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11. ADDRESSING UNCERTAINTIES IN WALL CORRECTIONS

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11.1 INTRODUCTION

The previous ten chapters of this report have served to indicate the progress in wind tunnel wall correction technology during the thirty years since the publication of AGARDograph 109 by Garner, et al. [14]. Although significant progress has been made, much of it due to the emergence of enhanced computational capabilities which have been used in conjunction with the wind tunnel and its data, the present status of the wall interference technology has certainly not yet matured for the high-speed, or high-blockage, or high-lift, or powered-lift, or time-dependent flows. These flows, discussed in Chapters 5-9, typically exhibit large gradients, may be particularly sensitive to small perturbations due to the critical non-linear transonic and/or viscous effects that are present, and perhaps are not even amenable to correction in conventional wind tunnels. On the other hand, the adaptive wall technology, developed during these intervening thirty years and discussed in Chapter 10, may provide a means for providing correctable data; but it has not yet become a "production-testing" capability. However, at its present level of development, this latter technology should prove to be an extremely valuable test bed for understanding and evaluating wall interference phenomena, concepts, procedures, and limitations, as indicated in sections 1.3.2 and 10.5, for example.

Chapters 2 through 10 have outlined models for estimating wall-induced interference effects for different types of aerodynamic tests and test section wall types; each chapter has most adequately presented associated model-specific limitations and constraints. Chapter 1 of this report addresses the major assumptions; many of the general limitations of wall corrections have necessarily been discussed there as prerequisite to selecting an appropriate correction methodology. Chapter 12 discusses limitations from the standpoint of "Future Necessary Work" required to eliminate them as constraints on current testing techniques and correction capabilities.

Customers are requiring better quality wind tunnel data; that is, data with its uncertainty quantified and reduced to acceptable levels. As already pointed out, current and continuing customer requirements for these data at higher Reynolds number, yet obtained faster and cheaper, places even greater demands on wall (and tunnel) interference corrections and the uncertainties in them. Understanding the limitations of wall corrections is relevant not only to the corrections themselves, but also to their range of applicability and their uncertainties. The present chapter focuses on understanding the sources of uncertainties and approaches to resolving those uncertainties associated with wall corrections and methodologies. These aspects will arbitrarily be divided into three groups, with a major section for each: the fundamental aspects, the experimental aspects, and the computational aspects, respectively.

The limitations discussion herein will be for a broad interpretation of the word. That is, one will not find quantitative information such as

Method A, based on pressure signatures measured at 25 stations along top and bottom wall centrelines in Wind Tunnel B, produces wake blockage corrections accurate to ±0.001 for solid model blockage ratios between 0.3 and 0.6 at Mach numbers between 0.1 and 0.4 and chord Reynolds numbers around 3 million, adding $20 K to the cost and two days to the tunnel test time. However, Method B ....

Ultimately, such quantitative information will be required for characterising both correction methods and facilities so that customers can make informed technical and economic decisions about wind tunnel testing options. As already seen in the previous chapters, such quantitative information is generally not available and it depends not only on the wall correction method, but also on the facility and its testing procedures, staff, instrumentation, data reduction, quality control, costing algorithms, ... ; i.e., aspects governed by institutional and customer commitments and resources. In the broader sense, limitations follow from assumptions, model sophistication, and physical or economic constraints.

11.2 FUNDAMENTAL ASPECTS

While Whoric and Hobbs [25] have cited wall interference effects as a significant source of uncertainty in wind tunnel data, in the past frequent discussions have occurred over whether or not to apply wall interference corrections. These discussions centred around the belief that no correction (a quantity of zero magnitude and unknown uncertainty) is better than a correction of known magnitude and unknown uncertainty, particularly when the experiments were conducted at transonic conditions where any verifiable correction was obtained with difficulty. Many times corrections were accepted based on whether they moved the test results in the perceived right direction; otherwise, they were rejected. It is now known that this perception can be false due to non-linear effects caused by compressibility and/or viscous interactions (see Chapter 5). Furthermore, a correction could overshoot the free-air solution (or the *"true value"*) even if the sign is correct. Classical and conventional wall correction ideas and procedures generally work well for most subcritical flows and do provide guidance in the higher speed flow regimes. However, uncertainties have not been established for most of these latter correction procedures by a formal error propagation technique.

11.2.1 CONSEQUENCES OF THE FUNDAMENTAL ASSUMPTION

As pointed out in section 1.1, the fundamental assumption underlying the theory and practice of most wind tunnel testing is that

there exists an "equivalent" free-air or unconstrained flow to or with which the aerodynamic quantities measured under strictly controlled and defined conditions can be associated or applied.

The goal of wind tunnel wall corrections is to find or deduce this association.

11.2.1.1 REGARDING " 'EQUIVALENT' FREE-AIR FLOW"

What must be recognised is that the equivalent free-air flow (i.e. the *"true value"* or the *"truth"*) can never be determined with exactness, whether this determination is made via experimental or theoretical means. Rabinovich [22] presents the following postulates of the theory of measurements:

(1) the true value of the measurable quantity exists;
(2) the true value of the measurable quantity is constant; and,
(3) the true value cannot be found.

The basis for these postulates is that the modelling or measurement of any physical system is an imperfect act and that the randomness in the result will cause it to scatter about the true value.

Analytical knowledge about or realistic assessment of this equivalent free-air flow (i.e., the "true value") remains elusive for all but perhaps streamlined flow about some simple shapes in physical flow regimes adequately described and approximated by linearised equations. Knowledge of this "truth" for realistic configurations is the desired goal of wind tunnel testing and CFD analysis. Both schools of endeavour have means for approximating the truth as discussed in the next two sections and Chapter 12. Not knowing this truth, for little more than some simple flows, is a fundamental limitation in assessing the practical limits of validity and uncertainty in wall correction methods. In fact, as noted in Chapters 1 and 5, rigorous definition of just what property (or properties) should be constrained or matched (the equivalence condition) in order to establish this correspondence between wind tunnel and equivalent free-air flows is open to discussion.

