceived that an axle will precess about an axis orthogonal to both the axis of rotation and the axis about which the torque is applied at the point where these two axes intersect.

An embodiment of the invention is illustrated in Fig B.

In Fig B a propulsion device of the invention comprises an axle rod 10 with a longitudinal axis Y_1Y_2 and is arranged to be able to rotate in high quality low friction bearings 11, 12. These bearings permit longitudinal movement to allow thrust produced to develop a working stroke. The axle 10 carries at one extremity an orthogonally disposed cross member or support 13 rigidly fixed to axle 10 by welding (not shown). The cross member 13 has slots 14_114_2 and pivots 15_115_2 that are normal to the orthogonal line Z_1Z_2 of the cross member 13.

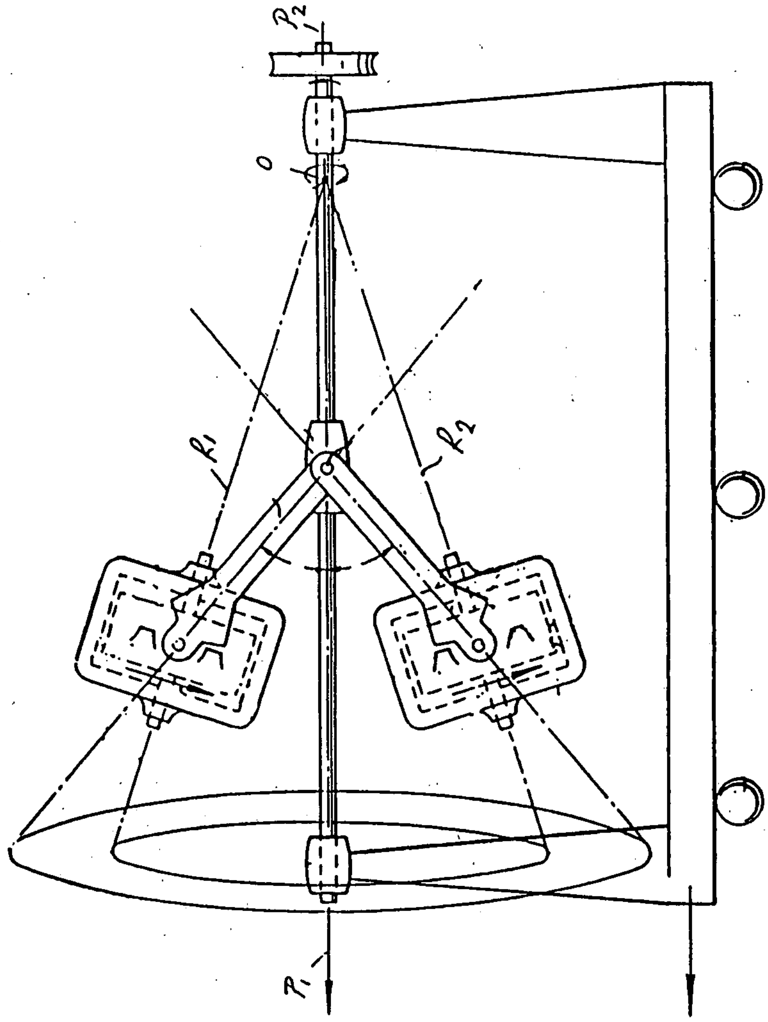


Figure 11.3.14

To these pivots $15_1 15_2$ are attached axles $16_1 16_2$ respectively terminating in free running rotor discs $17_1 17_2$ that spin in opposite directions as shown by arrows on the discs <u>per se</u>. If a torque T_1T_2 is now applied to the axle 10 then, since the axis $\overline{Y_1Y_2}$ about which the torque is applied cuts the axes Z_1Z_2 of both rotors, $17_1 17_2$ in point N_5, precession should take place about an axis through N_5 in the plane N_4 P_{9_1} P_{10_1} where P_{9_1} P_{10_1} is parallel to axis P_9P_{10} through pin 15_1. (It is the same <u>mutatis mutandis</u> for pin 15_2); but the construction is such that this is <u>not</u> possible without a movement of the whole device in the line Y_1Y_2 such that the point N_5 is made the apex of a frustro-conical volume shown by the dotted circles, the radius R_1 and as the generator G_1; the rotors $17_1 17_2$ rising (or lowering) as the resultant forces dictate to the dotted positions shown at $17_1 17_2$ to bring said apex A coincident with N_5. The identical integers of Fig B with identical references are shown in Fig C for greater clarity. The movement of the whole device is along Y_1Y_2 in this instance in the direction of arrow Y_3. If the motion along Y_1Y_2 is resisted by frictional forces or by gravity which generally is to be expected, the axle 10 will rotate under the applied torque, work will be done and an energy balance produced. It is to be appreciated, however, that in the form of the device shown in Figs B and C when the rotors $17_1 17_2$ move to positions $17'_1$ and $17'_2$ the cross member 13 moves <u>not</u> as is to be expected, in opposition to arrow Y_3, but with it in the <u>same</u> direction and matter is moved without reaction and a propulsion device is established.

To provide for continuous propulsion a multi-phase device is essential with a separation of the phases as in an analogous electrical machine.

In Fig C, it can readily be seen that if the distance l_1 be increased to l_2 then the apical distance of the fulcrum is increased from $\underline{a_1}$ to $\underline{a_2}$. Similarly if the torque T_1T_2 is increased in amplitude then the angle ${}_1$ will increase to ${}_2$.

The essential feature enunciated by Laithwaite can, I think, be seen to reside in the Jones' machine of Fig 11.3.10, hence its success in producing linear motion and presented by Jones, without adequate explanation, in his Fig 11.3.14.

