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10. ADAPTIVE WALL TECHNIQUES

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LIST OF SYMBOLS

AR	*aspect ratio*
b	width of test section
c	model chord
c_L	lift coefficient
c_p	pressure coefficient
d_t , d_b	wall displacement of top and bottom wall, positive when directed outwards
d_s , d_a	symmetric and anti-symmetric part of the wall displacement
h	height of test section
M	Mach number of free stream flow
p_e	wall pressure distribution of the (fictitious) exterior flow, Figure 10.5
p_i	wall pressure distribution of the interior flow, Figure 10.5
s	semi-span of wing
U_∞	free stream velocity
u, v	in two-dimensional flows: disturbance velocity components in flow direction (u) and upwards (v)
u_t , u_b	disturbance velocity at top and bottom wall
u_s , u_a	symmetric and anti-symmetric parts of wall velocity induced by the model, Equations 10.5a&b
u_{ws} , u_{wa}	symmetric and anti-symmetric parts of wall velocity induced by wall deflection
w	$=u - iv/\beta$, complex flow velocity
u,v,w	in three-dimensional flows: velocity components streamwise (u), spanwise (v), upwards (w)
u_{int}	in two-dimensional flows: interference velocity component in flow direction
v_{int}	in two-dimensional flows: interference velocity component upwards
w_{int}	$= u_{int} - iv_{int}/\beta$
x, y	in two-dimensional flows: co-ordinates streamwise (x) and upwards (y), Figure 10.5
z	$= x+i\beta y$ co-ordinate in the complex plane
x, y, z	in three-dimensional flows: co-ordinates streamwise (x), spanwise (y), upwards (z)
α	model incidence
β	$=\sqrt{1-M^2}$ Prandtl-Glauert compressibility factor
θ	wall angle
Θ	$= \cos^{-1}(1-2x/c)$, non-dimensional co-ordinate
δ	wall displacement, positive when directed outwards
δ^*	boundary layer displacement thickness
$\delta\alpha$	correction to model incidence
δy	wall displacement for two-dimensional wall adaptation, Equation 10.4
Δ_{upw}	normalised upwash variation
ϕ	disturbance potential, Equation 10.1
Ω, Γ	influence functions defined in Equations 10.6 a&b
X, M, Λ, N	influence functions defined in Equations 10.7 a&b

10.1 INTRODUCTION

In the context of wind tunnel wall corrections, adaptive wall techniques may be described as procedures which contrive to manipulate and control the levels and gradients of wall interference present in the test section by making appropriate adjustments to the wall boundary conditions. In so doing, they provide a measure of control over the nature of any primary corrections and residual variations that would not otherwise be available. As the adjustments to the walls are usually made in response to the information provided by conventional wall interference assessment procedures, adaptive wall techniques may be considered as extensions to many of the algorithms described elsewhere in this document.

The origins of these techniques can be traced back to the mid 1930's, when the activities of a group of engineers and scientists at the National Physical Laboratory (NPL) in England led to the use of two flexible walls being advocated as a means for alleviating wall-induced blockage effects in the high speed testing of two-dimensional models. Subsequently, the test section of a of closed wind tunnel was converted for adaptive use by the installation of a flexible roof and floor and techniques enabling three-dimensional models to be tested at high subsonic reference Mach numbers were eventually developed. One of the adaptive wall tunnels at NPL was used extensively for over a decade, providing, amongst other things, valuable information to support the British war effort. However, its use fell into decline in the early 1950's, when it was realised that ventilated test sections offered a simpler means of testing through the speed of sound.

By this time, a number of other agencies had become involved in adaptive wall activities. Nevertheless, the advent and rapid development of transonic ventilated test sections marked the demise of the first era of adaptive wall research. Aside from the fact that the new ventilated facilities appeared to provide more effective mechanisms for preventing test section choking and alleviating the intensity of the reflections of shock or expansion waves from the tunnel walls, the principal reasons for interest being transferred away from adaptive wall techniques were as follows:

i. The computational power required to conduct rigorous assessments of the wall interference on-line was not available. The methods used to adapt the walls at NPL were based on empirical correlations derived from potential flow analyses and required little or no on-line computation: the validity of this approach became increasingly uncertain as the reference Mach number was raised towards unity.

ii. The operation of flexible-walled tunnels was cumbersome: wall profiles were adjusted manually via systems of screw-jacks and the static pressure distributions along each liner were monitored (by eye) on arrays of U-tube manometers. The lack of sophisticated analytical adaptation algorithms made the process of adapting the walls even more laborious: adjustments were made primarily on the basis of past experience and, even with the most experienced tunnel operators, it could, on occasion, take several minutes to derive the final wall settings.

Coinciding with the substantial advances that had been made in the fields of automation and computational technology by the end of the 1960's, adaptive wall techniques re-emerged as a potential means for alleviating a number of the concerns about contemporary wind tunnel testing practice that, with the ever-increasing demands placed on the quality of test data, were being scrutinised with renewed vigour. One of the principal motivations behind this development was the desire to reduce the uncertainties associated with the effects attributed to wall interference in transonic wind tunnel test data. Thus, unaware of the previous activities at NPL, researchers at various establishments set about the task of developing techniques that would minimise the effects of wall interference.

Considerable progress has been made towards achieving this goal in two- and three-dimensional testing. However, to date the vast majority of adaptive wall research has been conducted at the "proof-of-concept" level. Relatively little effort has been directed towards resolving the issues of more practical concern that will ultimately determine the cost of wall adaptation. Consequently, while adaptive wall techniques have been utilised productively and with some confidence in research environments for many years, they have yet to make a major impact on the procedures used for project-based production testing in large industrial wind tunnels.

Rather than attempting to present a comprehensive review of the principal developments that have occurred in recent years, or to offer explanations for the limited extent of their current utilisation, the purpose of this Chapter is to describe the most powerful and widely used adaptive wall techniques, explain their limits of applicability and provide a perspective on the priorities for future development. To this end, the underlying principles of wall adaptation for steady flows are reviewed briefly in Section 10.2, their application to the reduction of wall interference in two- and three-dimensional testing is described in Sections 10.3 and 10.4, further capabilities are reviewed in Section 10.5 and an outlook on the future is provided in Section 10.6. Sufficient information is also provided to enable potential users to construct techniques suitable for use in the subsonic testing of two- and three-dimensional models.

For further information, the interested reader is directed to the following works of reference: NASA Technical Memorandum 87639 (Tuttle and Mineck [39]), the latest edition of a bibliography on adaptive wall wind tunnels, AGARD Advisory Report 269 (Hornung, ed., [16]), which provides a catalogue of the activities surrounding adaptive wall technique development prior to 1990 and the "Proceedings of the International Conference on Adaptive Wall Wind Tunnel Research and Wall Interference Correction" held in June 1991 in Xian, China. The Adaptive Wall Newsletter (Wolf, ed., [44]) is a valuable additional source of otherwise unpublished information. The most recent attempts to review the state-of-the-art have been produced by Taylor and Goodyer [35], [36] and Taylor [37]. These publications provide a more comprehensive coverage of the results of experimental tests and more extensive citations of the original documents than those provided here.

10.2 THE RUDIMENTS OF ADAPTIVE WALL TECHNIQUES

10.2.1 THE AIMS OF WALL ADAPTATION

As the technology has matured, two different approaches to wall adaptation have evolved. These are distinguished by the nature of the objectives of wall adaptation and the manner in which its success is measured.

The first contrives to control components of the wall-induced perturbations to the flow throughout the test section in such a way as to enable the success of wall adaptation to be gauged purely from assessments of the admissibility of the adapted wall boundary conditions. Thus, if the wall boundary conditions can be shown to exhibit a direct correspondence with those of a streamtube in an equivalent unconfined flow, it may be inferred that wall adaptation has eliminated the effects of wall interference from the test section. The achievement of this goal over a suitably broad range of test conditions in practice is, of course, extremely unlikely. Nevertheless, the basic principle - often referred to as the principle of wall streamlining - has proved to be remarkably useful, particularly in the development of two-dimensional testing techniques. For the reasons described in Section 10.2.3.1, below, procedures which invoke this principle are referred to as interface matching techniques.

The second and more recently devised approach to wall adaptation contrives to control components of the wall-induced perturbations to the flow in more localised regions of the test section. The success of wall adaptation in these circumstances is gauged primarily from the extent to which the desired conditions have been attained in these targeted regions - although the admissibility of the adapted wall boundary conditions must also be verified. No attempt is made to control the flow away from these regions: it is merely hoped that by controlling the flow there, the magnitude of any unwanted residual perturbations in the immediate vicinity of each region will also have been reduced to acceptable levels. Procedures which adopt this approach to wall adaptation usually contrive to control the flow along lines passing through the test section. These are referred to as target line techniques.

In principle, the control of the test section flowfield afforded by wall adaptation may take any form the tunnel operator wishes. To date, it has been used primarily to minimise the effects of wall interference identified by established interference assessment procedures and thereby reduce the uncertainties associated with these features of wind tunnel testing. However, by intentionally manipulating the flow in the test section - imposing global velocity gradients or some other controlled form of wall-induced perturbation - wall adaptation may be put to a number of other uses. This subject is discussed further in Section 10.2.4.1, below.

10.2.2 THE FORMS OF WALL ADAPTATION

Of the various mechanisms that have been devised for providing adjustments to the wall boundary conditions, the most widely used may be grouped into two broad categories:

i. those which modify the profiles of impervious flexible liners so as to manipulate the conditions at a surface - the displacement surface - within the adjacent boundary layers; and

ii. those which contrive to manipulate the flow at a fixed surface near each wall by providing appropriate modifications to the test section ventilation.

The surfaces at which the flow conditions are manipulated are referred to as control surfaces. The principal differences between the control surfaces adjacent to impervious and ventilated test section walls are illustrated in Figure 10.1.

In flexible-walled test sections, the profile of each flexible liner is adjusted via a system of jacks. In ventilated test sections, the wall boundary conditions are modified by providing spatial adjustments to the wall geometry or the plenum pressure: these adjustments may be made in isolation or they may be combined in some way. The wall geometry is most often adjusted by modifying the open area ratio. In perforated test sections, this is most commonly achieved by employing sliding cover plates behind the perforations, while individually adjustable slats have been used in slotted facilities. Localised adjustments to the plenum pressure are made by segmenting the plenum chamber surrounding the test section and plumbing each sub-plenum independently.

The degree of control afforded by these forms of wall adjustment will be determined by the following factors:

i. the precision with which each control surface may be defined;

ii. the accuracy with which the conditions at each control surface may be monitored;

iii. the nature of the relationship between the conditions at the control surfaces and the parameter used to adjust them; and

iv. the extent of the practical constraints imposed on the nature of these adjustments.

In flexible-walled test sections, the control surfaces are reasonably well defined and measurement of the appropriate boundary conditions is usually relatively straightforward: the magnitude and direction of the local velocity vector may be derived directly from a knowledge of the wall geometry, static pressure measurements and computations of the boundary layer thickness. (The principal exception to this occurs when shock waves impinge on the control surfaces, in which case more detailed flowfield measurements in the immediate vicinity of the interactions may be required to provide adequate descriptions of the local boundary conditions.) Moreover, in circumstances where the wall boundary layer displacement thicknesses may be computed to an acceptable degree of accuracy, there is a direct correlation between the wall jack settings and the profiles of the control surfaces.

In contrast, considerable problems have consistently been encountered in procuring reliable measures of the conditions at the control surfaces adjacent to ventilated test section boundaries, particularly the normal (cross-flow) velocity. These are, of course, not unique to the adaptive forms of ventilated wall facility - see, for example, Section 4.3. However, the attendant uncertainties in any subsequent assessments of wall interference clearly have severe implications for the prospects of prescribing appropriate adjustments to the wall settings. Furthermore, even assuming that accurate measures of the residuals may be derived, the relationships between them and the parameters utilised to adjust the ventilation are ill-defined and non-linear.

Thus, it may be seen that the control afforded by adjusting the profiles of flexible liners is substantially more powerful than that provided by spatial modifications to the test section ventilation. However, the extent to which this control may be exploited in practice will be determined by the constraints imposed on their design. For instance, a practical mechanism capable of providing fully three-dimensional adjustments to the profiles of flexible liners has yet to be devised. Instead the walls are usually profiled in single curvature, affording two-dimensional or quasi-three-dimensional control, depending on the number and orientation of the walls being adjusted. Aside from making the complete elimination of wall interference a practical impossibility, this makes it necessary to ensure that, when prescribing the loci of target lines, each has a streamwise component throughout its length.

Flexible wall adjustments are further constrained by the use of a finite number of jacks to modify the wall shapes, leaving the profiles of the liners between the jacks to be determined by factors such as their structural properties and the local wind-on pressure loading. The manner in which the jacks are distributed along each liner, together with the limits of their travel, may also constrain the extent to which the wall shapes may be manipulated. These factors will dictate the range of test conditions for which the walls are operating optimally - a potentially important consideration given that the requirements for wall displacement will inevitably be functions of the reference conditions (Mach number and model attitude) and model geometry: two- and three-dimensional (full- and half-span) models may need to be accommodated.

Yet another important constraint is the requirement to ensure that the walls blend smoothly with the contraction and the diffuser. The impact of this constraint - which is closely related to the limits placed on the length of the test section - is likely to be most apparent in high-lift testing, when the upwash ahead of large two-dimensional models and the downwash aft of two- and three-dimensional models will be most pronounced. If the flow about the model is separated, there may also be substantial amounts of blockage present in these circumstances.

There are a number of other factors that may need to be addressed when designing flexible liners - providing for optical access to the test section, the housing of model strut mountings and the co-ordination of the adjustments to the wall contours and the model attitude, for instance. However, not all of these apply only to the design of adaptive flexible-walled test sections. Furthermore, few of these issues have yet been addressed by research activities. Therefore, to avoid cluttering the text with undue speculation or details pertaining to the mechanical construction and operation of practical jacking systems, the emphasis of the remainder of this Chapter will be placed on describing the utility of closed test sections with two flexible walls. Despite the apparent limitations associated with the simplicity of their design, these facilities have proved to exhibit capabilities that surpass those of the alternative forms of adaptive wall test section in virtually all measures of quality and performance. A photograph of a typical research facility and a schematic illustration of its test section are presented in Figure 10.2.

10.2.3 THE PROCESS OF WALL ADAPTATION

The standard procedure for adapting the walls may be broken down into three stages, as follows.

i. The wall interference at the control surface(s) or along the target lines (as appropriate) is assessed with the flexible walls set to pre-determined contours - such as those which may have been derived at a previous data point.

ii. If, at any point at which it was assessed in (i), the indicated interference exceeds what are deemed to be acceptable limits, an appropriate algorithm is employed to prescribe improved wall contours.

iii. The wall boundary conditions are then adjusted accordingly and the procedure repeated until the residual interference satisfies established acceptance criteria.

It is intuitive to expect that the process of *minimising* the effects of wall interference will be iterative, its iterative nature being most pronounced when the consequences of wall adaptation are particularly difficult to predict - as may be the case when the flow about the model is transonic or separated. Thus, the sequence of events beginning with an assessment of the wall interference and concluding with adjusting the wall boundary conditions is considered to be an adaptation iteration. The process of minimising or otherwise controlling the effects of wall interference is referred to as an adaptation cycle.

However, as wall adaptation should not reduce the rate at which data is acquired if it is ever to be used routinely in large industrial wind tunnels, a reliance on iteration is unlikely to be acceptable in production testing. This will require the necessary adjustments to the wall settings to be derived and applied rapidly and in one step - although the option to refine the data further by additional adaptation should always be available assuming practicalities allow it. In turn, this implies that a degree of anticipation or prediction will be required in deriving the adjustments - although it is also conceivable that the walls will not always need to be adjusted between successive data points and that a given wall setting may be acceptable for use over a range of test conditions.

Regardless of its potential application, an adaptive wall technique will always possess the following components: a wall interference assessment procedure, a wall adjustment strategy and a set of completion criteria. Subsequent treatments of the test data, including the application of post-test corrections, are not usually viewed as being part of the adaptive wall technique - although for the reasons outlined in Sections 10.2.3.3 and 10.2.4.1, this position may change in the future.

10.2.3.1 WALL INTERFERENCE ASSESSMENT

The ease with which accurate measures of the wall boundary conditions may be acquired in closed test sections means that the use of two-variable interference assessment methods, as described in Section 4.1.4, is favoured in flexible-walled facilities. This allows wall adaptation to be completed without invoking any assumptions about the geometry or aerodynamic behaviour of the model under test. As is the case in conventional test sections, this is an important attribute as any errors in the assessment of interference will have a direct impact on the quality of the test data. It should also be noted that deficiencies in the assessment arising from random or systematic experimental error will influence the prescribed adjustments to the wall contours. Thus, the occurrence of such errors may often be identified by monitoring the admissibility of the control surface boundary conditions (flexible wall contours and pressure distributions) throughout the adaptation process. With experience, this information may allow appropriate corrective action to be devised.