Experimentally, aerodynamic quantities measured on very small models tested in large tunnels are generally deemed to be interference free and to represent "truth". Testing relatively small models at low speeds is deemed to produce a small (linear) perturbation from free-air flow with primary corrections for the freestream magnitude and direction. As the relative model size increases, gradients and other nonuniformities in the wall interference show up on the model; these residual variations in interference lead to residual corrections as discussed in Chapter 1. However, nonlinearities due to compressible and viscous effects can also occur and produce flow nonuniformities that are not readily separable from the residual wall interference determined by conventional correction methods. When viscous nonuniformities become more severe and begin to predominate, their effects must be modelled or accounted for in the correction procedure.

In addition, the decomposition of corrections into primary and residual is also influenced by the choice of equivalence condition. Concepts and ideas derived from linear flow and theory must be re-examined to find their limitations in correction methods for other flow regimes.

11.2.1.2 REGARDING "MEASURED AERODYNAMIC QUANTITIES"

Increasing demands on the accurate measurement of aerodynamic quantities of direct interest to the customer generally requires more accurate measurement of many other parameters and quantities such as those related to reference conditions, tunnel control, instrument limits, safety, model control, wall data, support data, statistical correlations and assessments, flow quality, etc. One must quantify the uncertainties with every measurement and procedure, formally propagating these errors in order to establish, via continued accumulation, a statistical estimate of accuracy of the measured aerodynamic quantities. In many processes, this uncertainty quantification may be achieved by end-to-end replication thereby capturing and accounting for variations in all environmental variables, which may not be included in the data reduction equations. Measurement calibrations must be done via fixed procedure in order to establish the repeatability and its credibility at strictly controlled conditions. In particular, use of boundary measurement correction methods, as introduced and discussed in Chapter 4 and in all those chapters that follow it, requires accurate data measurements near the test section walls.

11.2.1.3 REGARDING "STRICTLY CONTROLLED AND DEFINED CONDITIONS"

Strictly controlled and defined (flow) conditions means more than just producing repeatability; it implies a continuing statistical assessment and configuration control of the experimental techniques, procedures, and all related processes for both software and hardware. One does not produce accurate aerodynamic data without the strict control required to define the conditions and quantify the measurement uncertainties (Belanger [3]; Croarkin [13]).

11.2.1.4 REGARDING "CAN BE ASSOCIATED OR APPLIED"

The task of wall interference correction is to find or deduce the association between the aerodynamic quantities measured under strictly controlled and defined conditions and a corresponding equivalent free-air flow, if one exists! There is uncertainty (hopefully quantified) in the measured data, there is uncertainty (generally not quantifiable) in approximating the "truth" (equivalent free-air flow, however it is assumed to be known or represented), and there is uncertainty in satisfying the equivalence or matching condition. All of these uncertainties contribute to the absolute uncertainty in the wall correction. If one accepts the approximation of truth as the absolute truth, then some quantitative measure of the uncertainty in satisfying the equivalence condition must also be propagated with the measurement uncertainties through the correction procedure to get the uncertainty of the wall corrections. If this latter uncertainty is small, relative to the corrections themselves, then the measured flow could be considered correctable; i.e., the association can be made. If not, then one must be prepared to modify the fidelity of the correction procedure, adapt or shape the tunnel walls (see chapter 10), or perhaps even adapt the model itself and then try again or else quit! Criteria for assessing what flows are <u>not</u> correctable within a given facility's capabilities must be established and readily available on line during testing.

11.2.2 COMPATIBILITY OF HARDWARE, SOFTWARE, AND PROCEDURES

Conventional or classical wall correction methods, as discussed in Chapters 2 and 3, can be applied to measured wind tunnel aerodynamic data knowing only a few characteristic tunnel and model dimensions and flow conditions. These parameters size and locate the linearised flow singularities and their images, allowing one to calculate and superimpose flow solutions to obtain the interference field. Questions and concerns about the compatibility of the tunnel (hardware) and its procedures with the correction method (software) and its procedures were of little concern. However, with the advent of boundary measurement (Chapter 4) and adaptive wall (Chapter 10) methods for wall corrections, many compatibility issues appear and must be resolved. These run the gamut from the basic calibration and bookkeeping of corrections (discussed in section 1.2) through the automated, integrated control of adaptive wall tunnels. If the correctable-interference tunnel concept of Kemp [16], discussed in section 5.1.4, is to become a production reality, then the hardware, software, and procedures associated with tunnel operation, data acquisition, data processing, wall interference assessment, limited wall control, and wall corrections must not only be made compatible, but also integrated and automated. In view of the rate at which computer capabilities improve, modularization will be highly desirable. If one is to have a hierarchy of potential correction methods, each requiring different measured data, then optional hardware and procedures will also be needed.

Hardware, software, and procedural compatibilities are also required in regard to obtaining, quantifying, and maintaining the customer-specified uncertainties in the measured aerodynamic data and wall or tunnel corrections. Sloppy tolerances at only one point in the chain, whether due to hardware, software, or procedural uncertainties, lead to inaccurate results. Limitations in the wall corrections can result from incompatibilities in the hardware, software, or procedures. For example, if some required input data for a correction method is not measured, then it must be estimated, deduced, or effectively neglected. Solution of boundary value problems generally require boundary data on all the boundaries; and wall corrections are attempting to account for the wind tunnel's imposition of the constraining (wrong) far-field boundary values on the measured aerodynamic data.