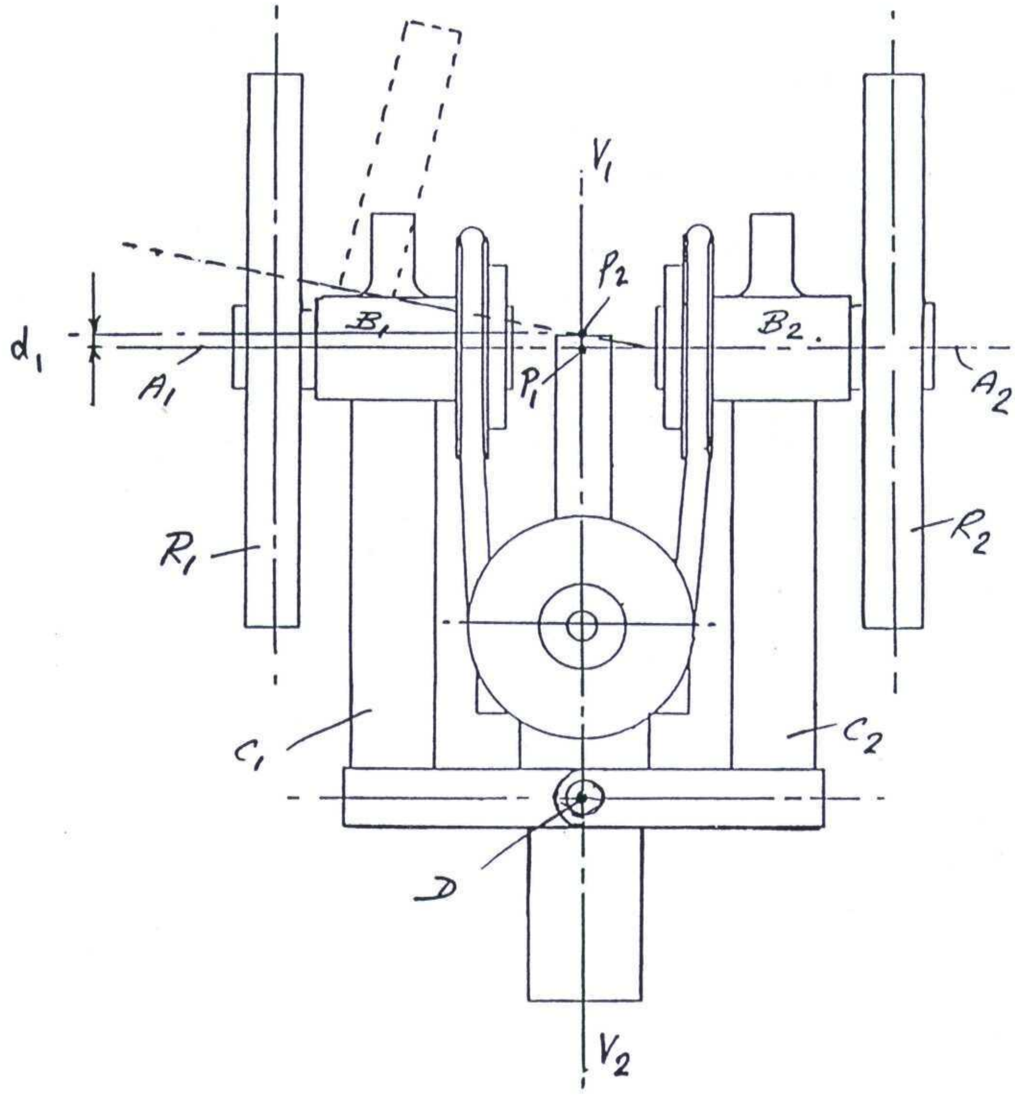


Figure 11.3.15

The latest gyroscopic apparatus said to produce a pulsatile* force is that of KIDD A.D. GB Patent Application 8629405. The apparatus has been tested at Dundee University and is now the subject of tests by an Australian Company (BNW) under the supervision of Mr. Kidd. The invention set out in the G.B. Patent Application is now the subject of two International Patent Specifications WO88/04363/4 published on the 16th June 1988.

The Kidd gyroscopic device was first demonstrated to the public in the GRAMPIAN television programme 'The Man who wants to change the World.' 21st May, 1987; later on 7th January, 1988 it was shown to a wide audience on CENTRAL Television.

Kidd's basic apparatus is shown in Fig 11.3.15, the essential feature enunciated by Laithwaite, and referred to above can again, I think, be used to explain the gyroscopic thrust, albeit very small. When the rotors $A_1.A_2$. move as shown in the figure for A_1 alone to the dotted line position A_3, then the change of point P_1, the true precessional point, to point P_2, a distance d_1, will produce a force along $V_1.V_2$. Figs. 11.13.16 and 11.13.17 show the later developments of the apparatus.

Suprisingly the experts at Dundee University believe the machine to move from vibrational forces. If this is so, the Kidd machine may be exhibiting forces demonstrated by DEAN US 2886976 (1959) and shown in Fig. 11.13.18, in which inward and outward swinging inertial masses may produce "a practically continuous lift". Conclusive tests are awaited.