To avoid the quality of wall adaptation being impaired by imperfections in the empty test section environment, the boundary conditions input to the wall interference assessment code are usually specified as perturbations from their empty test section or "aerodynamically straight" values: aerodynamically straight wall pressure distributions are nominally uniform and equal to the test section reference pressure; correspondingly, aerodynamically straight wall contours are monotonically divergent, in accordance with the requirement to accommodate the streamwise development of all four wall boundary layers.

The scope of the assessment conducted while adapting the walls is justifiably confined to addressing those components of interference that are controlled directly by wall adaptation. More rigorous analytical treatments - covering features such as sidewall interference - need only be completed post test. Thus, the principle of wall streamlining permits the quality of wall adaptation to be provisionally assessed purely in terms of the indicated mismatch between the "real" and "fictitious" flows: in two-dimensional testing, all that is required to minimise the effects of top and bottom wall interference is to match the flows at their interface - or to unload the control surfaces. As noted in Section 4.1.4, the fictitious flows need not be computed when their perturbation potentials are harmonic. In these circumstances, the relevant components of wall interference may be deduced directly from the measured boundary conditions. Examples of the methods currently used to assess wall interference in two- and three-dimensional testing are provided in Sections 10.3 and 10.4.

10.2.3.2 WALL ADJUSTMENT STRATEGY

The algorithm employed to prescribe improved wall settings is known as the wall adjustment strategy. In order to maximise productivity, this should allow the test programme to be completed with the minimum number of adjustments to the wall contours. Consequently, the formulation of a suitable strategy, together with its subsequent refinement, are amongst the most important tasks in the development of any adaptive wall technique - although it should be noted that, when contriving to minimise the effects of wall interference, deficiencies in their formulation only appear to impede the rate at which the walls converge to their optimum settings: they have no impact on the ultimate quality of the test data.

The effectiveness of a wall adjustment strategy will be determined by the extent to which the consequences of wall adaptation may be predicted. Thus, efficient strategies require the relevant components of wall interference to be related directly to parameters describing the wall setting and should also accommodate any aerodynamic coupling that may occur as a result of adapting the walls. When the flow in the test section is purely subsonic, linear theory has proved to be a powerful tool in predicting appropriate adjustments to the wall contours.

However, when the wall-induced perturbations cease to be harmonic, the consequences of wall adaptation become increasingly difficult to predict. As a result, the process of adapting the walls may be relatively inefficient. Nevertheless, the use of reasonably simple procedures - relaxation (adapting to a weighted mean contour somewhere between the current setting and an approximate prediction of an improved setting) or influence coefficient methods (which utilise theoretically derived quantities relating unit changes in wall setting to the resulting flow perturbations at a particular location in the test section), for example - has often proved to be effective in ensuring that the prescribed adjustments to the wall settings become progressively smaller as the adaptation process progresses.

Thus, recognising that wall adaptation usually follows a law of diminishing returns and by paying careful attention to the order in which tests are conducted (by ensuring that the changes in wall contour between successive data points are relatively small, for instance), it is conceivable that, with experience, highly productive wall adjustment strategies may be developed. Several schemes have been proposed. However, relatively little effort has yet been directed towards their evaluation in practice. Perspectives on the demonstrated capabilities of current wall adjustment strategies are provided in Sections 10.3 and 10.4.

10.2.3.3 COMPLETION CRITERIA

While the principle of wall streamlining constitutes a mechanism for eliminating or otherwise controlling wall interference, factors such as those outlined in Section 10.2.2 make the attainment of interference-free flow a practical impossibility. Consequently, in seeking to minimise the effects of wall interference in two-dimensional testing, the practice has developed of terminating adaptation cycles at the stage when experience has shown that further adjustments to the wall settings are unlikely to produce detectable modifications to the model performance. However, it is possible that similar levels of refinement will not be required in production testing since there is probably little to be gained from adapting the walls beyond the point at which the test data become amenable to reliable post-test (or on-line) analyses in these circumstances.

Strictly speaking, the flow over a model is currently only deemed to be fully "correctable" if there is a free-air flow about the same shape that corresponds exactly to that in the wind tunnel. As the wall-induced velocity components will always vary by a certain amount in the vicinity of the model, this raises interesting - and as yet unresolved - questions as to the acceptability of these variations. This comment applies to the quality of the data obtained in conventional as well as adaptive wall test sections (although the variations present in adapted test sections will usually be appreciably smaller than those likely to be found in conventional test sections).

The current absence of clear guidance on this matter has impeded the development of highly productive adaptive wall techniques, particularly in three-dimensional testing, where the residual variations appear to be most pronounced. However, by allowing perturbations to the flow to be introduced in a controlled manner, wall adaptation enables the effects of localised variations in the wall-induced velocity to be studied systematically. Thus, it would appear that adaptive flexible-walled test sections constitute suitable platforms for investigating the rationale of the application of wind tunnel corrections in more detail. This subject is discussed further in Section 10.5.

10.2.4 FURTHER POINTS OF CLASSIFICATION

Before moving on to describe the features of several two- and three-dimensional testing techniques in more detail, it is convenient to introduce two additional factors that are used to distinguish between the different types of adaptive wall technique.

10.2.4.1 THE TYPE OF FLOWFIELD BEING SIMULATED

In principle, the ability to control the flow at the test section boundaries allows a diverse range of flowfields to be simulated within flexible-walled test sections. For example, in addition to facilitating the simulation of free-air boundaries, the principle of wall streamlining allows the conditions in an open-jet test section to be simulated simply by setting the walls to contours exhibiting uniform static pressure distributions. Those simulations already found to be practical in two-dimensional testing are illustrated in Figure 10.3.

It may be seen that the introduction of controlled levels of wall interference may be used to some advantage. However, the utility of wall adaptation in these circumstances may be viewed in several ways. For instance, instead of regarding the scenario depicted in Figure 10.3f as simulating steady pitching motion, it may also be considered as simulating the steady-state conditions about a model of modified camber. Therefore, in order to reflect the different ways in which wall adaptation may be exploited, it is convenient to distinguish between techniques which merely contrive to minimise the effects of wall interference - or, more precisely, reduce them to acceptable levels - and those which intentionally introduce controlled perturbations to the flow. This is achieved in the following sections by describing the adaptation algorithms as being either reductive or manipulative. The most powerful reductive techniques are described in Sections 10.3 and 10.4, while the use of manipulative algorithms is reviewed in Section 10.5.

10.2.4.2 THE NATURE OF THE FLOW AT THE CONTROL SURFACES

As the principle of wall streamlining allows the wall adaptation process to be driven purely by information gathered at the flexible walls, it is convenient to classify the various types of interface matching technique by the nature of the flow at the control surfaces. The groupings adopted to describe the range of test conditions currently straddled in two-dimensional testing are illustrated in Figure 10.4.

Group 1 Flows : the range of test section environments for which the reference Mach number is subsonic and regions of supercritical flow near the model, if they exist, do not extend to the control surfaces. The flow at the control surfaces and throughout the fictitious flowfields is purely subsonic in these circumstances and may be modelled using potential flow theory or the linearised compressible flow equations.

Group 2 Flows : the range of test section environments for which the reference Mach number is subsonic and at least one supercritical tongue emanating from the model extends beyond a control surface. The flow along this control surface is transonic in these circumstances and the region of supercritical flow in the fictitious flowfield is usually terminated by a near-normal shock. The ability of passive solid-walled tunnels to simulate these flows is limited by test section choking.

Group 3 Flows : the range of test section environments for which the reference Mach number is subsonic and the model is almost completely immersed in supercritical flow. This extends far into both fictitious flowfields and may be (i) terminated by systems of oblique and normal shocks (when simulating subsonic freestream Mach numbers) or (ii) turned through oblique shocks before returning to its undisturbed state (when simulating sonic and very low supersonic freestream Mach numbers). Problems associated with test section choking, establishing appropriate reference conditions ahead of the model and starting the tunnel prevent passive solid-walled tunnels being used to simulate these flows.

Group 4 Flows : the range of test section environments for which the reference Mach number is supersonic and the strength of the bow shock is such that it precedes a region of subsonic flow which protrudes into one or both of the imaginary flowfields. The flow at the control surfaces and in the fictitious flowfields is, again, transonic in these circumstances. Difficulties in establishing appropriate reference conditions ahead of the model and starting the tunnel prevent passive solid-walled tunnels being used to simulate these flows.

Group 5 Flows : the range of test section environments for which the reference Mach number is supersonic and regions of subcritical flow between the bow shock and the model do not extend to the control surfaces. The flow at the control surfaces and throughout the fictitious flowfields is purely supersonic in these circumstances. Testing may proceed in passive solid-walled tunnels provided the model is safely within its test diamond.

The requirement to control the flow away from the control surfaces blurs the distinction between the different forms of target line technique. The nature of the flow at the control surfaces will, however, still play an important role in determining the most appropriate wall interference assessment procedure, wall adjustment strategy and completion criteria to employ.

10.3 TWO-DIMENSIONAL TESTING

A test section with flexible walls at top and bottom offers itself and appears to be ideal for the testing of two-dimensional models using the interface matching technique. Strictly speaking, when implemented in facilities with two flexible walls the (two-dimensional) interface matching technique can only eliminate the top and bottom wall interference. The influence of the sidewall boundary layers, being a three-dimensional effect, is not controlled and cannot be eliminated. A procedure, by which two-dimensional wall adaptation may alleviate the sidewall boundary layer effects is reviewed in Section 10.4.3 of this Chapter. Presently we assume that the flow past a two-dimensional wing, spanning the test section, is nearly two-dimensional.

In seeking to eliminate the effects of top and bottom wall interference, the aim of two-dimensional interface matching is to shape the flexible walls in such a way that the distribution of pressure and flow angle measured at the walls match those of a fictitious exterior flow resulting from computation. When this is achieved, within practical limits, the walls conform with the streamlines of the unconfined flow. The fictitious exterior flow is then the analytical continuation of the flow in the test section.

10.3.1 WALL ADAPTATION BY ITERATION

An iterative procedure for the wall adaptation may be contrived for example, in the following way:

Initially the walls may be straight or have any shape approximating the streamlines of the unconfined flow. During a test run, the wall pressures are measured along lines of pressure taps, usually the centrelines of the top and bottom walls. Next, a fictitious "external" flow is computed that is bounded by the test section walls and attains the conditions of undisturbed parallel flow at infinity (Figure 10.5). It should be noted that the "external" flow may be considered as an inviscid potential flow - in contrast to the flowfield adjacent to the model - and can be computed on the basis of inviscid or even linearised theory simply and fast with modern computers. The computed wall pressures of the external flow, p_e, are compared with the measured pressure distribution of the test flow, p_i. If p_e and p_i agree within prescribed error bounds, the external flow is the analytical continuation of the interior flow and the wall shape conforms with the streamlines of the unconfined flow. Otherwise, the difference $p_e - p_i$ is considered as remaining wall interference and the wall shape must be corrected in a second iteration cycle and so on.

This iterative procedure, as it was described in early publications (see, for example, Erickson & Nenni, [7]; Goodyer, [10]; Legendre, [22]; and Sears, [32], is quite intuitive. It has, however, caused much confusion, leading to the widespread belief that the wall adaptation must necessarily be iterative. It will be shown in the following that for Group 1 Flows an explicit computation of the fictitious external flow is not required and that full wall adaptation can be attained within one iteration step.

10.3.2 ONE STEP METHODS OF WALL ADAPTATION FOR GROUP 1 FLOWS.

The principles of one step methods are equally valid for two- and three-dimensional flows, for interface matching and target line methods. In all cases the procedure requires an assessment of the wall interferences by a two-variable method and a subsequent computation of the wall movement required to eliminate the interferences. In the case of interface matching, the component of the interference velocity normal to the wall is evaluated which gives immediately the flow angle to which the wall must be adjusted in order to extinguish the interference velocity. The assessment of wall interferences using Green's formula is discussed in Chapter 4. In the case of two-dimensional flows, Green's formula reduces to the Cauchy integral formula which is discussed in the following.

The Cauchy Integral

It is assumed that the flow near the test section walls deviates from parallel subsonic flow by small disturbances. The two-dimensional disturbance potential fulfils the equation:

$$\beta^2 \phi_{xx} + \phi_{yy} = 0 \quad \text{with} \quad \beta^2 = 1 - M^2 \tag{10.1}$$

(x , y) are the co-ordinates in the flow direction (x) and upwards (y) as shown in Figure 10.5.

The wall interference in two-dimensional tunnel flow is then computed by a Cauchy type integral :

$$w_{\text{int}}(z) = \frac{1}{2\pi i} \int_C \frac{w(\varsigma)}{\varsigma - z} d\varsigma \tag{10.2}$$

where the complex variable z is defined by $z = x + i\,\beta y$ and ζ by $\zeta = \xi + i\,\beta\eta$ with (ξ , η) as the running co-ordinates in the x - and y - directions. While introducing the variables z and ζ use is made of the Prandtl-Glauert transformation.

The complex integral is taken along a counter-clockwise oriented closed path (C) around the model - suitably along the lower wall from the upstream to the downstream end of the test section, from there across the test section to the upper wall and along the upper wall upstream and back across the test section to the starting point (Figure 10.6).

$w(\zeta) = \beta\, u(\xi,\eta) - i\, v(\xi,\eta)$ is the disturbance velocity in complex notation at a point (ξ,η) and

$w_{int}(z) = \beta\, u_{int}(x,y) - i\, v_{int}(x,y)$ is the interference velocity at a point with co-ordinates (x,y).

To evaluate the integral of Equation 10.2, the disturbance velocities (u,v) must be known along the closed path C. At the upper and lower wall of the test section the values of u and v are simply evaluated by measuring the wall pressure and the wall angle, which is the derivative of the wall displacement. Assuming that the linearised Bernoulli's equation may be applied to describe the disturbance velocity at the walls, then:

$$u/U_\infty = -c_P/2 \quad \text{and} \quad v/U_\infty = \theta$$

where θ is the wall angle and c_p the pressure coefficient measured at the walls.

The evaluation of u and v along those parts of the closed curve C that cross the test section at the upstream and downstream end is not as simple. However, if the test section is sufficiently long - a requirement for full wall adaptation - the disturbance velocities at the upstream and downstream ends are small and may safely be neglected.

Equation 10.2 is the wall correction formula of Smith (1982). A derivation of Equation 10.2 was given by Mokry [29]. The formula is the two-dimensional equivalent of Green's formula, introduced by Ashill & Weeks [2] for the computation of wall interferences in general three-dimensional flows (Equation 4.14 of Chapter 4). Because of the importance of Equation 10.2 both for the wall interference assessment and a wall adaptation strategy a brief derivation is reviewed in Appendix A.

Equation 10.2 may be used either to compute the wall interference velocity at the model or at any other point within the test section. When the Cauchy integral is evaluated for z-values on the wall, it must be regarded that the integrand is singular at $\zeta = z$. The proper integration is performed by taking the limit-value of the integral for z-values approaching the wall. For z-values at the wall, Equation 10.2 takes the form:

$$w_{\text{int}}(z) = \frac{1}{2} w(z) + \frac{1}{2\pi i} \int_C \frac{w(\zeta)}{\zeta - z} d\zeta \quad , \quad z \in C \qquad (10.3)$$

where the Cauchy principal value is to be taken for the integral.

Equation (10.3) leads immediately to a one step formula for the wall adaptation. The normal velocity at the walls for interference-free flow must be: $(v - v_{int})$. The wall angle is $\theta = (v - v_{int}) / U$ and the wall displacement δy is :

$$\delta(y) = \int \theta\, dx = \int (v - v_{int}) / U_\infty\, dx \qquad (10.4)$$

where v_{int} is the negative imaginary part of w_{int}, evaluated at the wall according to Equation (10.3).

Equation (10.4), as a one step formula for the wall adaptation, was first described by Kraft & Dahm [17]. The discovery that wall adaptation for group 1 flows in two dimensions need not be iterative is attributed to Lo [26] .