11.2.3 CONSEQUENCES OF PHYSICAL AND ECONOMIC CONSTRAINTS

Trade-offs and compromises have and will continue to be made with respect to our modelling the required physical phenomena and the cost in time and money for modelling. When it becomes important to meet a specified small uncertainty in a simulation, then both physical and economic constraints become even more contradictory. The more accurate data at higher Reynolds number obtained faster and cheaper places severe demands on tunnel testing and corrections. This scenario will be accomplished by producing less data, but of much better quality. More time and effort will be put into customising (or should we say "customerising") the wind tunnel tests for obtaining the required results from among readily available testing options at a facility. Both institutional and customer commitment and resources will be required; lack of either, seriously cripples what can be done to obtain accurate aerodynamic data.

As pointed out elsewhere (Chapter 12, in particular) much can be done with regard to implementing those wall correction methodologies (hardware and software) presented in the previous chapters herein. However, such implementation involves a commitment of people and money, both of which seem to be dwindling in the wind tunnel and CFD disciplines recently. This situation is a limitation and will remain so until such time as the stakeholders and/or customers can be convinced otherwise.

11.3 EXPERIMENTAL ASPECTS

This section presents experimental aspects for establishing uncertainty limits on wall interference corrections. While some of the topics are not usually grouped with wall interference discussions, they are required in the larger scheme to address the issue of obtaining valid uncertainty limits. For instance, discrepancies in wind tunnel data caused by flow nonuniformities or stream turbulence have many times been attributed to tunnel wall effects, so tunnel flow field surveys are briefly discussed. The first section (11.3.1) presents traditional approaches to determining interference. This section includes caveats that restrict their sole use for establishing uncertainty limits. Also presented is a recent, promising technique which may alleviate many of these restrictions under certain circumstances. Next, the requirement to establish data credibility is discussed in section 11.3.2. Here, use of modern methods of statistical quality control (SQC) typical of those advocated by national standards laboratories are suggested to enable the consistent achievement of the required level of measurement accuracy. Having presented the requirement for SQC, section 11.3.3 discusses how a measure of the true value of the wall interference correction may be realised. The subject of *"truth"* is, also, discussed in Chapter 12, "Future Necessary Work". The approach taken there is slightly different from that presented here; however, they are complimentary and the subject matter emphasises the perceived needs and approaches as viewed by the different authors. Finally, section 11.3.4 presents the characterisation of the National Transonic Facility as a case study in addressing these aspects of establishing uncertainty limits.

11.3.1 TRADITIONAL APPROACHES TO DETERMINING INTERFERENCE

Traditional experimental approaches to assessing wall interference effects have included (1) a single model tested in multiple test section geometries (i.e., solid and ventilated walls in the same tunnel), (2) multiple sizes of geometrically similar models tested in the same tunnel, and (3) a single model tested in multiple size tunnels.

The rationale for the first method is that solid wall boundary conditions are known with greater confidence than ventilated wall boundary conditions; therefore, more accurate corrections of solid wall data to free-air conditions can be determined for subsonic Mach numbers. Corrections to ventilated wall data are, then, empirically obtained by indexing these data to the corrected solid wall results. Unknown coefficients in ventilated-wall boundary conditions are obtained by tuning the numerical models to match the data at low Mach numbers. These boundary condition coefficients are assumed invariant with Mach number changes, and, subsequently, are used to extrapolate the computations to high Mach numbers. The approach is limited by non-linear, closed-wall model blockage at high speeds; numerical and physical modelling of the wind tunnel, test model and support system; and the formulation of the boundary condition and its performance across the facility test envelope. A representative example of this method is the procedure used by Steinle to establish the porous slot boundary condition for the Ames 11-Foot Transonic Tunnel (see section 5.2.5).

The second approach assumes that results from multiple size models can be extrapolated to zero model-span-to-tunnel-width ratio to yield interference free flow and that incremental corrections may be determined as a function of model size and test conditions. The method assumes that models may be fabricated with sufficient accuracy to assure geometric similitude (to negligible uncertainty), that deformation under load is the same for all models, and that model Reynolds number effects are negligible. In this method, extreme care must be exercised to insure the proper accounting of model mounting and support system effects because base effects are critical for drag computations and moment matching which is dependent on stream curvature over the model aft region. Wall Reynolds numbers may be significantly different for matched model Reynolds numbers; an implicit assumption is that the wall boundary conditions are insensitive to these changes. This fact alone can mask aerodynamic interactions and make proper comparisons difficult. Empirical interference corrections established by Crites and Rueger for the Boeing (formerly McDonnell Douglas) Polysonic and Transonic Wind Tunnels incorporate many of these ideas (see section 5.3.2.2). Additionally, the combined experimental and numerical approaches of Crites and Rueger (see section 5.3.2) and Sickles, et al. (see section 5.3.3) use this method.

The third approach assumes that large-tunnel aerodynamic tests of relatively small models may be assumed nearly interference free and, as such, may be used as the baseline from which to index smaller tunnel results obtained with the same model (see section 5.3.2 and 5.3.3). Besides being highly dependent on the Mach number, this approach is probably the most difficult to assess because it generally encapsulates any reference facility bias within the resulting corrections. However, this third approach is a subset to a process which will be proposed later in section 11.3.3.

Each of these approaches allow the determination of a wall correction; but, none of them allow a direct assessment of the associated uncertainty (or limitations). For example, in the first approach, mathematical and physical limitations preclude establishing uncertainty limits; in the second approach, multiple models imply multiple mounting systems and probable differences resulting from Reynolds number effects; and, in the third approach, the large facility may impose a bias different from that of the smaller facility. Reported attempts to assess the uncertainty limits on wall corrections are few.