According to the Sunday Express 23rd October, 1988 the Kidd apparatus is able to make a trip to Mars in 34 hours and experiments will now be made in California. Dr. Harold Aspden, senior visiting Research Fellow at Southampton University, has seen the results of early tests. "Scientifically speaking it is a bombshell" he says. "I would not have believed this if I had not seen it with my own eyes. It will totally revolutionise the travel industry. Taken to the ultimate, we will have planes without jet engines and helicopters without rotor blades". Personally, I find this rather hyperbolic.

A search of the literature discloses that similar devices have been proposed by:-

Louis KEKS	Australian 282020 (1966)
Toussaint SALINI	French 1486488 (1967)
A.C. JONES	German 2341245 (1975)
Alain SIRITZKY	French 2293608 (1976)
- GROSSMAN	German 2430605 (1976)
E.J.C RICKMAN	GB 1479450 (1977)
NORIYUKI IRIE	Japanese 56-106076 (1981)
Harald STANGER	German 3307298 (1984)
JIYUNICHI CONO	Japanese 60-56182 (1985)
TAKESHI FUMOTO	Japanese 62-103486

and

P. GREEN RESEARCH DISCLOSURE, No.140 (December 1975)

Finally, I return full circle to the prophetic statement of Telsa; for experiments are now proposed and a new machine is in existence to test a revolutionary type of gyroscope that may be able to tap the rotational energy in space and demonstrate the exotic and exciting phenomenon of inertial radiation.+

* Pulsatile (pulsatory) a term now almost obsolete; used from 1541 to 1872 in medical and musical fields of interest.