A final remark is due regarding the notion of one step methods. As mentioned above, the one step formula is limited to cases where linearised flow theory is applicable at least in a region near the wall (or the control surface about which the integral (Equation 10.2) is taken). In extreme cases a second iteration may be required even for the linearised flow. After adapting the wall to the computed wall shape the flow about the model will change by some small amount. The changed flow will produce slightly different wall interference. We may imagine that the singularities representing the model and consequently the images representing the wall interference are slightly modified by the wall adaptation. This second order effect is negligible in most cases. It may become significant e.g. when wind tunnel choking occurs at near sonic speeds. An initial adaptation step may bring the flow at the wall to subsonic conditions so that - in a second iteration step - the linearised flow assumption holds.

In wind tunnel practice, the test condition - angle of attack and freestream Mach number - will be changed in small steps so that at each step only small changes of the flow are encountered and, therefore, only small corrections to the wall displacements are required that can be done within a single iteration step. The wall adaptation procedure may not even slow down the model testing if wall pressure assessment and wall adjustments do not take more time than changes of the test condition (angle of attack or Mach number). The adapted wall shape for the succeeding configuration may be extrapolated from the previous values. During the test the wall pressure distributions will be measured and used to compute the proper wall shape that can be used for extrapolation to the next test configuration and so on. In this way a continuously self-correcting wind tunnel may be realised.

10.3.3 WALL ADAPTATION IN SUPERSONIC FLOW (GROUP 5)

In contrast with the situation for Group 1 Flows, the experience of adapting the walls in Group 5 Flows is rather limited. As a result, the testing techniques are far less refined. The most notable investigations were conducted on large aerofoils generating modest lift at Mach numbers up to 1.35. (Taylor, [37])

As the bow shock generated by the model was not attached at any of the conditions straddled during these tests, there was no obvious and immediately available means of procuring reliable estimates of the wall interference. Therefore, in the absence of suitable reference model data, it was necessary to rely entirely on the principle of wall streamlining to define the desired data quality: a Transonic Small Perturbation code was used to compute the fictitious external flows and the walls adapted until the mismatch between the real and fictitious flows, evaluated along the centrelines of the control surfaces, appeared to have been reduced to levels deemed to be acceptable in Group 1 Flows. A lag-entrainment method was used to compute the displacement thickness contours in the flexible wall boundary layers.

The wall adjustment strategy used differed from those employed in Group 1 Flows in several important respects. Aside from its lack of refinement, it reflected the fundamental differences between the elliptical and hyperbolic natures of subsonic and supersonic flow. For instance, recognising that supersonic disturbances cannot be propagated upstream, it was not used to adjust the full-length of the profiles of the flexible walls:

i. In the early stages of the adaptation process, its use was confined to adjusting the slope of the upstream portion of each control surface. It was only applied further downstream once the local mismatch between the real and fictitious flows had been reduced to an acceptable level.

ii. Adapting the walls ahead of the bow shock and beyond the point at which any reflected disturbances would pass downstream of the model and the subsonic portion of its wake was unnecessary.

Furthermore, as any wall-induced blockage adjacent to the model or the near-portion of its wake could force the bow shock stand-off distance to be appreciably larger than that in free-air, it was found that wall streamlines could only be approached from one direction: adjustments to the wall contours should, in general, be directed towards the tunnel centreline. Thus, without the freedom to iterate via progressively smaller overshoots, it seems that, for practical purposes, wall adaptation in supersonic flows will exhibit a one-sided asymptotic convergence to free-air streamlines. Aside from making it more difficult to ascertain the stage at which the walls have attained their optimum settings, this indicates that errors in estimating the modifications to the control surface profiles associated with shock - boundary layer interactions or the provision of insufficient local control over the wall contours may prevent the adaptation process from ever reaching free-air streamlines.

Consequently, although these investigations demonstrated that wall adaptation yielded substantial alleviations in the intensity of any reflected disturbances, it was not possible to quantify the extent to which top and bottom wall interference had been reduced. No direct attempts were made to address the alleviation of sidewall interference, or to investigate the issues associated with shock - wall boundary layer interactions in any detail. Moreover, although attempts were made to assess the sensitivity of the model data to any (aerodynamically) undesirable waviness in the flexible walls, as the test section was not designed for supersonic testing, the results of these studies were not conclusive.

It may be seen that two-dimensional Group 5 interface matching techniques are still in the preliminary stages of development. Much is to be done before they may be utilised competitively beyond the research environment.

10.3.4 WALL ADAPTATION IN TRANSONIC FLOWS (GROUPS 2- 4)

Again, the experience of wall adaptation in circumstances where the flow at the walls is transonic is rather limited. Lewis [23] conducted the most systematic evaluation of the prospects for simulating Group 2 Flows while the most experience of testing in Group 3 and 4 Flows has been accumulated by Taylor [37].

Faced with difficulties in obtaining accurate measures of the residual wall interference or reliable independent sources of reference model data, these activities adopted similar approaches to wall adaptation as that described in Section 10.3.3 above. The principal distinctions between the techniques occurred in the wall adjustment strategy.

As the maximum attainable Mach number ahead of the model with the walls set in their aerodynamically straight positions was approximately 0.75 in these tests, it was not always possible to initiate wall adaptation at the desired reference Mach number. Therefore, a policy of adapting the walls at a speed below that ultimately required and relying on the attendant blockage relief to provide the necessary increase in choking Mach number was adopted for Group 2 simulations.

The process of initiating wall adaptation from a Group 1 Flow condition meant that subsonic wall adjustment strategies could be employed at much higher reference Mach numbers than would otherwise have been possible. However, once the point had been reached where the supercritical patches of flow at the walls could not be removed by wall adaptation, local adjustments to the wall contours were not so easily prescribed and the process of minimising the local wall loading became more iterative. Although the supercritical wall loading could not be reduced to the desired levels within one or two iterations, the residual wall loading elsewhere in the test section was very low and the supercritical patches in the real and fictitious flows were well matched - see Figure 10.7. Moreover, the model data - most noticeably, the upper surface shock location - and the wall loading exhibited a double convergence. The test data therefore appeared to be of a reasonably high quality. Nevertheless, with the effects of sidewall interference unaddressed, further experience is required to assess the extent to which the technique must be refined if the most demanding contemporary standards for residual interference are to be guaranteed in production testing.

When simulating Group 3 Flows, wall streamlines derived from Euler free-air computations were initially used as the starting point in each adaptation cycle. As with Group 2 Flow simulations, subsequent wall adaptation proved to be relatively ad-hoc and was occasionally prolonged by difficulties in establishing appropriate reference conditions ahead of the model whilst simultaneously unloading the portions of the control surfaces adjacent to it.

However, once generated, the adapted wall contours for a given model incidence appeared to be valid for a wide range of freestream Mach numbers - a direct consequence of the Mach freeze phenomenon. Thus, once the walls had been adapted for a range of model incidences when simulating one freestream Mach number, it appeared that data for a range of neighbouring freestream Mach numbers (extending from about 0.96 to 1.15 in this case) could be obtained on a one-shot basis - although, as the flow in the adapted portion of the test section was remarkably insensitive to the freestream Mach number, there would have been little point in completing a detailed test matrix in these circumstances. With the influence of the sidewall boundary layers likely to be appreciably less important than in Group 2 Flows, this was a particularly refreshing discovery. Nevertheless, further experimental evidence and technique refinements - including reductions in the time required to compute the fictitious flowfields - will probably be required before this radically different approach towards near-sonic testing may be employed with confidence in production testing.

Initial experiences of adapting the walls to simulate Group 4 Flows also proved to be laborious. Following the general pattern established for Group 5 simulations, the first iterations were dedicated to improving the location of the bow shock - as judged by the progressive confluence of its positions in the real and fictitious flowfields. Effort was then focused on relieving the mismatch in the region of subsonic flow aft of the shock before moving on to address the supersonic flow further downstream. However, unlike the situation in Group 5 simulations, these phases of the adaptation cycle were not distinct as the region of subsonic flow on each control surface provided a path by which disturbances could be propagated upstream.

Thus, in many respects, the procedure for simulating Group 4 Flows currently appears to be the least refined of all two-dimensional interface matching techniques. Nevertheless, a measure of encouragement for future development was gained from the observation that the quality of the adapted test data appeared to be remarkably insensitive to model incidence. It remains to be seen whether the wall contours derived for a particular model incidence will be capable of producing data of an acceptable quality over a range of model incidences in Group 4 Flows.

10.4 THREE-DIMENSIONAL TESTING

10.4.1 WALL ADAPTATION BY INTERFACE MATCHING

Interface matching in ventilated test sections

Shaping the walls into a three-dimensionally curved surface meets, obviously, extreme technical difficulties. In an early study at AEDC Kraft et al. [18] restrained from using solid walls for three-dimensional wall adaptation, but rather investigated the capability of adaptable ventilated walls. The configurations investigated featured variable porosity in conjunction with suction through the walls. In this way significant reduction of wall interference could be obtained. Nevertheless, the method has not found much spread for the reasons outlined in Section 10.2.2. (See also Chapter 4.3 for more details).

Interface matching in test sections with impervious flexible walls

A configuration using eight flexible walls formed to an octagonal test section was investigated by Ganzer et al. [8] at the University of Berlin. Each of the walls was deflected by a set of jacks to accomplish a nearly continuous three-dimensional wall adaptation. Special attention was given to the sealing of the gaps between the flexible plates by lamellas manufactured from spring steel.

A cylindrical test section constructed from a thick walled rubber tube of 80 cm diameter was used at the DFVLR Göttingen by Wedemeyer et al. [42]. Full three-dimensional wall adaptation was achieved by deformation of the rubber wall with a set of 64 jacks, 8 jacks each at 8 cross sections. In conjunction with the rubber tube test section a one-step adaptation algorithm for three-dimensional flows was developed that takes advantage of the cylindrical geometry of the test section. A universal algorithm based on the two variable method of Ashill & Weeks [2] and capable of computing interference-free wall contours for arbitrary test section shapes as well as residual wall interferences in three-dimensional flows was developed by Holst [15]. The latter is particularly useful when the wall adaptation is imperfect as in the case of two-dimensional adaptation for three-dimensional flows, which is discussed in section 10.4.2.

It was demonstrated that interference-free flow can be achieved in the octagonal test section in Berlin as well as in the rubber tube test section of the DFVLR Göttingen. In spite of this success, the full three-dimensional wall adaptation methods were no longer pursued when it was shown that wall adaptation for three-dimensional flows can be accomplished within acceptable approximations in test sections with two flexible walls that had so far found prevailing use in two-dimensional testing. Since this two-dimensional wall adaptation for three-dimensional flows has become a standard method, a detailed description will be given in the following section.

10.4.2 TARGET LINE METHODS: TWO-DIMENSIONAL WALL ADAPTATION FOR THREE-DIMENSIONAL FLOWS

Interface matching by means of two flexible walls is, of course, not conceivable when the flow is three-dimensional. It is, however, appealing to use test sections with two flexible walls to relieve wall interferences in three-dimensional testing because of their relatively simple construction and their convenience. It was shown by Wedemeyer [41] that blockage and upwash interferences can be eliminated at the centreline of the test section by suitable adaptation of the flexible walls of a two-dimensional adaptive wall test section. An algorithm for the two-dimensional wall adaptation for three-dimensional flows was developed at the VKI by Wedemeyer [41] and Lamarche & Wedemeyer [20]. Similar methods have also considered to eliminate the interferences along some well defined "target line" that need not be the centreline of the test section. (In principle, target line methods are not limited to test sections having two flexible walls, although, for practical reasons, only these have been used so far).

Presently we consider the case where the model is mounted in a test section with two flexible walls so that its axis coincides with the centreline of the test section. It is supposed that the model is symmetrical or nearly so to the vertical plane of symmetry of the test section, i.e. a symmetrical model at zero or small yaw angle is assumed. The lateral extensions of the model are supposed to be not a large fraction of the lateral extensions of the wind tunnel so that the model is exposed only to the flow near the centreline. Although the wall interferences are eliminated strictly only along the target line, it is expected and was proven by numerical simulations (see Section 10.4.2.6) that the residual interferences are relatively small throughout the remainder of the test section. If the centreline is used as the target line, the residual interferences are of second order small in terms of the distance from the centreline. In half-model testing the model is usually mounted with its axis along the centreline of a sidewall. Such an arrangement may be conceived of as a model mounted at the centreline of a duplex test section. For wall adaptation a line of pressure taps should be provided near the reflection plane on top and bottom wall.

For a symmetrical model at zero yaw angle the interference velocity along the centreline has a longitudinal component $u_{int}(x)$ and a vertical component $w_{int}(x)$. The walls are adapted now in such a way that the interference velocities ($u_{int}(x)$, $w_{int}(x)$) are extinguished at the centreline. By deflecting the upper and lower wall in a suitable symmetrical way (Figure 10.8a) a distribution of longitudinal velocity $u(x) = - u_{int}(x)$ is induced. Similarly, by deflecting the upper and lower walls anti-symmetrically (Figure 10.8b), a distribution of vertical velocity $w(x) = - w_{int}(x)$ can be induced. Combining symmetrical and anti-symmetrical wall deflections any wall interference can be extinguished along the centreline. The wall interference velocities at the tunnel centreline may be computed by the two variable method of Ashill & Weeks (see Chapter 4) which requires a detailed pressure measurement at all test section walls. The method of Lamarche & Wedemeyer [20] rests upon pressure measurements at the centrelines of top and bottom walls only. The wall interferences at the tunnel centreline can be inferred from the top and bottom wall pressure distributions under the supposition of symmetrical models with small lateral extensions (precisely, under the condition that the model can be represented approximately by singularities distributed along the tunnel centreline). With these assumptions the wall interference assessment and the wall adaptation algorithm are largely simplified to the evaluation of one-dimensional integrals.

10.4.2.1 Assessment of the Wall Interferences

The u-component of the disturbance velocity at the walls is evaluated via measurement of the pressure coefficient in the usual way: $u / U_\infty = - c_p /2$. The *w*-component (normal to the wall) is inferred from the wall angle θ or, alternatively, the derivative of the wall displacement $d\delta/dx = \theta = w/U_\infty$.

Let us assume, at first, that the walls are aerodynamically straight, i.e. $\theta = 0$ at the top and bottom walls. Defining the symmetrical part u_s and the anti-symmetrical part u_a of the disturbance velocity at the walls:

$$u_s = (u_t + u_b)/2 \qquad (10.5a)$$

$$u_a = (u_t - u_b)/2 \qquad (10.5b)$$

where u_t and u_b are the velocities at the top and bottom walls, the interference velocities u_{int}, w_{int} at the centreline are related to the wall velocities by linear integral operators:

$$u_{\text{int}}(x) = \int u_s(\xi)\,\Omega(\xi - x)\,d\xi / \beta h \qquad (10.6a)$$

$$w_{\text{int}}(x) = \int u_a(\xi)\,\Gamma(\xi - x)\,d\xi / h \qquad (10.6b)$$

where the integration is, nominally, from $- \infty < \xi < + \infty$. As the wall velocities die out quickly with increasing distance from the model location the integrals encompass, in practice, only a finite path. Ω and Γ are functions of the normalised variables ($\underline{\xi}$ -$\underline{x}$), with $\underline{x} = x/\beta h$, $\underline{\xi} = \xi /\beta h$, and the aspect ratio b/h of the test section.

Equations 10.6a and 10.6b are similar to (the real and imaginary parts of) the Cauchy-integral (Equation 10.2), but the influence functions Ω and Γ are, of course, more complicated. They are computed once for a given test section geometry b/h. The form of the influence functions and their computation is discussed in Appendix B.

10.4.2.2 WALL ADAPTATION ALGORITHM

As mentioned above, the wall adaptation strategy aims at eliminating the wall interferences along the centreline of the test section by displacing the flexible top and bottom wall so as to generate velocity distributions $u_c = -u_{int}$, $w_c = -w_{int}$ that counteract the interference velocities. The wall displacements are again divided into a symmetrical part $d_s = (d_t + d_b) / 2$ and an anti-symmetrical part $d_a = (d_t - d_b) / 2$ where d_t and d_b are the displacements of the top and bottom wall, positive in the outward direction. In the following the notation $\boldsymbol{u}_s$ and $\boldsymbol{u}_a$ will be used for the non-dimensional velocities $\boldsymbol{u}_s = u_s / U_\infty$, $\boldsymbol{u}_a = u_a / U_\infty$ and $\boldsymbol{d}_s$, $\boldsymbol{d}_a$ for the non-dimensional wall displacements $\boldsymbol{d}_s = d_s/h$, $\boldsymbol{d}_a = d_a/h$ where U_∞ is the freestream velocity and h the test section height.