One promising approach to directly addressing the issue of allowable or acceptable variations in wall induced interference is that recently presented by Ashill, Goodyer, and Lewis [2] and Lewis and Goodyer [19], [20]. They used the two-dimensional adaptive wall tunnel at the University of Southampton in conjunction with Ashill's correction method (see Chapter 4). In this approach, the tunnel walls are iterated to convergence for flow about an airfoil. Then, known levels of blockage, blockage gradient, upwash, and stream curvature are experimentally introduced into the tunnel flow via appropriate incremental positioning of the top and bottom walls. The recorded data are used in Ashill's correction method to assess the theory's ability to properly recover the converged-wall solution. An important distinction is made that proper interference assessment doesn't necessarily imply the ability to correct wind tunnel data, but only the possibility that the data may be correctable. It should also be noted that, while they specifically address issues of the correction methodology, their focus is not the issue of establishing interference free flow. Uncertainties due to tunnel bias such as tunnel flow angularity and blockage due to sidewall boundary layers and tunnel calibration coefficients are ignored.

11.3.2 ESTABLISHING DATA CREDIBILITY

Wind tunnel customers are presently demanding absolute transonic cruise drag accuracies of 1 count ($\Delta C_d = 0.0001$) or less, which may only be obtained if proper accounting of all dominant error sources is realised. To place this number in economic perspective, for aircraft such as the proposed High Speed Civil Transport, 1 drag count equates to 8 passengers or 60 miles of range. To place this number in technical perspective, experimentally resolving 1 drag count requires measuring angle of attack to 0.01 degree or Mach number to 0.001 (see Table 1 in Chapter 1). Note the use of "or" in the preceding sentence for if all uncertainty is assumed to reside in one variable, then the contributions of all other possible uncertainty sources must be negligibly small. Therefore, the actual resolution of the angle of attack and Mach number must be less than the cited values. A root-mean-square analysis shows that "minor" uncertainties must be the order of one half the value of the "major" (or dominant) uncertainty to contribute to the total uncertainty. Any experimentalist who has measured back-to-back polars recognises the achievement of this level of measurement precision as a particularly daunting task, requiring much care and adherence to standardised testing procedures. The task is further complicated when comparing data obtained in different tests of the same model in the same tunnel. Even the smallest of changes in the tunnel circuit (such as contamination on or a tear in a turbulence screen), modifications made in the tunnel plenum, or something as simple as changing the data sampling period and/or rate may yield results which can bias test results and generate greater than the allowable differences between the repeat tests.

Achieving this required high level of test-to-test consistency mandates the implementation of statistical quality control (SQC) methods to establish "data credibility" (Belanger [3]; Croarkin [13]; Taylor and Oppermann [24]). Implementation of SQC methods are atypical of past practices generally applied in aerodynamic laboratories, but implementation of SQC methods are now being addressed (Anon. [1]). SQC in a given wind tunnel implies that the mean values of aerodynamic measurements made on the same model over widely separated repeat tests will compare to within the required accuracy at a specified level of confidence (typically 95 percent). Credibility of test results implies ongoing statistical assessment and configuration control of the experimental techniques, procedures, and all related processes. It is important to note that even though they are part of the same rigorous treatment, measurement corrections such as those due to tunnel calibrations and those due to wall interference effects have not yet been explicitly considered in this discussion. If SQC has been achieved, and if the bias uncertainty effects of the walls are approximately constant for small configuration changes to the model, then traditional methods of incremental testing may confidently be pursued with only minimal impact of the walls on the test results.

11.3.3 DETERMINING THE TRUE VALUE OR "TRUTH"

Assuming that statistical quality control (SQC) has been achieved in the wind tunnel environment, the aerodynamicist may begin to rationally consider tunnel-to-tunnel, tunnel-to-computation, and tunnel-to-flight comparisons at the required accuracy levels. At this point, each data set must be referenced to an absolute baseline; therefore, each data set will require an assessment of and correction for the bias imposed by the tunnel walls and other tunnel specific effects such as differences in mounting systems, dynamic loads, stream turbulence, and flow angularity. The obvious question is "How good are the corrections?" In actuality what is being asked is "How much uncertainty is attached to the corrections?" or "What are the limitations?" This is a most difficult question to resolve because it requires the true value and it implicitly poses the question "How is truth determined?".

The subject of truth can be directly approached in at least three ways, each of which has limitations. First, truth can be approached via direct analytical or numerical computational fluid dynamic (CFD) solutions of the Navier Stokes equations or approximations to them. While valuable for establishing model problems and for looking at gross effects on very simple geometries, the analytical approach is extremely limited because of present day mathematical capabilities. Truth from a numerical perspective via CFD solutions of the Navier Stokes equations is limited by grid resolution, computational algorithms, computer power, and a fundamental lack of understanding in fluid physics areas such as transition, turbulence, shock wave/boundary layer interactions, and separated flow.

The second approach to attaining truth is via experimental simulations in the wind tunnel. While attractive on the surface, experimental methods are probably the most difficult of the approaches to execute. Proper experiment design must consider the facility and its ability to accurately simulate the flight environment, including setting and maintaining test conditions, stream steadiness and turbulence, acoustic environment, and flow uniformity. Instrumentation types, accuracies, and locations are critical, particularly in three-dimensions where obtaining the required amount of data may be prohibitively expensive or destructively intrusive to the flow. Test models for wind tunnel experiments designed to capture truth will be very expensive due to any special fabrication materials, the required machining accuracies, surface finish specifications, and the required onboard instrumentation. As an example, it is not unusual for models designed for the cryogenic high-Reynolds number environment of the National Transonic Facility at NASA Langley Research Center to cost on the order of a million dollars. The actual ability to simulate the flow as desired may be an issue. For example, it is known that the wall boundary conditions for ventilated wall tunnels are sensitive to Reynolds number (see Binion [5]); in fact, in section 5.2.3.2 it can be seen that the wall Reynolds number is explicit in the boundary condition. Ventilated-wall interference studies which test, for instance, geometrically similar full-size and half-size models must consider that the wall Reynolds number for the half-size model is double that of the full-size model at matched model Reynolds numbers. Additionally, consideration must be given to different dynamic loads at matched test conditions resulting in different model deflections and different force balance uncertainties.