\+ Gunn. S. Professor's++ invention finds no room in Britain. The Times. London. Saturday 1987 December 18th (1987) p.2

++ Professor Eric Laithwaite

A gyroscopic apparatus (100), having application as a prime mover, comprises a pair of discs (102) disposed opposite one another with arms (104) rotatably supporting the discs (102) connected at a pivot point, the pivot axis therof lying in a plane midway between the discs (102). A drive arrangement (124, 126, 180) operates to spin the discs (102) in opposite directions whilst simultaneously rotating the whole assembly of discs (102) and arms (104) about a second axis in the same plane as, but perpendicular to, the pivot axis. A camming arrangement (144, 146, 148, 152) working in conjunction with the rotation about the second axis periodically forces the spinning discs (102) to pivot about the pivot axis to thereby generate a force along the second axis which can be used to perform useful work.

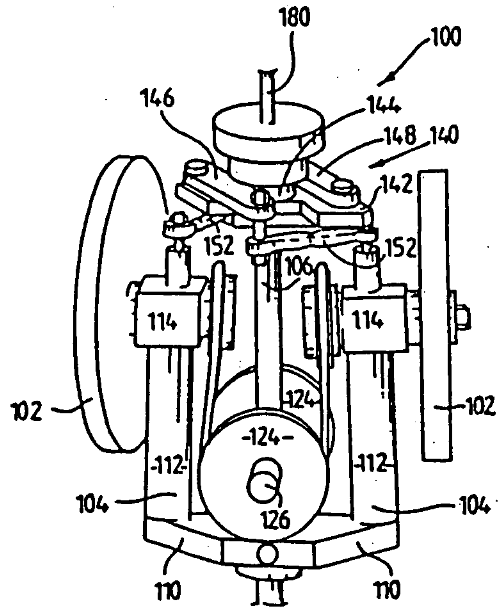
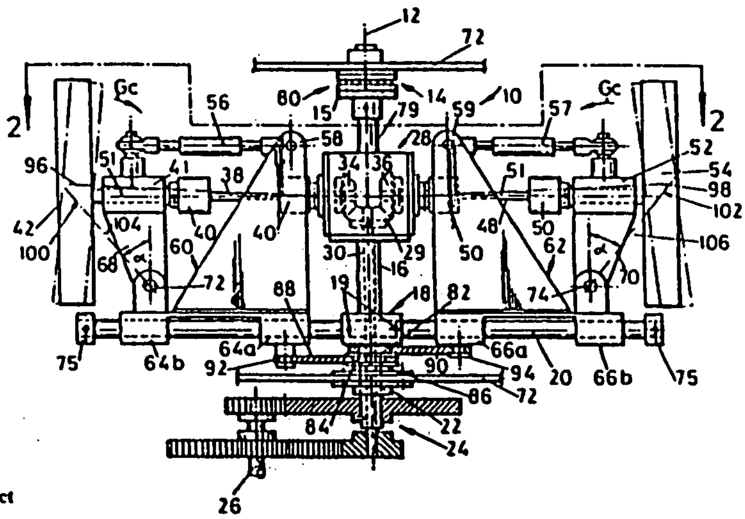


Figure 11.3.16



(57) Abstract

A gyroscopic thrust apparatus which releases rotational energy of gyroscopic mass along the axis of precession via a gyroscopic couple, to generate thrust. In particular, the gyroscopic thrust apparatus has two gyroscopic diametrically opposed discs (42, 54) adapted to be rotated in a vertical plane about common horizontal axis, but rotated in different directions. The discs are pivotably mounted on shafts (38, 48) and supported on a frame (60, 62) which is coupled to drive means (24) for driving the gyroscope about a spin or precession shaft axis (12). As the gyroscope rotates the disc (42, 54) also rotate and, as speed of rotation increases, the gyroscopic movements overcome the centripedal force and the discs (42, 54) lift to transmit an upward force along the spin shaft (79).