The wall displacements $\boldsymbol{d}_s$ and $\boldsymbol{d}_a$ required to eliminate the interference velocities in this way are related to the wall signatures, $\boldsymbol{u}_s(\xi)$, $\boldsymbol{d}_{s0}(\xi)$, $\boldsymbol{u}_a(\xi)$, $\boldsymbol{d}_{a0}(\xi)$ by linear integral operators:

$$\boldsymbol{d}_s(x) = \int \beta^2 \boldsymbol{u}_s(\xi)\, X(\xi - x) + \boldsymbol{d}_{s0}(\xi)\, M(\xi - x)\, d\xi \,/\, \beta h \qquad (10.7a)$$

$$\boldsymbol{d}_a(x) = \int \beta^2 \boldsymbol{u}_a(\xi)\, \Lambda(\xi - x) + \boldsymbol{d}_{a0}(\xi)\, N(\xi - x)\, d\xi \,/\, \beta h \qquad (10.7b)$$

$\boldsymbol{d}_{s0}$ and $\boldsymbol{d}_{a0}$ denote the pre-set wall displacements, usually the wall setting of a previous test condition. The influence functions X, M, Λ, N depend only on the normalised variable $(\underline{\xi} - \underline{x}) = (\xi - x) / \beta h$ and the aspect ratio b/h of the test section. A method for computing the functions X, Λ, M, N is discussed in Appendix B. An algorithm based on Equations 10.7a&b for the wall adaptation is used routinely in adaptive wall tunnels at ONERA/CERT Archambaud & Mignosi, [1] and at DLR (Holst & Raman, [14]).

It is important to note that the time required to perform the calculation of the adapted wall contours need not be an obstacle to fast wall adaptation procedures in production testing. A computational code used at the DLR Göttingen requires about 0.1 second on a 133 MHz Pentium computer to compute the wall contours from Equations 10.7a&b. In comparison, the algorithm employed by Holst [15] using full boundary measurements requires about 3 seconds for the calculation of wall contours in three-dimensional flow.

The method described above has been extended by Rebstock and Lee [31] who considered the more general case, that the model is not necessarily mounted at the centreline of the test section. The wall interferences are then computed from flow measurements at the full boundary of the test section and can be eliminated on a given target line that is, for example, the model axis. The generalised adaptation algorithm was used in the TCT wind tunnel at NASA Largely to verify the method. A distinct advantage of pressure measurements at the full boundary is, that residual interferences can be computed at once. The full wall interference assessment requires more testing time and, of course, a sufficient number of pressure taps distributed over the whole of the test section walls. The quality of the wall adaptation can, however, hardly be improved by using wall signature information on the full boundary because the elimination of the wall interferences is still limited to the target line.

Several approaches have been developed independently of these activities. Lewis and Goodyer [24] combined the two-variable method of Ashill & Weeks [2] with the influence coefficient method of Goodyer [11] and have employed various target lines, for example, a straight target line for blockage interferences following the fuselage and a target line for upwash interferences following the forebody of a model and subsequently a swept line following the wing geometry.

Another approach due to Le Sant and Bouvier [21] is used in the S3Ch adaptive wall wind tunnel at ONERA Chatillon. The ONERA S3Ch method may be seen as an improved version of the VKI method, using the same principles and ideas:

i. A model representation is identified according to pressure measurements on the top and bottom wall.

ii. Wall interferences are assessed on a target line.

iii. A wall shape is predicted so as to cancel the wall interferences on the target line.

The application of these principles is, however, different in a two-fold respect:

i. The location of the model is user defined, i.e. it is not necessarily aligned with the centreline. The singularities representing the model are set at the model location, including the model support sting. The model attitude is taken into account as well as the model support.

ii. The target line on which wall interferences are assessed is not necessarily straight but it may follow the fuselage and continue along the 3/4-cord line of the wing.

The ONERA method is more complicated than the VKI method. Its use is less easy as information about the model is required. On the other hand the user may control model representation and target line. Large models may be used and support interferences may be taken into account.

It should be noted that severe restrictions exist on the extent to which a target line may be swept if wall interferences are to be eliminated along it. These arise because the perturbations to the flowfield introduced by wall adaptation are - for subsonic flows - analytic throughout the test section implying similar constraints on the form of the target line and the wall interferences to be eliminated. However, it should also be noted that elimination of wall interferences along a target line is not necessarily the best approach to take when adapting the walls. This point is discussed in Section 10.4.2.4.

10.4.2.3 COMPLETION CRITERIA

The importance of completion criteria, particularly in three-dimensional testing has been outlined in section 10.2.3.3. In the case of Group 1 Flows the wall adaptation may - as a rule - be completed by a one-step iteration. If in exceptional cases (or Group 2-5 Flows) more than one iteration is required, the iteration procedure may be terminated if further iterations do not produce detectable modifications to the adapted wall contour. In all cases an assessment of residual interferences may be desirable after completion of the wall adaptation. If wall pressure measurements on the full boundary are available residual interferences may be calculated by the method of Ashill & Weeks [2]. If pressure measurements at the full boundary are not available, residual interferences may still be calculated by conventional wall interference assessment methods (see Chapter 2). An example of the numerical assessment of residual upwash variations for various wingspan ratios is given in section 10.4.2.6.

10.4.2.4 EXPERIMENTAL RESULTS FOR GROUP 1 FLOWS

Since two-dimensional wall adaptation for three-dimensional flows can only be approximate, it is desirable to verify the methods experimentally. To this end a number of wind tunnel tests have been performed in wind tunnels at NASA Langley, ONERA/CERT, DLR Göttingen, the University of Southampton and at the Northwestern Polytechnical University (NPU) in Xian, China. Representative results of experimental tests are presented in Chapter 4 of AGARD Advisory Report 269 (Hornung, ed., [16] and original publications (citations of which are also found in AGARD-AR-269). In all cases where experimental results have been compared with interference free data from larger wind tunnels good agreement was found, even in cases where the ratio of wingspan to test section width and the blockage ratio were not small.

Figures 10.9 a&b show experimental results obtained in the T2 tunnel at ONERA/CERT for an axis-symmetric body (Figure 10.9a) and an aeroplane half model (Figure 10.9b). The results at M=0.84 show convincingly that wall adaptation is achieved within one iteration step since no significant changes are obtained for further iterations. Comparison with results for not adapted walls and with interference-free results from a large wind tunnel (NASA Ames 11 ft X 11 ft) gives an impression of the quality of the wall adaptation. The results for the half model, spanning 80% of the test section, show that wall interferences are largely reduced although spanwise variations of the wall induced upwash could not completely be eliminated.

As another example of the quality of wall adaptation in an extreme case, results obtained in the high speed wind tunnel of the DLR Göttingen for an aeroplane model spanning 75% of the test section width are given in Figure 10.10. Figure 10.10a shows a plan view of the model in the test section and Figure 10.10b the pressure distributions at two wing sections (the most outboard section y/s=0.925 could accommodate only three pressure taps because of its limited thickness). The Figure shows the improvement of the test data after wall adaptation and the agreement with interference free results at the wing section y/s=0.6. A small deviation is apparent for the wing section y/s=0.925 and this deviation agrees well with the predicted residual interferences.

By referring to the data presented in these Figures, it is possible to describe some more general observations that have been made about the capabilities of two-dimensional wall adaptation for three-dimensional Group 1 Flows :

i. As, prior to adapting the walls, the distribution of wall-induced blockage in the plane of the model is remarkably one-dimensional (Figure 10.9b), two-dimensional wall adaptation is effective in reducing it throughout the test section, not merely in the vicinity of the target line: the residual variations in Figure 10.9b really are rather small. This situation appears to prevail at high reference Mach numbers, even for relatively large models.

ii. In contrast, the wall-induced upwash is not distributed so evenly across the test section prior to adapting the walls, especially in regions aft of a lifting surface. Thus, while wall adaptation may be effective in eliminating it along its target line, the residual variations over the remainder of the model are much larger. When a target line that does not deviate far from the streamwise direction is employed during tests on models of high aspect ratio, the dominant residual usually takes the form of velocity gradients in the spanwise direction over the wing - in effect, wall-induced wing twist. This is evident in both Figure 10.9b (where, while wall adaptation has clearly reduced the overall levels of upwash - and hence the magnitude of any primary correction to incidence - it has actually increased the effective wing twist, for which there is currently no correction) and in Figure 10.10 (where the differences in the residual interferences calculated at the two spanwise stations may be interpreted as evidence of wall-induced twist).

Three ways of reducing wall-induced wing twist have so far been identified: (i) reducing the overall model size, (ii) changing the cross sectional proportions of the test section (this way is discussed in section 10.4.2.6), and (iii) sweeping the upwash target line adjacent to the wing, foreplane or tailplane.

An example of the effects of employing a swept target line is provided in Figure 10.11. As mentioned above, target lines on which wall interferences are to be eliminated are subject to certain restrictions. In lack of definite rules a tentative line was assumed as depicted in Figure 10.11 and wall adaptation was aiming at the best possible reduction of the wall induced upwash along this line. Figure 10.11 and Table 10.1 show that wall induced wing twist could be reduced in this way to comply with the criteria proposed by Steinle & Stanewski [34] although at the expense of residual camber. Moreover, as a result of directing the target line along the root chord before sweeping it outwards towards the wing tip, and then aligning it with the tip chord, the penalties associated with sweeping the target line - the production of residual camber at the wing root and tip - have also been kept small. Consequently, the residual variations over the wing, the principal components of which are summarised in Table 10.1, are sufficiently small as to comply with the criteria proposed by Steinle and Stanewsky [34]. Having said this, the benefits of employing swept target lines will need to be balanced against the costs - the additional expense associated with acquiring wall pressure data at the full boundary and solving Equation 4.14 (as opposed to Equations 10.6 and 10.7), for instance - if they are to be used in routine production testing.

10.4.2.5 Wall Adaptation for Non-Linear and Supersonic Flows (Groups 2-4)

In the case of two-dimensional flow and generally in cases using interface matching techniques, the strategy of streamlining the walls could easily be extended to non-linear flows as discussed in Section 10.2.4.2, the main difference being that a computation of the fictitious external flow is required for non-linear flows. An extension of the target line technique to non-linear flows is not as straightforward because the described method depends on the assumption that the effects of the wall constraints and the wall displacements can be superimposed. The superposition principle is, however, not applicable for non-linear flows. It is difficult to define a strategy of two-dimensional wall adaptation for three-dimensional flow, if the superposition principle does not hold. A way that was investigated by Lamarche [19] depends on the "transonic area rule". To alleviate the blockage effect the two flexible walls were shaped in such a way that the area distribution of the test section equals the area distribution of a corresponding streamtube around the model in free-flight. The streamtube was computed for an "equivalent body of revolution". The equivalent body of revolution is an imaginary model that generates a wall pressure signature equal to that of the real model. The equivalent body was determined, more or less, by a method of trial and error, which is laborious and time consuming. It was shown, however, that nearly interference-free flow could be achieved in this way. For a lifting model only the symmetrical part of the wall pressure distribution was used to define the equivalent body of revolution. For the anti-symmetrical part of the wall pressure, which is related to lift, it was shown that the linear algorithm is still valid.

Theoretical and experimental investigations of the possibility of two-dimensional wall adaptation for three-dimensional supersonic flows have been performed at the DLR by Heddergott & Wedemeyer [13] and at NPU by He et al. [12.

10.4.2.6 LIMITATIONS AND OPEN QUESTIONS

Two-dimensional wall adaptation for three-dimensional flow is necessarily imperfect. The strategy in which the wall interferences are eliminated at the centreline of the test section is subject to the following assumptions:

i. The lateral extension of the model is not a large fraction of the test section width.

ii. The asymmetry of the flow with respect to the vertical centre plane of the test section is small.

In the following, we address the limitations imposed by these assumptions.

Limitations due to model size

It is common to all target line methods that wall interferences are eliminated only along the target line. It is expected, however, that the remaining residual interferences are small although they increase with distance from the target line. If the target line coincides with the centreline of the test section the residual interferences near the target line remain small to second order in terms of the distance from the target line. As a consequence the limitations due to model size are far less restrictive than might otherwise be anticipated. In order to have an estimate about the residual interferences to be expected, numerical studies have been performed for blockage and upwash interferences.

Blockage interferences - concerning the u-component - are caused mainly by the large volume of the fuselage of an aeroplane model. Residual blockage interferences on the fuselage are small just because the target line is chosen to run near the fuselage. This does, however, not imply that they are small elsewhere in the test section. It has been shown by numerical simulations that residual blockage interferences remain extremely small throughout the test section so that they are negligible even for large blockage ratios (for example Verte [40]. Unfortunately this is not the case for upwash interferences.

Upwash interferences - concerning the w-component - are caused by lifting bodies i.e. mainly by the wing of an aeroplane model. As the wing spans a large portion of the test section width, spanwise variations of the wall-induced upwash are to be expected while only a constant level of upwash can be eliminated at any streamwise position.

It was noted by Wedemeyer and Lamarche [43] that test sections with aspect ratios other than square can be advantageous as the spanwise upwash variations are reduced or even become negligibly small. Depending somewhat on the wingspan ratio, the ideal test section should be rectangular with a width to height ratio of about b/h=1.4. The upwash level at the tunnel centreline may also be reduced as was noted already by Glauert [9] in the context of wall interference corrections. An extensive numerical analysis by Lewis & Goodyer [24] has generalised these findings to cover a wide range of model spans and test section proportions. The results are summarised in Figure 10.12. The contour lines show the root-to-tip variation of the normalised upwash: $\Delta_{upw} = AR/(8c_L)\ (w_{int}(max)-w_{int}(min))/U_\infty$ (in degrees). For a wing with a typical aspect ratio AR=8 and lift coefficient of c_L=1 the factor AR/8c_L becomes 1 and the upwash variation is shown directly on contour lines in a plane $2s/\sqrt{bh}$ versus b/h where $2s/\sqrt{bh}$ is the wing span ratio and b/h the width to height ratio of the test section. For a square test section $(b/h=1)$ and a wing span ratio $2s/\sqrt{bh} = 0.7$ the normalised upwash variation is about 0.15°. More favourable conditions are found for a test section ratio of $b/h \approx 1.4$ where the upwash variations are less than 0.025°. It appears that residual upwash interferences remain relatively small if the wing span does not exceed 70% of the test section width for square test sections.

Another limitation due to model size concerns the capability of the flexible walls to be adjusted to the computed wall contour in that relatively large wall displacements are required to accommodate the downwash field. The flow downstream of a three-dimensional lifting model follows about a constant downwash angle that may be as large as 2 or 3 degrees for high-lift configurations. Consequently, the flexible walls have to be displaced significantly downstream of the model. As the tunnel flow must be directed back to the collector, significant deflections of the flexible walls are required that may set limits to the model size or maximum angle of incidence. These circumstances may be relieved using a rotated system of co-ordinates, i.e. the wall setting is rotated by about half the downwash angle. The model angle of incidence is then corrected by this amount.

Limitations in asymmetric flow

Obviously, two-dimensional wall adaptation cannot cancel sidewash interferences. This is a serious limitation of the method whenever sidewash interferences are significant. Such situations are, however, very rare. Objects tested in wind tunnels are, with few exceptions, designed to produce only small side force per unit yaw angle, while the opposite is true of the normal force. Interferences are proportional to the forces experienced by the model. Hence the relative sidewash interferences are usually very much smaller than the upwash interferences and may be neglected in most circumstances. Exceptions may arise in cases where model yaw is accomplished by rotation of the model about the support sting because in these cases the requirement of nearly symmetric flow (point ii above) is eventually violated. In cases where sidewash interferences are significant they may be corrected by classical correction formulas (see Section 2.2).

10.4.3 EFFECTS OF THE SIDEWALL BOUNDARY LAYER

In two-dimensional testing, the sidewall boundary layers are affected by the model and may cause serious interferences. These are not wall interferences in the classical sense, but it is appropriate to discuss these boundary layer effects in the context of target line techniques for three-dimensional flows. A method to reduce sidewall boundary layer interferences is presently developed at the DLR Göttingen in co-operation with ONERA/CERT (Michonneau [28]). The idea is briefly as follows.

Based on potential flow calculations about the wing section, the pressure distributions and subsequently the sidewall boundary layers are computed. The displacement thickness of the boundary layers induces interference velocities at the model which are computed either by linear flow theory or, in transonic flows, by means of a three-dimensional potential flow solver. Finally, the flexible top and bottom walls are adapted so that the interference velocities are eliminated along the central section of the model where pressure measurements are performed.

The wall adaptation is superior to global correction methods in cases where boundary layer interferences vary along the chord of the model. In this way models of larger chord length may be used.