The third approach to truth is also experimental via actual flight demonstration tests. The cost of the flight program may be prohibitively expensive because of availability and operational costs associated with the aircraft, its required support staff, and the instrumentation requirements. Additionally, the required measurement accuracy may be unobtainable due to an inability to adequately resolve flight conditions such as dynamic pressure and aircraft attitude. As another flight test example, drag on a single "representative" vehicle selected from the fleet is determined by measuring fuel flow and consumption in the engine. In multi-engine aircraft, the single engine fuel-consumption results are assumed to hold for all engines. Currently, drag in flight can only be measured to within a few percent (ex., Paterson [21]).

The previously cited and limitation-filled ways of establishing truth in actuality point to a fourth approach which is not intuitively obvious; that is, to set a standard or to simply *declare truth*. In this approach, a consortium of test facilities/organisations (for example, those which conduct transonic performance testing on transports, or those which test fighters) would establish representative test conditions where a common check-standards model(s) would be tested. Each organisation would then analyse and correct their data using the techniques and the boundary conditions (empirical, analytical, or measured) which best describes their facility. All participants would be required to document their data, test procedures, correction methodology, and results for scrupulous examination by the consortium members. Strict adherence to SQC standards would necessarily be required to ensure data credibility. Upon acceptance by the standards committee, the results from all participants would be averaged and declared as truth. The variation about this *truth standard* would be used to establish associated uncertainty limits. Significant deviations from the mean could, then, be used as a measure of goodness and used to allow the critical assessment of where correction methods breakdown and where further research is warranted. The most significant limitations associated with this approach are institutional. This approach requires long-term management support and commitment in terms of funding, and, most importantly, the investment in knowledgeable personnel who will develop, implement, and maintain both SQC and wall interference correction methods. Additionally, technical limitations such as the installation of sufficient instrumentation and standardised data reduction techniques must be addressed. As a side note, results from these studies could be used to establish an advocacy position for facility funding and further investment in testing techniques.

11.3.4 CHARACTERISATION OF THE NATIONAL TRANSONIC FACILITY

After the occurrence of any significant change to a wind tunnel circuit, facility calibrations are in order to verify/establish the tunnel performance envelope and fan map. At the time of this writing (January 1998), the National Transonic Facility (NTF) is coming on line after a major upgrade which included the installation of a new drive system. This section presents an overview of the action plan which is currently underway in the NTF for defining the operational envelope, evaluating the system and aerodynamic uncertainties, and ensuring data quality. Ensuring data quality requires that all identifiable uncertainties be quantified, including those introduced by the presence of the wind tunnel walls. Obtaining the desired outcome of fewer data of higher quality at a higher rate (see section 11.2.3) emphasises the establishment of this approach. The previous section presented a procedure for establishing the true value of an interference free flow; this section discusses the activity which initiates that procedure in the NTF as an example of the process which must be undertaken to ensure the acquisition of high quality aerodynamic data.

11.3.4.1 THE TEAM

A team was formed prior to the 1997 facility upgrades to calibrate the NTF when it returned to active status; the team ultimately expanded to 13 full members and 5 consulting/specialist members as requirements were developed. NTF customers were invited to participate fully in all phases of the process, including planning, review, testing, and analysis. Weekly team meetings were held to formulate goals, to establish realistic objectives, and to define areas of responsibility. Most importantly, these weekly meetings were necessary to build a cohesive working relationship from a diverse range of technical backgrounds, to obtain individual buy-in to the process, and to establish working-level communications. A wide range of disciplines was included early in the planning to ensure that as many issues as possible would be addressed and to minimise surprises which typically occur in this type of activity. Included were managers, test engineers, research engineers, scientists, technicians, and data systems personnel; the areas of expertise covered were statistical quality control and measurement uncertainty, wall interference, tunnel calibration, tunnel flow quality measurement, models, instrumentation, tunnel simulation and scheduling, and dynamics. Team communications and getting everyone understanding the same technical language is extremely important to success; for example, in this project, each discipline had a different unique definition of *tunnel empty* and this greatly affected test planning.

11.3.4.2 THE APPROACH

The approach of the team was to create a virtual future by defining the desired outcome, then to build backwards to determine how the outcome was achieved. This simple approach focused the team on actual requirements in test planning, priorities, and implementation. Data accuracy requirements for performance testing were established in partnership with the customers; these requirements are given in the following table.

TYPE OF TEST	INCREMENTAL	ABSOLUTE
High lift	0.2% C_L and C_D	0.4% C_L and C_D
Transonic cruise	1/2 count C_D	1 count C_D

The team was forced to recast its mission in the light of data quality upon recognition that a traditional wind tunnel calibration combined with wall interference corrections was insufficient to meet these requirements and to produce certifiable world-class results on a continuing basis.

In reality, a *characterisation of the facility* was required to achieve the overall goals. This characterisation was composed of many individual tests grouped in four general categories, or thrusts. These thrusts, which are distinguished in the next section, are (1) the standard, centreline calibration, (2) flow quality, (3) measurement uncertainty assessment, and (4) tunnel wall interference corrections. Implementation of statistical quality control methods was recognised as the only viable approach to achieving and maintaining the goal of certifiable data quality. By its very nature SQC is an ongoing, periodic process; it, therefore, allows and mandates continuous improvement. Recognising this distinction allowed a very success oriented approach to be assumed since problem areas which will occur can be re-addressed by the ongoing commitment to periodic testing.

11.3.4.3 CHARACTERISATION

As previously stated, the tunnel characterisation is divided into four categories or thrusts which are described in this section. Activities and tests in each of these thrusts are to be repeated on a continuing basis, some more frequently than others. It is recognised that many of these activities are not traditionally related to either wall interference or wind tunnel calibration; however, in order to establish uncertainty limits on corrections, it is necessary that each of the areas be considered in the process. Additionally, if statistical quality control is to be achieved, all of these aspects of the facility must be documented, and improvements and changes must be made in the light of their impact on data quality.