Figure 11.3.17

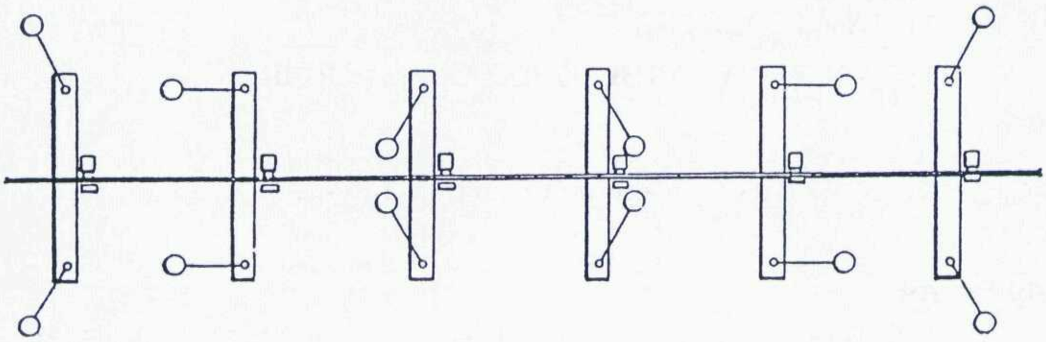


Figure 11.3.18

On going to press I have been made aware of an experiment by Hayasaka and Takeuchi which cannot be explained by the usual theories and which was reported by Pease, R. in 'The Times', January 2, 1990, under the title: Gyroscope Study That Will Put Newton and Einstein in a Spin.

The weight change of each of three spinning mechanical gyroscopes the rotor masses of which are 140, 175, and 176 g has been measured during inertial rotations, without systematic errors. The experiments show that the weight changes for rotations around the vertical axis are completely asymmetrical: The right rotations (spin vector pointing downward) cause weight decreases of the order of milligrams (weight), proportional to the frequency of rotation at 3000—13000 rpm. However, the left rotations do not cause any change in weight. The weight change for the 175 g rotor was a decrease of 0.91 mg per thousand rpm increase in rotation rate and that of the 140 g rotor, 0.59 mg.

See: Hayasaka, H. and Takeuchi, S. *Anomalous Weight Reduction on a Gyroscope's Right Rotations around the Vertical Axis on the Earth.* Physical Review Letters. 63 No.25. (1989) p.2701—2704, and Salters, S.H. *Nature.* 343 (1990) p.509.

REPORT DOCUMENTATION PAGE

1. Recipient's Reference	2. Originator's Reference	3. Further Reference	4. Security Classification of Document
	AGARD-AG-313	ISBN 92-835-0549-2	UNCLASSIFIED

5. Originator Advisory Group for Aerospace Research and Development
North Atlantic Treaty Organization
7 rue Ancelle, 92200 Neuilly sur Seine, France

6. Title THE ANATOMY OF THE GYROSCOPE — PART III

7. Prepared at the request of the Guidance and Control Panel of AGARD.

8. Author(s)/Editor(s)	9. Date
F.W.Cousins and edited by John L.Hollington	March 1990

10. Author's/Editor's Address	11. Pages
See Flyleaf.	298

12. Distribution Statement This document is distributed in accordance with AGARD policies and regulations, which are outlined on the Outside Back Covers of all AGARD publications.

13. Keywords/Descriptors

Coriolic effects	Gyroscope forces
Tops	Boomerangs
Laser gyros	General relativity
Inertial drives	Gyroscopic gears

14. Abstract

Prepared at the request of the Guidance and Control Panel of AGARD, this report is Part III of the report, Parts I and II of which were issued in February 1988. It gives historical notes including pre-cursors in the animal kingdom, nuclear physics and astronomy. It also gives a detailed consideration of early mechanical gyroscopes. It further includes inter-alia a discussion of gyroscopic gears, vibratory gyroscopes, interferometric gyroscopes and the use of the gyroscope in optical and related devices and deals finally with the difficult problem of gyroscopic inertial drives.