10.5 MANIPULATIVE ALGORITHMS

The control of the test section flowfield afforded by wall adaptation may be exploited for purposes other than reducing the effects of wall interference in free-air simulations. For instance, the principle of wall streamlining allows the conditions at several different types of flowfield boundary to be simulated via interface matching techniques merely by imposing appropriate constraints in the fictitious flows. The range of two-dimensional Group 1 Flowfields already simulated in this way within flexible-walled test sections (Goodyer, [10]; Benvenuto and Pittaluga, [5] is illustrated in Figure 10.3.

Moreover, the facts that the wall boundary conditions are well defined and may readily be adjusted mean that the effects of introducing a variety of controlled perturbations to the flow may also be studied systematically in flexible-walled test sections. However, while the ability to actively manipulate rather than simply reduce the wall-induced perturbations to the flow may be expected to yield a number of novel freedoms to the practice of adaptive wall wind tunnel testing (Taylor and Goodyer, [35], [36], relatively little effort has yet been directed towards exploiting this interesting feature of wall adaptation.

The most notable attempts to exploit the manipulative nature of wall adaptation were made during a recent series of investigations by Lewis and Goodyer [24], [25]). Here, streamwise gradients of wall-induced blockage and upwash were intentionally introduced in order to gain an improved understanding of the effects of residual variations on wind tunnel test data and, wherever possible, to establish appropriate methods for correcting the data for these variations.

The scope of these studies was confined to the manipulation of two-dimensional Group 1 Flows. Although the effects of residual variations in blockage were investigated (by providing appropriate collective displacements of the flexible liners), efforts were focused on studying the seemingly more important effects of residual upwash gradients. This was accomplished by super-imposing displacements in the form of circular arcs onto wall contours that had been derived to minimise the effects of top and bottom wall interference. In this way, reasonably linear variations in upwash were generated over the model. The linearity of the gradient was, on occasion, refined by subsequent wall adaptation. Wall-induced blockage was kept to a practical minimum throughout.

The influence of linear variations in upwash along the tunnel centreline was studied by manipulating its gradient (via the radius of curvature of the circular arcs) and the point on the model chord at which the upwash was zero (via the streamwise location of the centre of wall curvature). The desired modifications to wall curvature were deduced using linear theory. A sample of the results is presented in Figure 10.13. This shows the sensitivity of the model lift coefficient at fixed geometric incidence. A clear pattern in the data is evident, namely that the lift coefficient is insensitive to the magnitude of the wall-induced camber provided it is centred at or near the three-quarter chord point. Therefore, this data provides evidence to support the validity of Pistolesi's three-quarter chord theorem (Thwaites, [38]), a widely used method for deducing corrections to model incidence for the effects of streamwise linear residual variations in upwash.

This theorem was subsequently invoked to construct portions of the lift-curve, the values of upwash at the three-quarter chord point being used to derive conventional corrections to the model incidence. In this way, the systematic manipulation of wall-induced upwash described above enabled the effective incidence of the model to be varied without adjusting its geometric attitude. The agreement between the resulting lift-curve and various independent sources of reference data provided further experimental corroboration of Pistolesi's theorem. The theory was then extended by Ashill et al. [3] to encompass the general case of non-linearity in the residual upwash variation as follows:

$$\delta\alpha = w(x_p)/U = (1/\pi)\int_0^{\pi}(w/U)(1-\cos\Theta)d\Theta \qquad (10.8)$$

where $\Theta = \cos^{-1}(1-2x/c)$ and x_p is the point on the model chord at which the residual upwash is used to produce the incidence correction, $\delta\alpha$. Note that linear variations in upwash yield $x_p=3c/4$. When the integral in this equation (rather than the upwash at point x_p) was used to construct the lift-curve, the agreement between the lift-curve and the reference data was improved.

During these experiments, it was found that the model's pitching moment coefficient appeared to exhibit similar trends in its sensitivity to wall-induced upwash as those illustrated in Figure 10.13 - although, in this case, the curves appeared to converge on a point towards the model's trailing edge. This observation suggested that a similar correction to incidence could be deduced from the residual wall-induced upwash. Subsequent analysis (Ashill et al., [3]) yielded:

$$\delta\alpha = w(x_p)/U = (1/\pi)\int_0^{\pi}(w/U)(1-2\cos\Theta+\cos 2\Theta)d\Theta \qquad (10.9)$$

an equation applicable to linear and non-linear residual variations. It may be verified that $x_p=c$ for a wall-induced upwash that varies linearly with chordwise position. In other words, for linear residual variations of wall-induced upwash, the appropriate point for making a correction to incidence on plots of pitching moment against incidence is the trailing edge.

Although derived in flexible-walled test sections, the conclusions of these investigations may be exploited in all wind tunnels where the assessment of wall interference is reasonably detailed. It is also interesting to note that the data presented in Figure 10.13 was obtained at conditions where a portion of the flow on the upper surface of the model was supercritical. This implies that the wall interference assessment procedure used throughout these investigations - a two-variable method based on Equation 4.14 - may be used with some confidence in the analysis of non-linear flows. Although further experimental evidence is required to substantiate this claim, it would appear that similar comments might also apply to the use of Equations 10.8 and 10.9.

The fact that the residual variations in wall-induced blockage and upwash about three-dimensional models may not be eliminated by two-dimensional wall adaptation would appear to make similar studies of their effects in three-dimensional testing extremely attractive. Aside from supporting the development of correction procedures that could be used in conventional, unadapted test sections (as reviewed in Section 1.3.2), these types of investigation would yield valuable guidance on the balance that will need to be made between data quality, its rate of acquisition and the mechanical complexity of the test section in designing adaptive wall facilities for production testing. They would also enable the extent to which some of the more novel potential uses for wall adaptation may be exploited in practice to be established. The limited data to hand (Lewis et al., [25]) suggest that the use of swept target lines may enable wall-induced camber and twist to be controlled almost independently about wings of high aspect ratio.

10.6 PRIORITIES FOR THE FUTURE

Even though more than two decades have elapsed since the dawn of the modern era of adaptive wall research, the pace of technique development continues to be rapid. With the range of potential applications also continuing to expand, there are many ways in which the technology could develop. For convenience, these are reviewed under three broad headings: data quality, rate of data acquisition, and complexity and cost. With the walls of the test section being adapted, these factors are clearly interrelated. Each exerts an important influence on wind tunnel productivity.

10.6.1 DATA QUALITY

There is now considerable evidence to suggest that with the walls of the test section adapted to minimise the effects of top and bottom wall interference in Group 1 free-air simulations, the quality of the test data produced in facilities with two flexible walls is superior to that obtained in any other type of wind tunnel. This appears to be true of two-dimensional (Elsenaar, ed., [6]; McCroskey, [27]) and three-dimensional (Lewis et al., [25]) testing. However, further work is required to establish whether flexible wall adaptation can yield similarly tangible benefits in Group 2-5 flows. If these cannot be established, there may be no alternative but to persist with a reliance on test section ventilation in these simulations - although, for the reasons outlined in Section 10.2.2, it should be noted that ventilated wall adaptation is not always successful in removing uncertainty from the test data (Neyland, [30]).

In view of the fact that in production testing, there is probably little to be gained from adapting the walls beyond the point at which the test data become amenable to reliable analyses, there is also a need to establish appropriate completion criteria, particularly for three-dimensional testing where residual variations are inevitable. The ability to study controlled perturbations to the flow in a systematic manner would appear to make flexible-walled test sections suitable platforms for developing these. Aside from supporting the development of correction procedures that could be used in conventional wind tunnels, these types of study would provide valuable guidance towards resolving important test section design issues (number and distribution of wall jacks, etc.). They would also enable the extent to which active manipulation of the flow may enable novel forms of wind tunnel wall correction to be devised. For instance, Taylor and Goodyer [35], [36] have suggested that the wall-induced wing twist present in three-dimensional testing might be tailored to simulate the aeroelastic deformations that occur during flight or to compensate for the distortion of the model under load. If proven, the latter feature would provide an alternative means for separating the effects of Mach number, Reynolds number and dynamic pressure in pressurised wind tunnels - a capability that is currently only available in cryogenic facilities.

10.6.2 RATE OF DATA ACQUISITION

Once suitable completion criteria have been established, it will be possible to determine the degree of wall adaptation required to provide acceptable levels of residual variation. The rate at which correctable test data may then be produced will be determined by the number of adaptation iterations required to produce acceptable data and the time required to complete each iteration.

Procedures capable of reducing residual variations to very low levels in one step have already been developed for simulations of two- and three-dimensional Group 1 free-air flows. It is also conceivable that a single wall setting may, on occasion, be acceptable for use over a range of test conditions. Several schemes have been proposed (Taylor and Goodyer, [35]) which seem likely to ensure that the rate of data acquisition is unlikely to be impeded by iteration, at least when simulating Group 1 Flows. Furthermore, recognising that future requirements for wind tunnel testing may be modified to accommodate the increasingly important role of CFD in the aerodynamic design process, there may be greater demand for some of the more novel uses of wall adaptation: it is possible to conceive of tests being conducted in twist (or camber) sweeps in addition to the more conventional longitudinal and lateral polars, for instance. There is clearly plenty of scope for research in developing highly productive wall adjustment strategies.

However, even if a reliance on iteration can be overcome, the requirements for rapid and accurate on-line assessments of wall interference and adjustments of the wall settings will need to be addressed if wall adaptation is ever to be used routinely in large industrial wind tunnels. These topics raise a number of test section design issues - such as the capabilities of the data acquisition system, the mechanical design and operation of the wall jacks and details of the overheads associated with the control logic required to safeguard against undesirable wall adjustments - that are beyond the scope of this AGARDograph. The following observations provide perspectives on the prospects for synchronising wall adaptation with changes to the test conditions (reference conditions and/or model attitude):

i. Methods have been devised which reduce the computational overheads associated with wall interference assessment and the prescription of improved wall settings in Group 1 free-air flow simulations to reasonably low levels. For instance, in test sections with twenty jacks on both the top and bottom walls, Equations 10.3 and 10.4 may be solved to govern the adaptation required in two-dimensional testing in fractions of a second on a modern personal computer. Equations 10.7a and 10.7b may be solved at similar expense in three-dimensional model tests - although the time required to assess the wall interference will rise if swept target lines are required (see Section 10.4.2.2).

ii. Mechanisms have been devised which allow reasonably rapid adjustment of the flexible walls. For example, in the test section of the pressurised and cryogenic T2 wind tunnel of ONERA/CERT, controlled adjustments to the wall contours are usually completed in under 0.5 seconds (Archambaud and Mignosi, [1]). It remains to be seen how far this type of timescale will be protracted in larger facilities. (The test section in T2 is 1.32m long, 0.39m wide and 0.37m deep.)

10.6.3 COMPLEXITY AND COST

It stands to reason that an adaptive flexible-walled test section will be more complex and costly to design, commission and operate than a conventional closed test section. Unfortunately, suitable measures of the value of these additional costs (and therefore acceptable limits for test section complexity) have proved rather difficult to establish. The reasons for this are reviewed briefly alongside suggestions by which the vagaries of the current situation may be resolved in Chapter 12. The additional complexity also implies additional risk - of system malfunction or failure to maintain acceptable levels of data repeatability, for instance. However, by appealing to the admissibility of the measured flow conditions at the control surfaces, schemes that may alleviate these concerns have been proposed (Taylor and Goodyer [36]).

The relative complexity of the techniques themselves (wall interference assessment and wall adjustments are required on-line), coupled with the fact that there is no one way of adapting the walls (the operator actually has a choice as to how the flow should be controlled) may also be viewed as sources of uncertainty and confusion amongst those unfamiliar with the technology. Therefore, once techniques have been developed to the point at which they may be used in production testing, effort in the forms of education and the development of robust, user-friendly operating sequences, will be required to install confidence in the user's mind that wall adaptation is actually removing important sources of uncertainty from the test data and providing additional capabilities that would not otherwise be available. In this respect, it may be helpful to note that several direct parallels may be drawn between the utilisation of aerodynamic control surfaces on aircraft and in flexible-walled test sections. Both are mechanically complex and costly to install and maintain and both provide improvements in aerodynamic performance that would not otherwise be available.

Paradoxically, precedent suggests (Barnwell et al., [4]) that concerns about the relative costs and benefits of wall adaptation will only be fully resolved when it is utilised in large-scale industrial facilities. With this in mind, it is interesting to note that most industrial tunnels actually already utilise some form of wall adaptation - while calibrating the test section (wall divergence), controlling the reference Mach number (second throat) or generating supersonic reference conditions (flexible nozzle), for instance. In these circumstances, its use is justified, presumably, on the basis that (i) the majority of the cost associated with determining the optimum wall settings is only incurred once, (ii) the walls are not usually adapted during a typical production test, they are merely adjusted to pre-determined settings and (iii) adjustments are not made very often during a typical test programme.

Wall induced upwash parameter	Straight walls	Adapted Walls		
		Root	Tip	Swept
Root camber	0.38	0.00	0.12	0.03
Mid-span camber	0.28	0.01	0.05	0.07
Tip camber	0.09	0.00	0.00	0.00
Mean twist	- 0.22	0.13	0.11	0.02
Max. to min. spread	0.50	0.13	0.18	0.07

Table 10.1 Summary of residual upwash variations for test case presented in Figure 1.11. (Straight walls v. three upwash target lines. All values quoted in degrees)

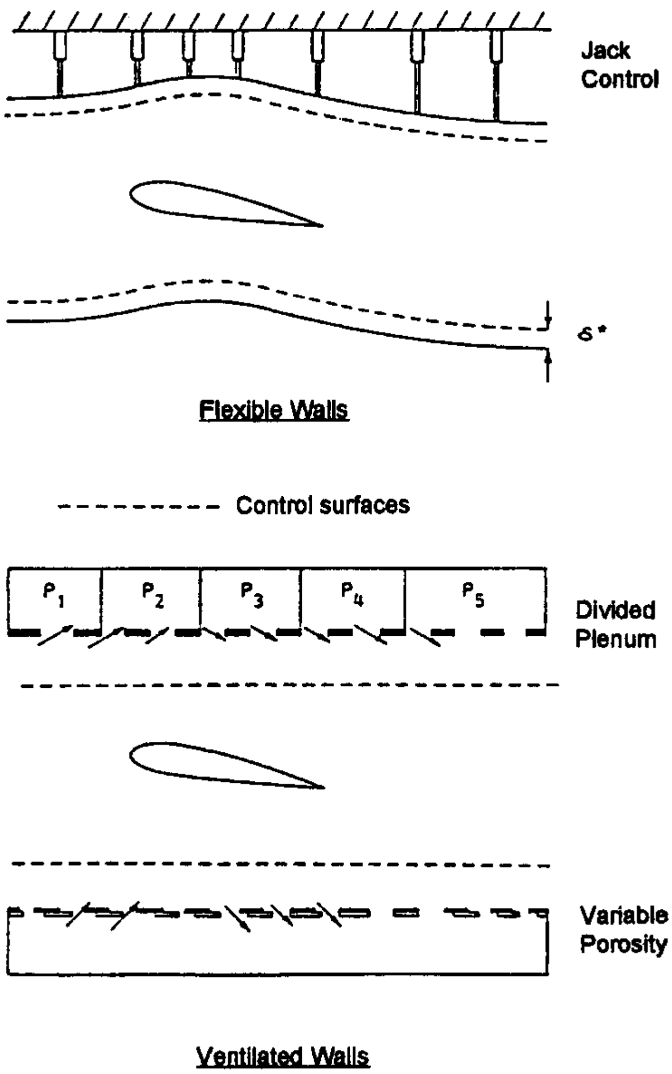
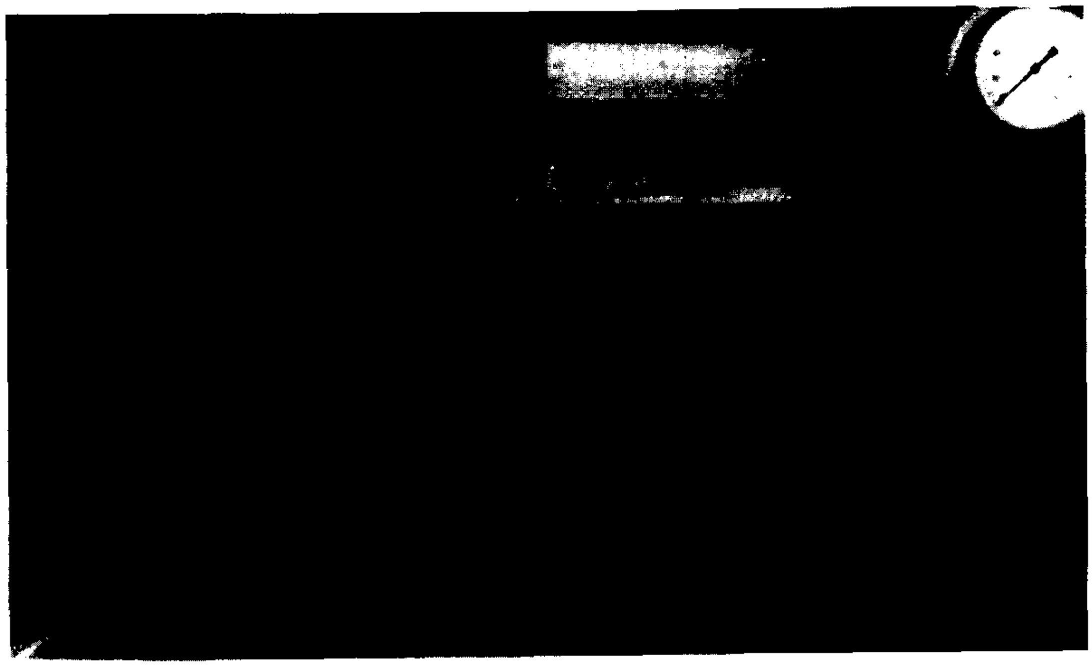
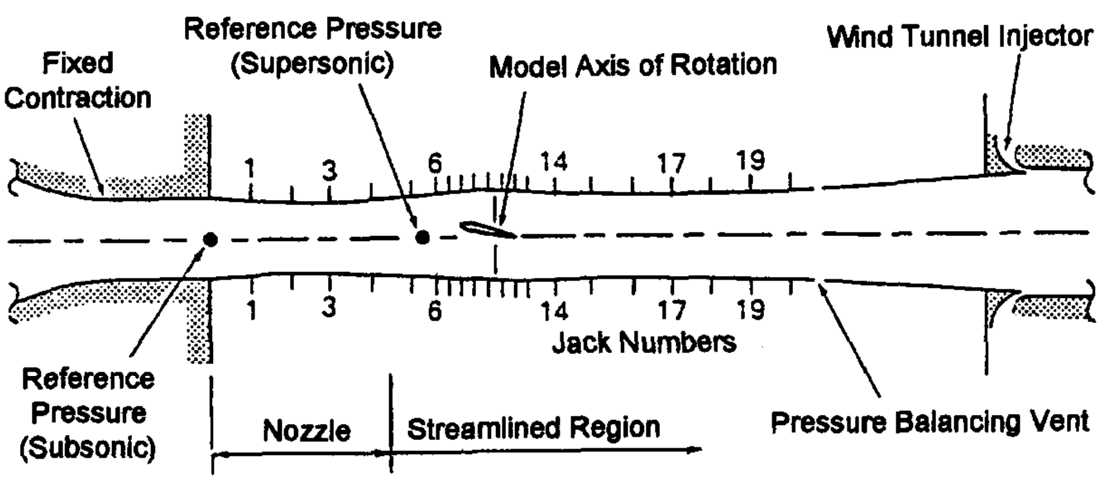