11.3.4.3.1 Thrust I – Tunnel Calibration.

The objective of this thrust is to perform a traditional calibration of the tunnel over the test envelope. This is accomplished by measuring the static pressure distribution over the length of the test section using a centreline pipe and along the tunnel walls using pressure orifices. Measurements of total temperature and total pressure from which flow conditions are established will be made in the settling chamber, as will static pressure in the plenum. The results will be used to obtain the longitudinal Mach number distribution. In the future, this activity is anticipated to occur every three to four years, or as significant changes to the tunnel mandate.

11.3.4.3.2 Thrust II – Flow Quality.

Thrust II is a multi-test series of experiments designed to assess the uniformity and steadiness of the flow at the tunnel cross section corresponding to the model centre-of-rotation. In the first test, a rotary rake will be used to determine distributions of temperature, pitot pressure, and flow angularity. While this is an important first step for quantifying any flow nonuniformities, the complete numerical modelling of the test section will ultimately require both upstream and downstream surveys to be performed for use as farfield boundary conditions. Performing these types of surveys is most difficult, particularly in a cryogenic nitrogen environment, and it is hoped that future advances in non-intrusive flow diagnostics will progress at a rate sufficient to aid this task.

In the second test, turbulence and flow unsteadiness will be measured via hot wire and fluctuating pitot pressure sensors in the settling chamber and test section. When scaling high-Reynolds number tunnel data to flight, a mismatch between shock locations can occur if the facility Mach number is incorrect due to wall-induced blockage or an inappropriate choice of reference pressure. Shock location may also be erroneous if the turbulence level is too high, resulting in premature transition to turbulence, thereby changing effective body shape. These measurements provide quantitative data upon which an assessment can be made.

Finally, in the third test, the tunnel wall boundary layers will be obtained using pitot pressure boundary layer rakes. These measurements will be made ahead of the test section in the contraction and on a solid sidewall and on a slotted wall in the test section. Additional future experiments are anticipated such as the development of a check standard model which is sensitive to variations in stream turbulence.

11.3.4.3.3 Thrust III – Measurement Uncertainty Assessment

Measurement uncertainty will be regularly evaluated two to four times each year by testing two different check standard models. The data generated by a single test of the check standard models will be combined with data from previous test entries to generate control charts for statistical assessment of data quality. The first check standard model is a pitot-static probe used to provide a single-point measurement of total, static, and dynamic pressures at the model centre-of-rotation. Measurements made during this test and during frequent, periodic re-tests will be used to determine the stability of the tunnel calibration and to establish its reproducibility (variation over time), thereby characterising uncertainty limits on the dynamic pressure. When the NTF was built, two geometrically similar models (60-inch and 30-inch wing spans) of a generic transport configuration known as the Pathfinder I (PFI) were built to evaluate tunnel wall interference effects. The larger model, which is instrumented with pressure orifices on the wing, is being removed from inventory as a general test bed for aerodynamic studies and is being reserved as the second check standard model. Frequent periodic testing of this model will be used to create an aerodynamic database to monitor all processes and subsystems associated with wind tunnel testing; including model installation, tunnel processes, instrumentation, data acquisition and reduction software, integrated tunnel flow angularity, pressure and aero-data repeatability.

11.3.4.3.4 Thrust IV – Wall Interference

The wall interference thrust is divided into several continuous improvement phases which, initially, are application and implementation of current technologies, followed by phases which concentrate on quantifying uncertainties and extending assessment techniques. The objective of the first phase is to tune the tunnel systems to enable on-line, post-point wind tunnel wall interference assessment and correction (WIAC) of standard performance aerodynamic tests. Typically, wall pressure measurements have been second tier measurements which were acquired if available and only if their acquisition did not inhibit the rapid acquisition of first tier data such as tunnel parameters and model forces and pressures. Because of their lower status, little attention was given to the quality of the measurement. Orifices were not protected and instrument calibrations were not monitored. With the implementation of boundary measurement methods for determining the interference effects of the walls, wall pressures must be elevated in importance to obtain wall corrections of the required accuracy. A significant effort is being expended to bring the NTF wall pressure system to first tier instrument status. This system includes an electronically-scanned pressure measurement system, temperature-controlled containers for the pressure scanner modules, and over 500 wall pressure orifices on 16 rows around the test section periphery. Raising this system to first tier status includes properly identifying, cleaning, and repairing all orifices, performing a leak-check verification, and continually monitoring the instrumentation calibrations. Empty tunnel pressure signatures will be obtained to ensure proper symmetry exists in the wall data, and to establish a pressure-signature baseline from which to assess orifice bias effects and generate tares. The model support system will be exercised over the angle of attack range to evaluate its effects on the tunnel-empty pressure field and to enable proper separation of these effects from those generated by the model.

In the second phase, preliminary data with the large PFI model installed will be obtained to assess model-plenum interaction effects on the tunnel calibration. These data will allow the proper specification of the tunnel reference pressure, whether the plenum pressure is sufficient or whether a more stable upstream value on the tunnel wall must be used (see section 5.2.4.3.5). All these data are required to rationally implement the PANCOR code (Kemp [17], [18]); see, also, section 5.3.1.1) in an on-line, post-point computational mode. Preliminary data to assess the effects of compressibility on wall pressure and drag rise in the NTF for a body of revolution will be generated by removing the wings from the PFI model and testing only the fuselage through Mach 1.

A third-phase series of experiments is being planned to advance the state of the art to the point where uncertainty limits can be placed on the interference corrections. This third phase will include tests of both the full-scale and the half-scale PFI models with the test section in both solid-wall and slotted-wall configurations.