AGARDograph No.313
Advisory Group for Aerospace Research and Development, NATO
THE ANATOMY OF THE GYROSCOPE — PART III
F.W.Cousins and edited by John L.Hollington
Published March 1990
298 pages

Prepared at the request of the Guidance and Control Panel of AGARD, this report is Part III of the report, Parts I and II of which were issued in February 1988. It gives historical notes including pre-cursors in the animal kingdom, nuclear physics and astronomy. It also gives a detailed consideration of early mechanical gyroscopes. It further includes inter-alia a discussion of gyroscopic gears, vibratory gyroscopes, interferometric gyroscopes and the

P.T.O.

AGARD-AG-313

Coriolic effects
Tops
Laser gyros
Inertial drives
Gyroscope forces
Boomerangs
General relativity
Gyroscopic gears

AGARDograph No.313
Advisory Group for Aerospace Research and Development, NATO
THE ANATOMY OF THE GYROSCOPE — PART III
F.W.Cousins and edited by John L.Hollington
Published March 1990
298 pages

Prepared at the request of the Guidance and Control Panel of AGARD, this report is Part III of the report, Parts I and II of which were issued in February 1988. It gives historical notes including pre-cursors in the animal kingdom, nuclear physics and astronomy. It also gives a detailed consideration of early mechanical gyroscopes. It further includes inter-alia a discussion of gyroscopic gears, vibratory gyroscopes, interferometric gyroscopes and the

P.T.O.

AGARD-AG-313

Coriolic effects
Tops
Laser gyros
Inertial drives
Gyroscope forces
Boomerangs
General relativity
Gyroscopic gears

AGARDograph No.313
Advisory Group for Aerospace Research and Development, NATO
THE ANATOMY OF THE GYROSCOPE — PART III
F.W.Cousins and edited by John L.Hollington
Published March 1990
298 pages

Prepared at the request of the Guidance and Control Panel of AGARD, this report is Part III of the report, Parts I and II of which were issued in February 1988. It gives historical notes including pre-cursors in the animal kingdom, nuclear physics and astronomy. It also gives a detailed consideration of early mechanical gyroscopes. It further includes inter-alia a discussion of gyroscopic gears, vibratory gyroscopes, interferometric gyroscopes and the

P.T.O.

AGARD-AG-313

Coriolic effects
Tops
Laser gyros
Inertial drives
Gyroscope forces
Boomerangs
General relativity
Gyroscopic gears

AGARDograph No.313
Advisory Group for Aerospace Research and Development, NATO
THE ANATOMY OF THE GYROSCOPE — PART III
F.W.Cousins and edited by John L.Hollington
Published March 1990
298 pages

Prepared at the request of the Guidance and Control Panel of AGARD, this report is Part III of the report, Parts I and II of which were issued in February 1988. It gives historical notes including pre-cursors in the animal kingdom, nuclear physics and astronomy. It also gives a detailed consideration of early mechanical gyroscopes. It further includes inter-alia a discussion of gyroscopic gears, vibratory gyroscopes, interferometric gyroscopes and the

P.T.O.

AGARD-AG-313

Coriolic effects
Tops
Laser gyros
Inertial drives
Gyroscope forces
Boomerangs
General relativity
Gyroscopic gears

use of the gyroscope in optical and related devices and deals finally with the difficult problem of gyroscopic inertial drives.

ISBN 92-835-0549-2

use of the gyroscope in optical and related devices and deals finally with the difficult problem of gyroscopic inertial drives.

ISBN 92-835-0549-2

use of the gyroscope in optical and related devices and deals finally with the difficult problem of gyroscopic inertial drives.

ISBN 92-835-0549-2

use of the gyroscope in optical and related devices and deals finally with the difficult problem of gyroscopic inertial drives.