Figure 10.1 Two types of adaptive wall test section



a. Photograph of the two-dimensional testing arrangement, near sidewall removed.



b. Schematic layout of the test section

Figure 10.2 The adaptive flexible-walled test section of the Transonic Self-Streamlining Wind Tunnel of the University of Southampton

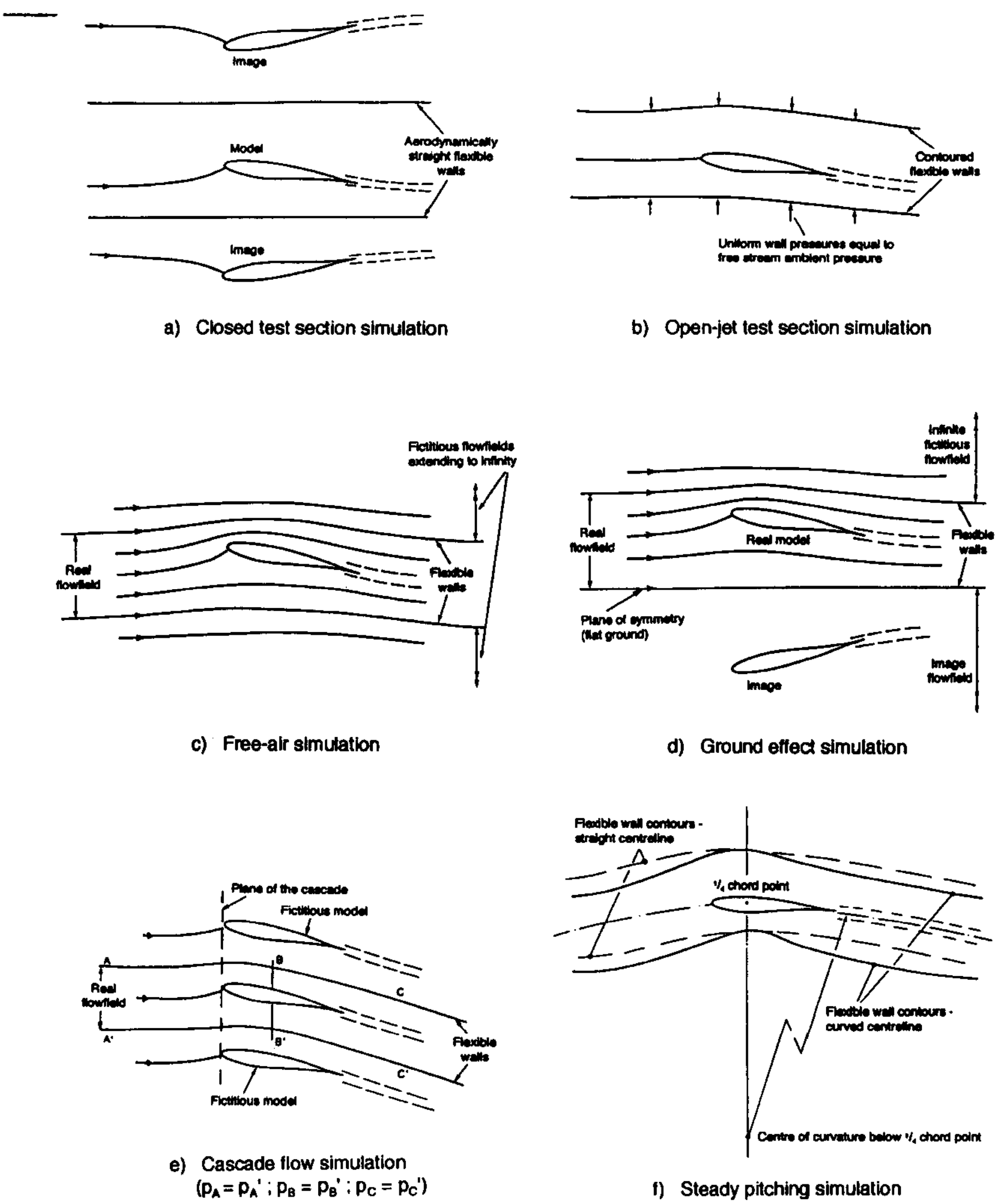


Figure 10.3 Examples of the ways in which the boundary conditions may be prescribed for two-dimensional testing in flexible-walled test sections.

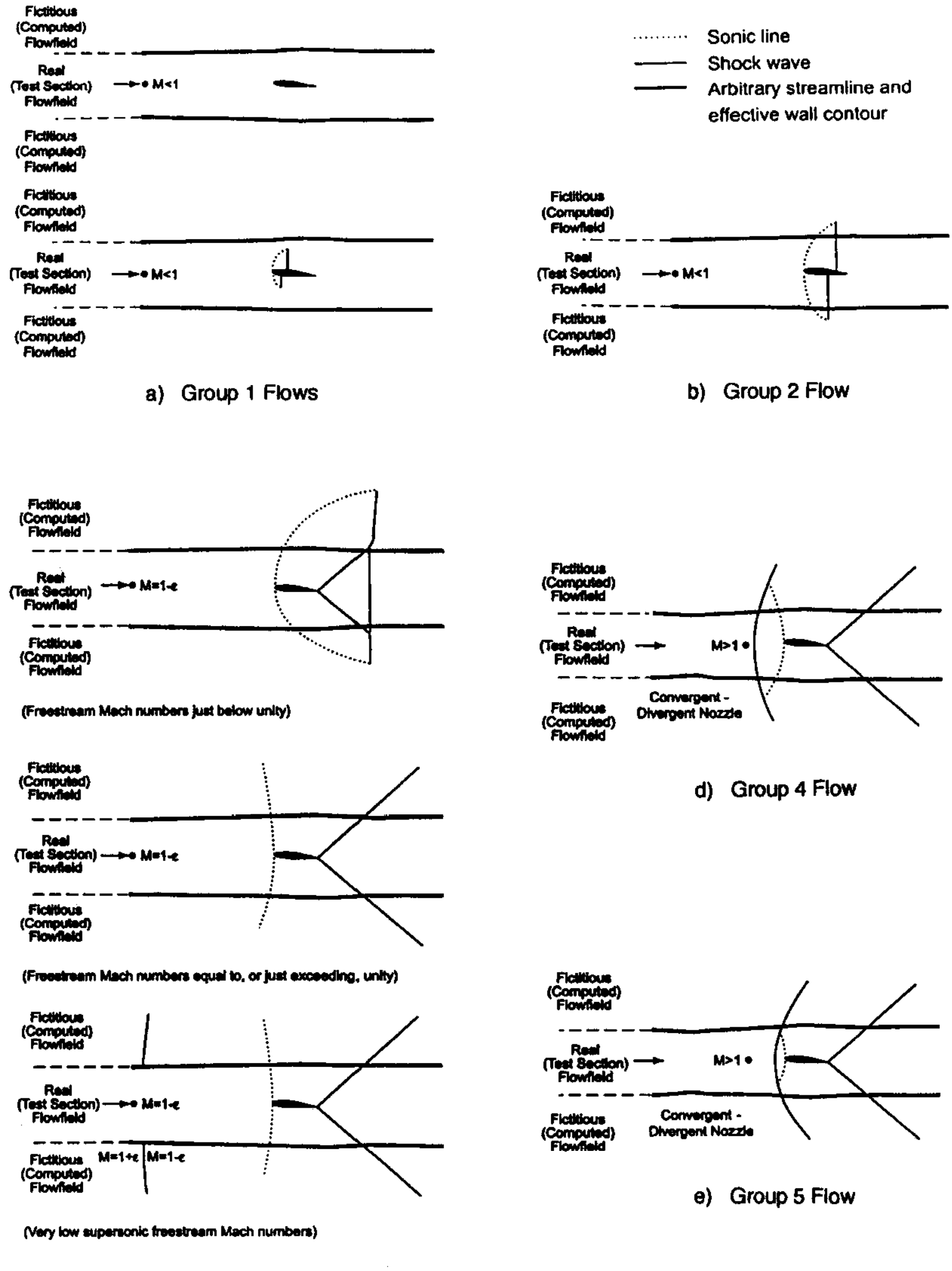


Figure 10.4 Ways in which the free air flowfield may be partitioned for two-dimensional interface matching in flexible-walled test sections

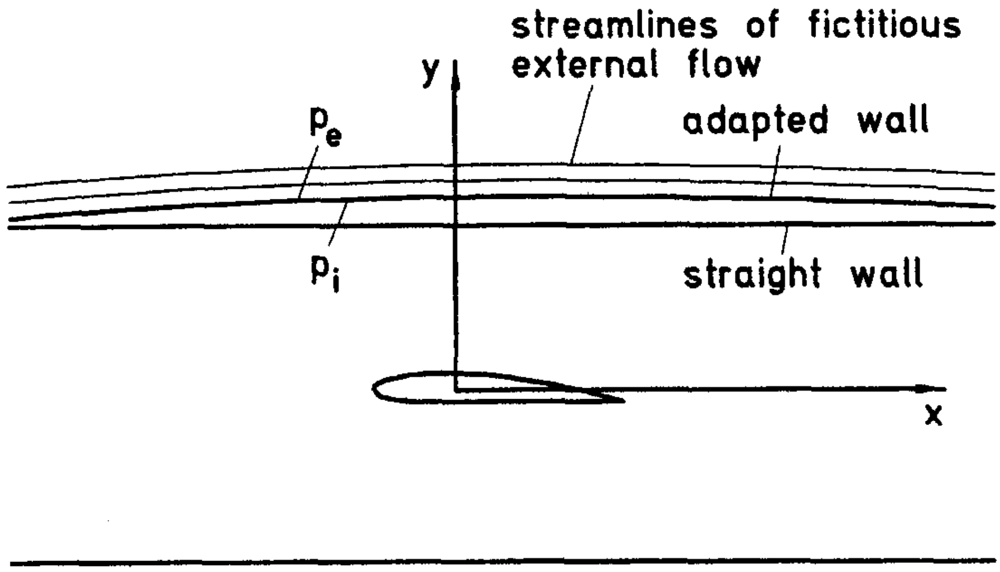


Figure 10.5 Internal and fictitious external flowfield

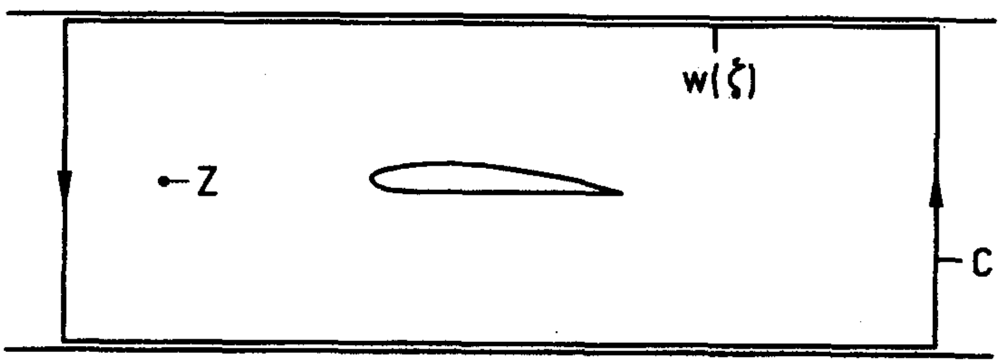


Figure 10.6 Path of Integration (Equation 10.2)

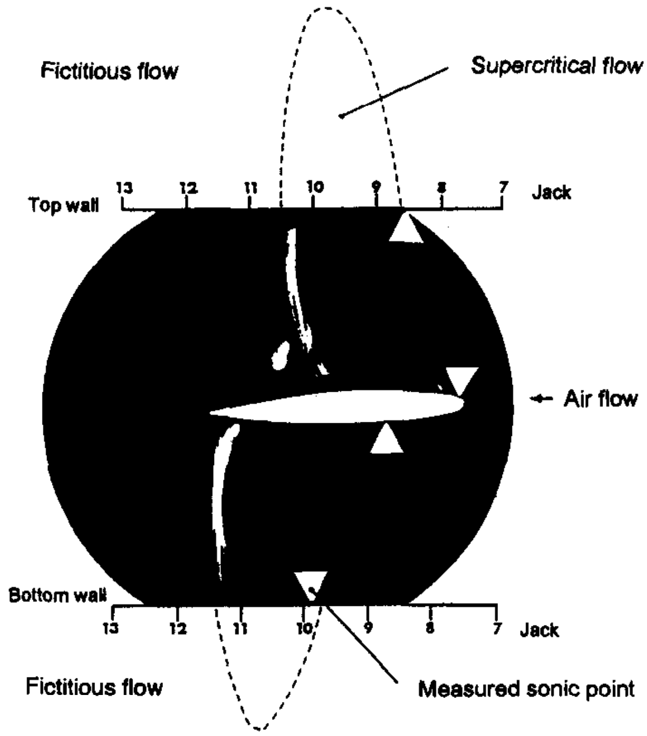


Figure 10.7 Montage of real and fictitious portions of a group 2 Flow after adapting the walls. NACA 0012-64 Section, M = 0,87, $\alpha = 4{,}0^{\circ}$ (Real flow : schlieren photograph; fictitious flow : TSP computation)