11.4 COMPUTATIONAL ASPECTS

A number of fundamental aspects related to limitations in computation of wall corrections have already been discussed in section 11.2 and will not be repeated here. The major computational thrusts related to wind tunnel wall interference correction technology will be summarised in the first sub-section. Much has been said throughout this text about the importance of and need to define uncertainty in measured aerodynamic data for the customer. Since, at high Reynolds number, this desired data may well be subject to significant wall corrections, then their uncertainties must be assessed. The second sub-section discusses sources of uncertainty in CFD based interference correction procedures. The third sub-section suggests how formal sensitivity analysis via automatic differentiation (AD) may be used to aid in assessing these correction method uncertainties quantitatively for the modern measured-boundary-data interference procedures which are frequently CFD based.

11.4.1 MAJOR COMPUTATIONAL THRUSTS

The rapid development and advancement in computational capabilities, with respect to both hardware and software, have certainly found application in the wind tunnel testing and interference correction communities. These capabilities have created the possibility for better pre-test wall interference prediction, rigorous post-test wall interference assessment and correction, and greatly reduced interference testing in adaptive wall wind tunnels. The major computational thrusts in wall interference since the time that AGARDograph 109 was published by Garner, et al. [14] have paralleled developments in CFD and those technologies supporting the adaptive wall concept. These thrusts have been to provide:

(a) rapid calculation of conventional corrections;

(b) more realistic analytical modelling of tunnel wall geometry and boundary conditions, test article geometry, and model support systems;

(c) initial application of these more realistic analytical models in both numerical tunnel simulations and wall interference assessment/correction methods;

(d) prediction and control of wall adaptation in adaptive wind tunnels;

(e) design assessment of ventilated test section walls; and

(f) research studies related to correctablity and its limits.

All of these computational thrusts have been discussed throughout the previous ten chapters. The assumptions, approximations, and empirical or analytical models used in specific computational approaches have generally been concisely stated in the first section or so of each chapter. The various results presented have essentially served to illustrate a given computational capability and its status. For many of the traditional models (simple wall, support, and configuration representation) and linear or full potential CFD approximations, a number of these capabilities have been reasonably well investigated and applied to real tunnel data, as seen in Chapters 2, 3, and 4 and in a few examples given in later chapters. However, for more advanced CFD algorithms and complex flow regimes, few of these capabilities have been extensively exercised or verified using real tunnel data. Limitations with respect to range of application for reasonable corrections and uncertainty in these corrections, therefore are not known.

Most of these major computational thrusts involving advanced CFD algorithms have been exploratory applications and investigations which have emphasised the physical possibility of performing the computational task as opposed to reducing it to practical feasibility. As pointed out in Chapter 12, much of the stage is set for implementation of many of these major computational tasks into what is to be the production testing environment. As already seen in the previous chapters, the modern measured-boundary-data correction methods for 3-D flows have not been verified using extensive data; there are very few adequate data bases and more are needed. Experimental uncertainty in the measured-boundary-data must be assessed, as discussed in section 11.3, for example, and propagated through the entire correction procedure to obtain an uncertainty for the correction.

11.4.2 UNCERTAINTY SOURCES IN CFD BASED CORRECTION PROCEDURES

Analytical or numerical models, at one degree of complexity or another, are used in all wall correction methods. In those that employ CFD, the levels and interaction of models are compounded so that establishing sources of inadequacy or uncertainty may be very tedious and, if located, may also be difficult to assess, modify, or correct. Typically, models are constructed empirically or analytically, guided by first principles, basic conservation laws, or assumed basis functions. The parameters in these models are determined by approximately matching or reproducing basic experimental or observed data. Then these models, generally with the determined parameters fixed, are used to predict or analyse (i.e., interpolate or extrapolate) the "fitted", dependent, output data for varying independent input data. The model may be a solution procedure or algorithms, or contain an algorithm, or require one for computation. These latter models also require parameters and input for controlling the procedure (such as discretisation, convergence, etc.). The variety of models already included in a CFD flow analysis code of interest here, for example, might include those for boundary conditions (such as tunnel walls, far-field free-air, test article geometry, and support geometry), fluid-flow conservation laws, solution algorithms, turbulence modelling, and elastic response. A wall correction procedure, particularly a non-linear CFD-based one, will then link two or more numerical CFD solutions subject to an equivalence or matching condition in order to compute corrections.

For a numerical model, the sources of uncertainty or error can be ascribed to those in the input data and those of the model. Model uncertainties arise due to inadequacies in the model's approximations (i.e., assumptions, rules, conservation laws, basis functions, etc.) to mimic physical reality and the uncertainties in the parameters which characterise the model (for instance, size and location of singularity strengths, coefficients of basis functions, and observed data). Assessing the model prediction uncertainty due to the inadequacy of the model approximations is a validation exercise requiring a measure of the physical truth or reality. If one has the latter and the model predictions are deemed inadequate, then, either bounds are established for acceptable tolerances or another model is obtained. These bounds for basic models, are assumed established at their development; however, when many basic models are coupled together, verification or validation is more difficult to assess or obtain. A number of the computational thrusts referred to in section 11.4.1 have involved such studies. As indicated elsewhere herein, a given model (or a collection of models) may be defined as truth for assessing relative prediction effects and uncertainties in the context of wall corrections where the truth is elusive.

Quantifying the uncertainty in a model's input data, or its parameters, is assumed to be done experimentally, for example, in a characterisation of the facility and its instrumentation (as discussed in section 11.3), or reasonably estimated sufficiently well. Some spatial or temporal dependence or functional form, or modelling, of the uncertainty may also be known and required in order to propagate the uncertainty. For a method which determines wall corrections, using input data and parameters for many models, one needs to understand and be able to numerically quantify uncertainties in these output wall corrections for given (known) uncertainties in the input data and model parameters. Conversely, perhaps, it is desirable to be able to estimate allowable input and parameter uncertainties required to obtain a desired wall correction uncertainty.