ISBN 92-835-0549-2

AGARD

NATO OTAN

7 rue Ancelle · 92200 NEUILLY-SUR-SEINE

FRANCE

Telephone (1)47.38.57.00 · Telex 610 176

DISTRIBUTION OF UNCLASSIFIED AGARD PUBLICATIONS

AGARD does NOT hold stocks of AGARD publications at the above address for general distribution. Initial distribution of AGARD publications is made to AGARD Member Nations through the following National Distribution Centres.Further copies are sometimes available from these Centres, but if not may be purchased in Microfiche or Photocopy form from the Sales Agencies listed below.

NATIONAL DISTRIBUTION CENTRES

BELGIUM
Coordonnateur AGARD — VSL
Etat-Major de la Force Aérienne
Quartier Reine Elisabeth
Rue d'Evere, 1140 Bruxelles

CANADA
Director Scientific Information Services
Dept of National Defence
Ottawa, Ontario K1A 0K2

DENMARK
Danish Defence Research Board
Ved Idraetsparken 4
2100 Copenhagen Ø

FRANCE
O.N.E.R.A. (Direction)
29 Avenue de la Division Leclerc
92320 Châtillon

GERMANY
Fachinformationszentrum
Karlsruhe
D-7514 Eggenstein-Leopoldshafen 2

GREECE
Hellenic Air Force
Air War College
Scientific and Technical Library
Dekelia Air Force Base
Dekelia, Athens TGA 1010

ICELAND
Director of Aviation
c/o Flugrad
Reyjavik

ITALY
Aeronautica Militare
Ufficio del Delegato Nazionale all'AGARD
3 Piazzale Adenauer
00144 Roma/EUR

LUXEMBOURG
See Belgium

NETHERLANDS
Netherlands Delegation to AGARD
National Aerospace Laboratory, NLR
Kluyverweg 1
2629 HS Delft

NORWAY
Norwegian Defence Research Establishment
Attn: Biblioteket
P.O. Box 25
N-2007 Kjeller

PORTUGAL
Portuguese National Coordinator to AGARD
Gabinete de Estudos e Programas
CLAFA
Base de Alfragide
Alfragide
2700 Amadora

SPAIN
INTA (AGARD Publications)
Pintor Rosales 34
28008 Madrid

TURKEY
Milli Savunma Bakanlığı (MSB)
ARGE Daire Başkanlığı (ARGE)
Ankara

UNITED KINGDOM
Defence Research Information Centre
Kentigern House
65 Brown Street
Glasgow G2 8EX

UNITED STATES
National Aeronautics and Space Administration (NASA)
Langley Research Center
M/S 180
Hampton, Virginia 23665

THE UNITED STATES NATIONAL DISTRIBUTION CENTRE (NASA) DOES NOT HOLD STOCKS OF AGARD PUBLICATIONS, AND APPLICATIONS FOR COPIES SHOULD BE MADE DIRECT TO THE NATIONAL TECHNICAL INFORMATION SERVICE (NTIS) AT THE ADDRESS BELOW.

SALES AGENCIES

National Technical
Information Service (NTIS)
5285 Port Royal Road
Springfield
Virginia 22161, USA

ESA/Information Retrieval Service
European Space Agency
10, rue Mario Nikis
75015 Paris, France

The British Library
Document Supply Centre
Boston Spa, Wetherby
West Yorkshire LS23 7BQ
England

Requests for microfiche or photocopies of AGARD documents should include the AGARD serial number, title, author or editor, and publication date. Requests to NTIS should include the NASA accession report number. Full bibliographical references and abstracts of AGARD publications are given in the following journals:

Scientific and Technical Aerospace Reports (STAR)
published by NASA Scientific and Technical
Information Branch
NASA Headquarters (NIT-40)
Washington D.C. 20546, USA

Government Reports Announcements (GRA)
published by the National Technical
Information Services, Springfield
Virginia 22161, USA

SPS

Printed by Specialised Printing Services Limited
40 Chigwell Lane, Loughton, Essex IG10 3TZ

ISBN 92-835-0549-2