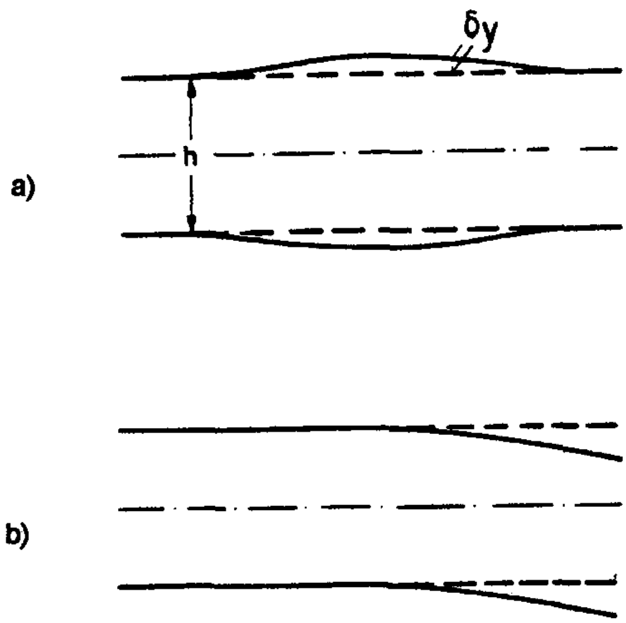
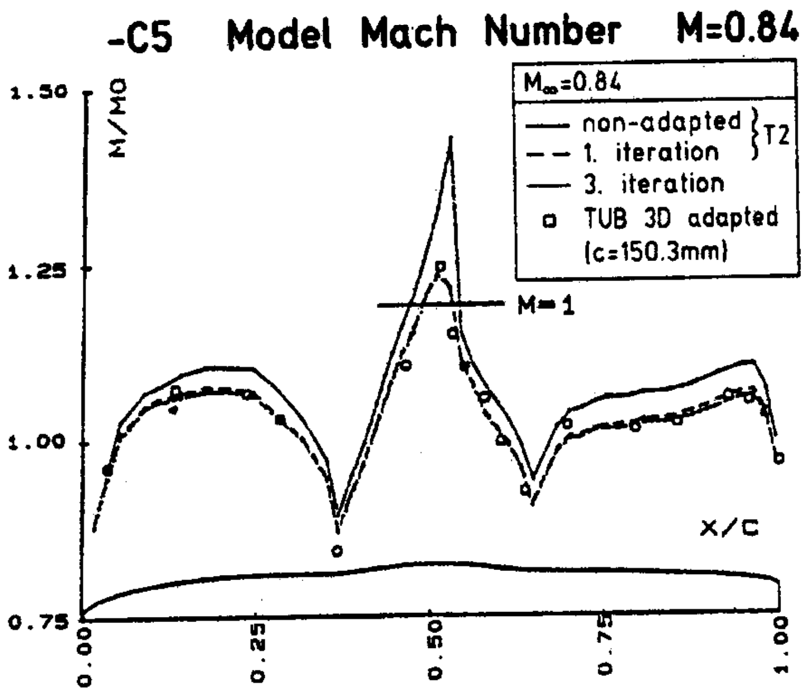


Figure 10.8 a) symmetrical and b) anti-symmetrical wall displacement



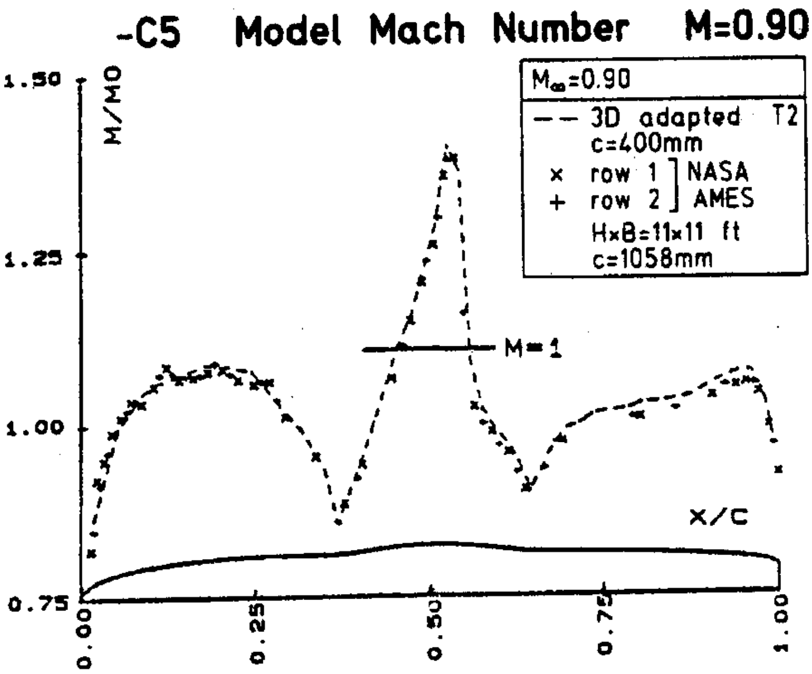


Figure 10.9a Pressure distribution on an axissymmetric body with 2 % blockage in the ONERA T2 adaptive wall wind tunnel. Comparison with interference free results.

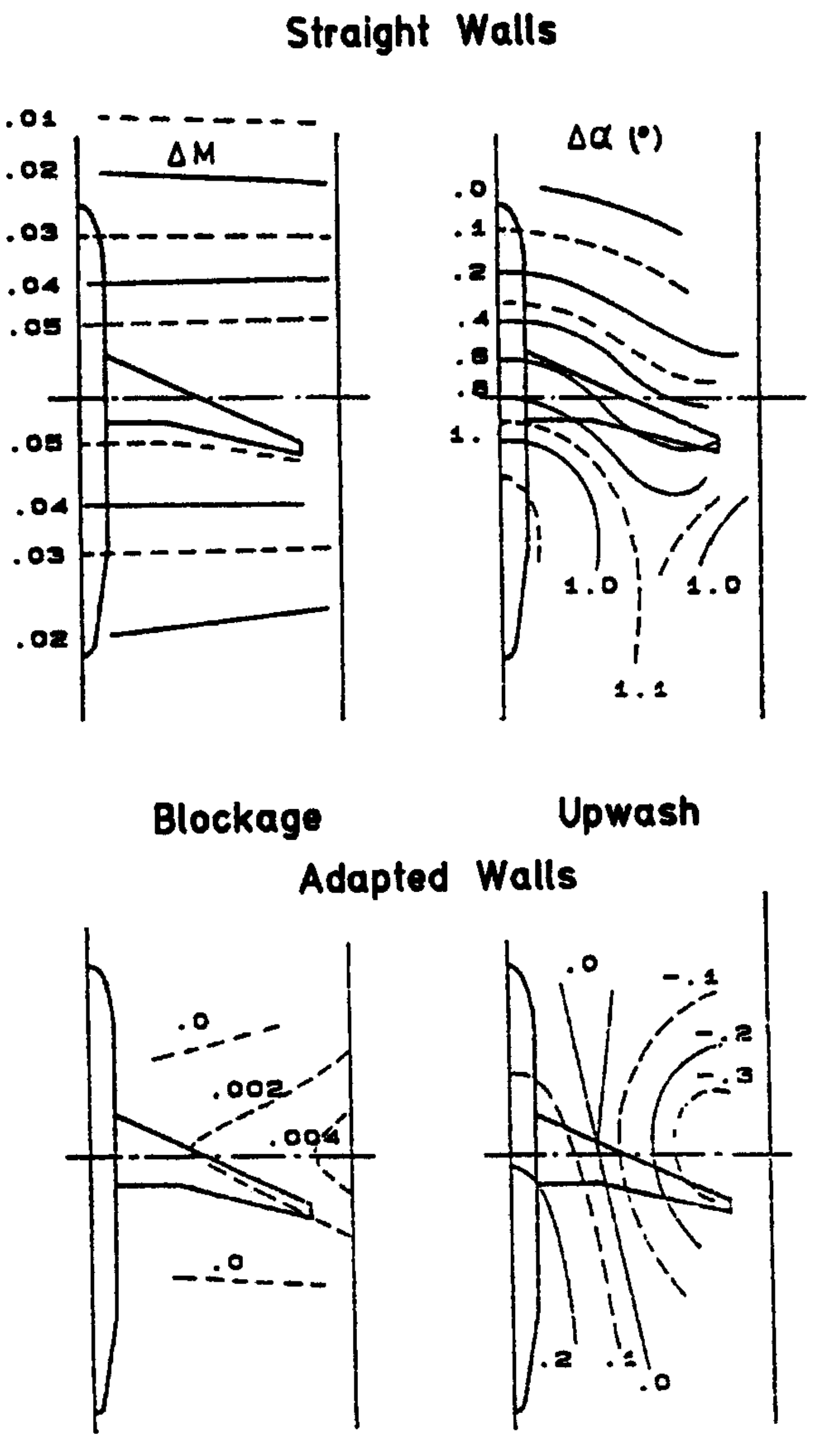
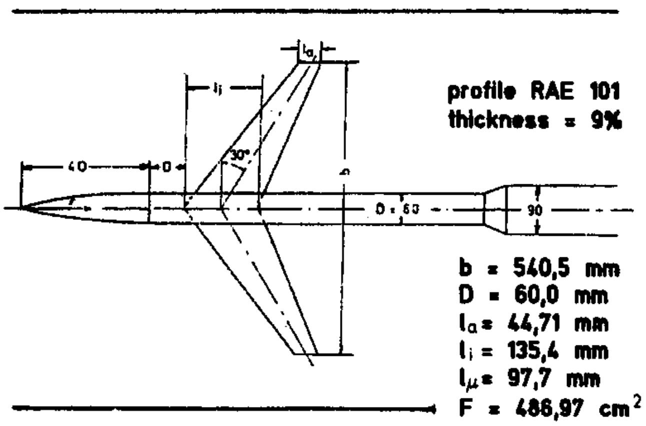
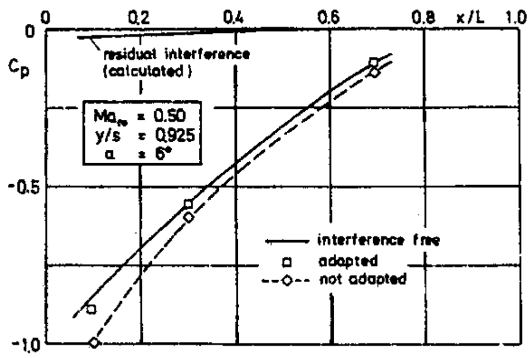


Figure 10.9b T2 half model. M = 0,78, α = 3.25 °. Contour map of wall induced blockage and upwash in the plane of the model.



a)

b)



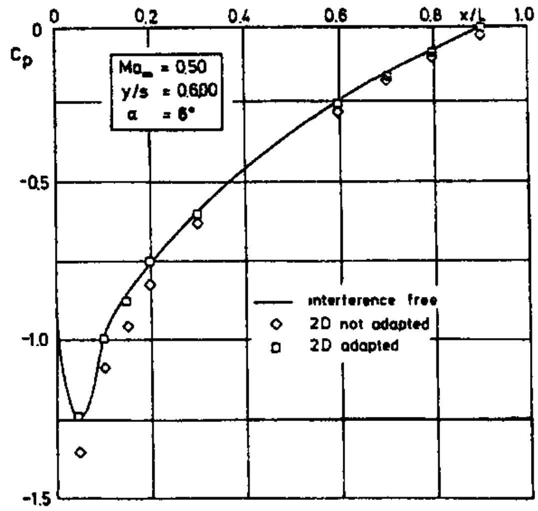


Figure 10.10 a) Planview of a model spanning 75% of the 0,75m x 0,75m adaptive wall test section of the HGK wind tunnel at DLR Göttingen.

b) Pressure distribution at wing sections y/s = 0,6 and y/s = 0,925 for the models shown in a)

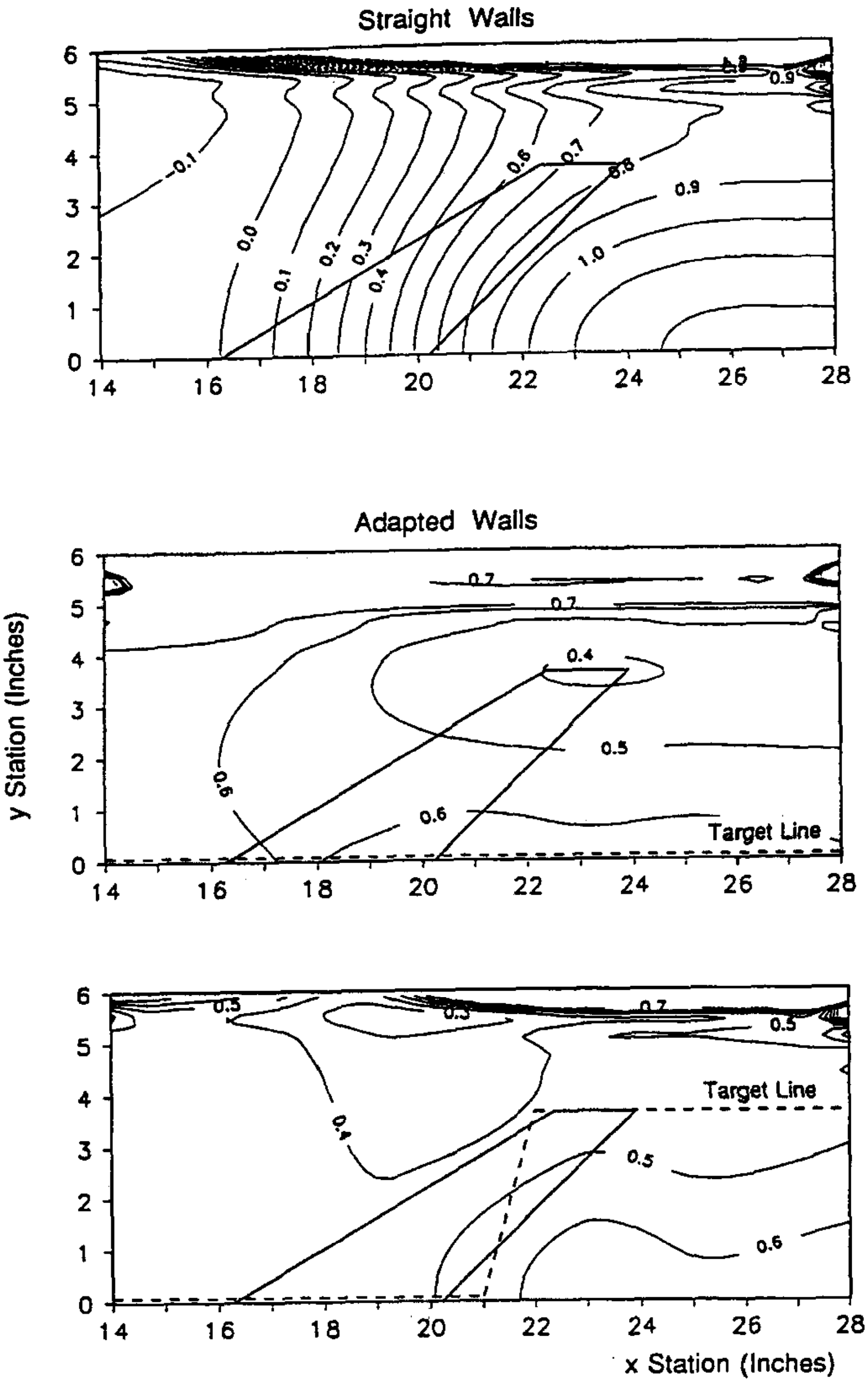


Figure 10.11 The effect of adapting the walls and sweeping the target line on the levels and distribution of wall-induced upwash in the plane of a simple half model ($M = 0{,}7$, $\alpha = 4°$, $b/h = 1{,}0$, contours of w/U, %)

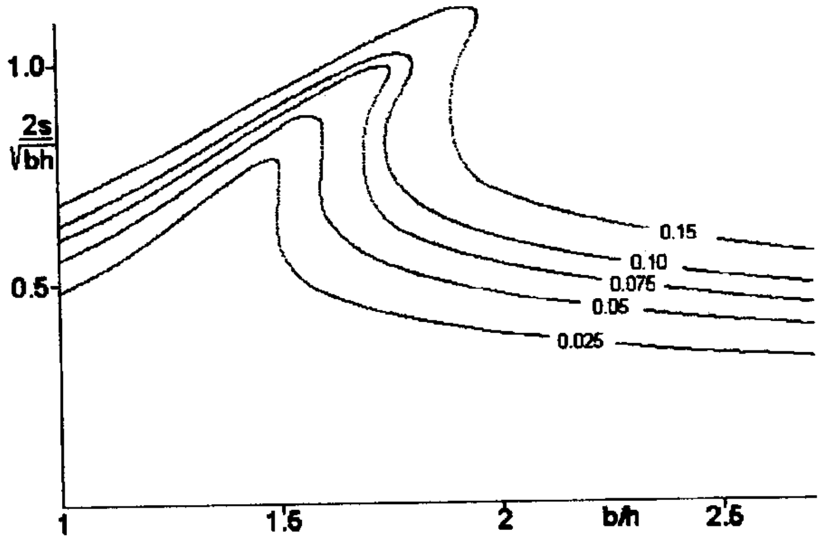


Figure 10.12 Contour lines of constant upwash variation in solid wall test sections of width to height ratio b/h