Assessing the model prediction uncertainties due to those in model input data or model parameters is very tedious if errors are formally propagated and may be computationally very expensive if done by numerical perturbation, whether finite difference or statistical based. Jitter programs, as discussed by Coleman and Steele [12] for example, have recently been used to generate finite difference approximations to the partial derivatives needed in uncertainty analysis of experiments. Essentially, the data reduction computer program (a model) is perturbed with respect to each of its input and parameters (by the uncertainties in each) in order to obtain the individual influence of each on the output result(s). For a wall correction procedure (a model) which is not too computationally complex nor expensive to execute, this jitter procedure, which is a finite difference sensitivity analysis, may be feasible.

When a number of numerical models, within a single computer code or several computer codes are sequentially linked or iteratively coupled (i.e., one model's output is another model's input and vice-versa), then assessing the uncertainty in the ultimate predicted output due to those of an intermediate model's input and parameters is extremely difficult. In addition, the linking and coupling algorithms will introduce more uncertainty through their input and parameters, for example the tolerance required to satisfy the matching or equivalence condition.

11.4.3 SENSITIVITY ANALYSIS FOR CORRECTION UNCERTAINTY ASSESSMENT

Sensitivity analysis is a method of assessing the sensitivity of a model's output with respect to its input data or internal parameters. It involves obtaining an estimate of the partial derivative of the output with respect to a given input or parameter and can be accomplished experimentally, analytically, numerically, or by some combination, depending on the nature and complexity of the model. If the model is in the form of a computer code (i.e., FORTRAN or C), then automatic differentiation (AD) or computational differentiation (Griewank and Corliss [15]; Berz, et al. [4]) is a practical, robust means for obtaining sensitivity derivative (SD) information. As can be seen from the papers included and references cited in these two SIAM conference proceedings, this mathematical/computational technology has a well established history (Rall, [23]) and is a continuing interdisciplinary activity with many varied current applications. Our interest here is in what has been done with realistic CFD models and how this information can be used in the wall correction methodologies, particularly in regard to the models and the uncertainty in their predicted output results.

The interest in multidisciplinary design optimisation of aerospace vehicles prompted the initial applications of AD to CFD codes by Bischof, et al. [6] using the emerging AD tool ADIFOR (Automatic DIfferentiation of FORtran) developed by Bischof, et al. [7], [8], [9]. In design optimisation, derivatives of CFD code output functions with respect to design variables are required. These design variables are generally parameters which specify boundary data or transformations to body-oriented co-ordinate systems. They become inputs to the CFD code through both inner boundary conditions such as geometric model shape and outer boundary conditions such as non-geometric flow variables. A brief summary of the early ADIFOR applications to a realistic, iterative CFD solver to obtain SD of lift, drag, and pitching moment with respect to non-geometric flow variables, CFD algorithm parameters, turbulence modelling parameters, and geometric model shape parameters is presented by Carle, et al. [11]. Recent applications have extended these ideas and techniques to other complex CFD flow solvers used in the aerospace enterprise. To our knowledge, however, no one has yet applied AD to ventilated wall simulation models, wall interference prediction codes, or wall correction procedures to obtain the sensitivities of the interference field, corrections, etc. (i.e., the output) with respect to Reynolds number, porosity parameters, measured wall signatures, wall slope, etc. (i.e., the input). Such sensitivity analyses are essentially just different AD applications to CFD codes that have been demonstrated as being differentiable by ADIFOR; these computational sensitivity exercises should be done. However, with respect to the propagation of uncertainties in model input and parameters, a somewhat different approach is, also, suggested and outlined below.

AD has also been used to obtain error bounds or estimates for the function and its derivatives as can be seen from several papers included in Griewank and Corliss [15] and Berz, et al. [4]. This interest, although originating in rounding-error estimation, is of importance in the data assimilation for improved weather prediction models and also in beam physics stability and control. Typically, second derivative information has been utilised. However, an idea that should be of interest in propagating uncertainties for wall correction applications was demonstrated by Bischof, et al. [10] for an initial-value problem where they showed:

"By differentiating the output of a model with respect to its parameters, one can quantify how sensitive or robust the model's predictions are relative to variations of that parameter, as well as gain insight into how to adjust parameters that are poorly known. Questions regarding the sensitivity of the model output to more abstract quantities involving many model variables can also often be rephrased in terms of derivatives, either directly or by embedding the problem of interest into a larger parameterised framework … . Our approach is an example of this latter approach: we obtain the TLM" (Tangent Linear Model) "evolution of a perturbation in the initial-value data by introducing a parameter that linearly interpolates between the unperturbed and perturbed initial states. We shall show that formal perturbation theory with respect to the parameter yields the TLM and can be shown to be equivalent to evaluation of the derivative with respect to the interpolating parameter."

For the CFD boundary value problems in wall correction procedures, it is suggested that interpolating parameters, scaling the (known) uncertainties in model data input and model parameters, can be introduced and that differentiation of the model output with respect to these interpolating parameters would produce SD that directly provide a first-order estimate of the propagated uncertainty. That is, derivatives with respect to the model input and parameters provide the output sensitivity to those quantities at that solution; where as, derivatives with respect to the interpolating parameters, which scale the respective uncertainties in model input and parameters, can be related to the uncertainty propagated to the output at that solution.

11.5 CONCLUDING REMARKS

Traditionally, theoreticians/CFD code developers and wind tunnel test engineers have not always communicated well with each other. Wind tunnel corrections have typically resided in one camp or the other because they were either theoretical or empirical. However, a new paradigm is emerging wherein the determination of wall corrections is smearing the dividing line between these two different cultures. The analyst must now take the best from each, the theory and computational capabilities of the theoretician and the measurement techniques of the experimentalist, and combine them into a rational methodology for reducing the wall-induced uncertainty in the test data. With this blending, the analyst must also recognise the limitations of each method and actively work to establish and refine the measure of truth. The increased demands for high accuracy data with well-defined uncertainty specifications and the push to scale wind tunnel data to flight Reynolds numbers require that CFD and SQC play definitive roles in wall correction methodology.

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