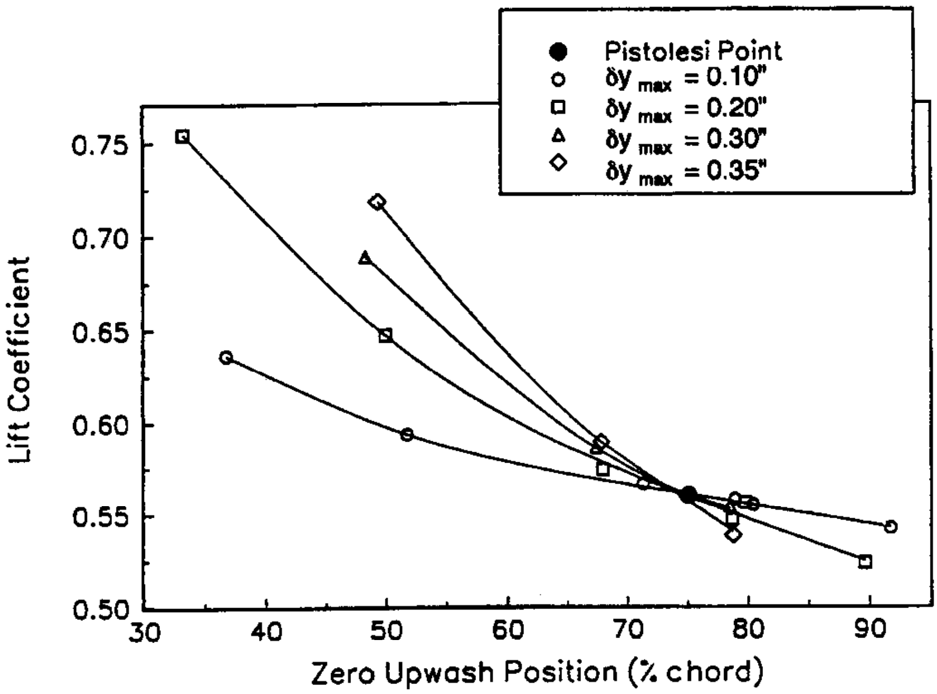


Figure 10.13 The influence of the magnitude and streamwise location of a linear gradient of wall-induced upwash on the lift coefficient of a two dimensional model (NPL 9510, M = 0,7, $\alpha \approx 2^{\circ}$)

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Appendix A

Cauchy's integral formula states that the integral $\int f(\zeta) / (\zeta - z)\, d\zeta$ taken about a positive oriented closed path in the complex plane containing z in the interior has the value $2\pi i\, f(z)$ if $f(z)$ is analytic in the interior, while the integral is zero if $f(z)$ is analytic in the exterior and zero at infinity.

In accordance with the assumption of linear flow theory, the flow within the test section may be decomposed into one part w_m due to the model in free air and one part w_{int} which is due to the wall interference. Thus $w = w_m + w_{int}$. The part w_{int} may be viewed as being generated by the images of the model reflected at the test section walls. This part has singularities only in the "exterior" flowfield while it is analytic in the interior flow (within the test section). The part w_m, however, has singularities only within the interior, while it is analytic in the exterior part and zero at infinity.

Applying Cauchy's integral formula to an interior point z and choosing $f(\zeta) = w_{int}(\zeta)$ the result is:

$$w_{int}(z) = \frac{1}{2\pi i} \int_C \frac{w_{int}(\zeta)}{\zeta - z}\, d\zeta$$

while :

$$0 = \int_C \frac{w_m(\zeta)}{\zeta - z}\, d\zeta$$

Adding the integrals and recognising that $w = w_m + w_{int}$, Equation 10.2 is obtained.

Appendix B

Computation of the influence functions

If u_s and u_{int} are known functions in a special case, Equation 10.6a may be considered as an integral equation for the unknown function Ω and, equally, Equation 10.6b as an integral equation for the function Γ. As an example, the computation of the influence function Ω is discussed in the following.

Writing $(\underline{\xi} - \underline{x}) = \underline{\eta}$ Equation 10.6a reads:

$$u_{int}(\underline{x}) = \int u_s(\underline{\eta}+\underline{x})\, \Omega(\underline{\eta})\, d\underline{\eta} \qquad \text{(A1)}$$

Discretisation and approximation of the integral by a sum leads to:

$$u_{int}(i) = \Sigma_k\, u_s(k+i)\, \Omega(k)\, \Delta\underline{\eta} \qquad \text{(A2)}$$

which is a system of equations for $\Omega(k)$ at discrete points $\underline{\eta} = k$. When solving Equation A2 numerically, care must be taken that the matrix $u_{s,i,k} = u_s(k+i)$ is not singular. For the present, the influence function Ω

is determined so as to solve Equation A1 for a special choice of functions u_{int} and u_s. The general validity of Equation A1 will be shown in the following.

Proof of the general validity of Equation A1

The functions u_{int} and u_s must be computed in a special case. They may be derived from the velocity field generated by a source doublet and its images (see for example Equation 2.60). Let $u_s^0(x-\xi)$ be the velocity at the wall and $u_{int}^0(x-\xi)$ the interference velocity generated by a source doublet at the location ξ and its reflections. The influence function Ω is determined so as to solve the equation:

$$u_{int}^0(x) = \int u_s^0(\eta+x)\, \Omega(\eta)\, d\eta\ . \qquad \text{(A3)}$$

The most general symmetric flow can be generated by a distribution of doublets of strength $q(\xi)\, d\xi$. With this the wall velocity and interference velocity at a point $\eta+x$ become:

$$1)\ \ u_s(\eta+x) = \int q(\xi)\, u_s^0(\eta+x-\xi)\, d\xi \qquad 2)\ \ u_{int}(x) = \int q(\xi)\, u_{int}^0(x-\xi)\, d\xi \qquad \text{(A4)}$$

Multiplying the first Equation A4 by $\Omega(\eta)$, integrating by η and using Equation A3 yields:

$$\int u_s(\eta+x)\, \Omega(\eta)\, d\eta = \int q(\xi) \int u_s^0(\eta+x-\xi)\, \Omega(\eta)\, d\eta\, d\xi = \int q(\xi)\, u_{int}^0(x-\xi)\, d\xi = u_{int}(x) \quad \text{q.e.d.}$$

Computation of the wall displacement

The wall adaptation strategy aims at eliminating the wall interferences along the target line by displacing the walls so as to generate velocity distributions $u_c(x) = -u_{int}(x)$ and $w_c(x) = -w_{int}(x)$. In the following, the target line is assumed to be the centreline of the test section. $u_c(x)$ is generated by a symmetric wall displacement $d_s(x)$ as shown in Figure 10.8a and $w_c(x)$ by an anti-symmetric wall displacement $d_a(x)$ (Figure 10.8b). In order to derive a relation between $u_s(x)$ and $d_s(x)$ the disturbance potential generated by displacing the wall symmetrically may be expanded in a power series:

$$\Phi(x,z) = a_0(x) + a_2(x)\, z^2 + a_4(x)\, z^4 + \ldots \qquad \text{(A5)}$$

Considering that Φ must be a solution of the disturbance equation $\beta^2 \Phi_{xx} + \Phi_{zz} = 0$ it is found that: $a_2 = -\beta^2 a_0''/2$ and $a_4 = \beta^4 a_0''''$, where a_0'' and a_0'''' are the second and fourth derivatives.

The axial velocity at the centreline is (using non-dimensional quantities $\mathbf{u_s} = u_s/U_\infty$, $\mathbf{d_s} = d_s/h$ etc.):

$$(\partial\Phi/\partial x)_{z=0} = a_0'(x) = \mathbf{u_c}(x) = -\mathbf{u_{int}}(x) \qquad \text{(A6)}$$

The normal velocity at the wall position $z=h/2$ is:

$$(\partial\Phi/\partial z)_{z=h/2} = -\beta^2 h\, a_0''/2 + \beta^4 h^3 a_0''''/48 + \ldots = \mathbf{w_c}(x)\ . \qquad \text{(A7)}$$

As $a_0' = \mathbf{u_c} = -\mathbf{u_{int}}$ is a slowly varying function of x, the higher derivatives in the power series of Equation A7 may be neglected. The wall displacement is then: $d_s(x)/h = \mathbf{d_s}(x) = \int \mathbf{w_c}(x)\, dx = -\beta^2 a_0'/2 + \beta^4 h^2 a_0'''/48$, or, with Equation A6:

$d_s(x) = \beta^2/2(u_{int} - (1/24)u_{int}'')$ (A8)

where $''$ is used to denote the second derivative with respect to the normalised variable $\underline{x}=x/\beta h$.

Equation A8 may be used immediately to compute the required wall displacement when $u_{int}(x)$ is known by evaluation of Equation 10.6a. Computing the second derivative of $\underline{u}_{int}(x)$ numerically is, however, intrinsically inaccurate if u_{int} is given at discrete points. Higher accuracy and more convenience is attained by taking the second derivative of Equation 10.6a:

$u_{int}''(x) = \int u_s(\xi)\, \Omega''(\xi-x)\, d\xi/\beta h$ (A9)

Inserting Equation A8 into Equation A7 gives:

$d_s(x) = \int \beta^2\, u_{s0}(\xi)\, X(\xi - x)\, d\xi\, /\beta h$ with: $X = 0.5\, (\Omega - \Omega''/24)$ (A10)

Equation A10 is the wall adaptation formula (Equation 10.7a) for the case that the walls are initially straight ($d_{s0}=0$).
If the wall adaptation is performed from a state of pre-adapted walls ($d_{s0} \neq 0$, $d_{a0} \neq 0$), the influence functions M and N must be known (Equations 10.7). M and N may be computed in the following way:

We consider the flow in an empty test section (test section without model). In this case the adapted walls will be straight, i.e. $d_s = d_a = 0$. We further assume that the walls are pre-adapted, but so that $d_{s0}(\xi)=X(\xi)$ and $d_{a0}(\xi) = \Lambda(\xi)$. With these assumptions Equation 10.7a becomes:

$$0 = \int \beta^2\, u_s(\xi)\, X(\xi-x) + X(\xi)\, M(\xi-x)\, d\xi/\beta h. \quad \text{(A11)}$$

Using the transformation $\xi \rightarrow -\underline{\xi} + x$ on the second part of the integral and considering that $X(-\xi)=X(\xi)$ and $M(-\xi)=M(\xi)$ we obtain:

$$0 = \int \beta^2\, u_s(\xi)\, X(\xi-x) + M(\xi)\, X(\xi-x)\, d\xi/\beta h \quad \text{(A12)}$$

Equation (A12) suggests that $M(\xi) = -\beta^2\, u_s(\xi)$. Thus, by computing the flowfield in an empty pre-adapted test section with $d_{s0}(\xi) = X(\xi)$, the wall velocity $u_s(\xi)$ is generating the function $M(\xi)$. The computation of $N(\xi)$ is analogous.

For the case of a square test section ($b/h = 1$) the functions Ω, Γ, X and Λ were computed numerically and tabulated by Lamarche & Wedemeyer [20]. Plots of these functions and functions M and N are shown in Figures 10.B1 to 10.B3. The symbols indicate computed values, the lines are analytic interpolations fitted to the numerical curves. For fast computations of wall interferences and wall deflections it is advantageous to use analytic functions rather than tables of the influence functions. (Using tabulated values necessitates time consuming interpolations). The use of algebraic functions approximating the numerical curves have reduced the computing time by orders of magnitude. The approximating formulas are shown within the figure captions. They are derived with consideration of global conditions as the correct value of the integral and asymptotic behaviour at infinity. The parametric constants have been adjusted to yield the best fit.

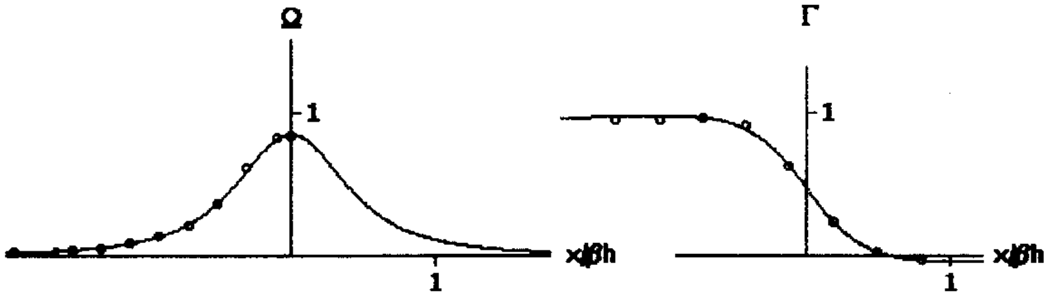

Fig.10.B1 Influence functions Ω and Γ. Symbols: numerical values, lines: interpolation formulas: $\Omega = e/2/(e+x^2)^{3/2}$, with: $e=0.346$; $\Gamma = e_1 + e_4\, x/(e_3+x^2)^{1/2} - (e_1+e_4)\, x/(e_2+x^2)^{1/2}$ with: $e_1=0.47$, $e_2=0.40$, $e_3=0.60$, $e_4=2.0$. $x = x/\beta h$.

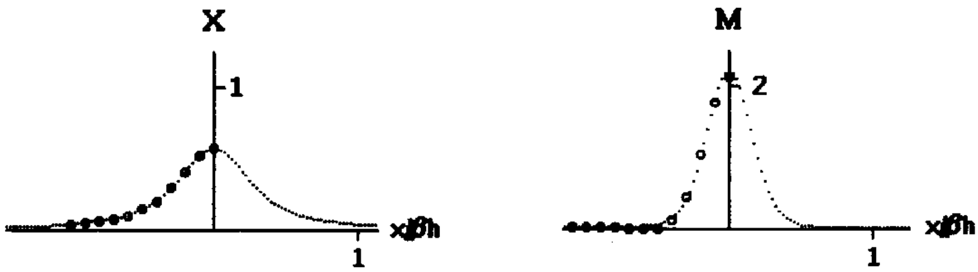

Fig.10.B2 Influence functions X and M. Symbols: numerical values, lines: interpolation formulas: $X = e/4/(e+x^2)^{3/2} + e(e-4x^2)/32/(e+x^2)^{7/2}$ with $e=0.346$, $M = e_1/2/(e_1+x^2)^{3/2} + e_3(e_2-4x^2)/(e_2+x^2)^{7/2}$ with $e_1=0.22$, $e_2=0.22$, $e_3=0.026$. $x = x/\beta h$.

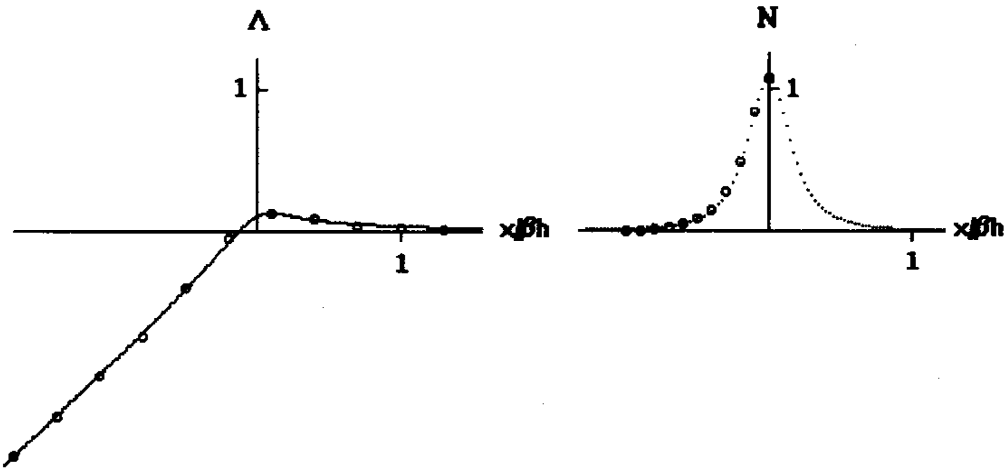

Fig.10.B3 Influence functions Λ and N. Symbols: numerical values, lines: interpolation formulas: $\Lambda = c_1 x + c_4(c_3+x^2)^{1/2} - (c_1+c_4)(c_2+x^2)^{1/2}$, with: $c_1 = 0.47$, $c_2 = 0.03$, $c_3 = 0.09$, $c_4 = 1.5$.
$N = e_1/4/(e_1+x^2)^{3/2}$, with $e_1 = 0.055$. $x = x/\beta h$.