ADVISORY GROUP FOR AERONAUTICAL RESEARCH AND DEVELOPMENT

REPORT 90

SOME REMARKS UPON THE PROBLEM OF TEMPERATURE SHOCK IN AIRCRAFT

by

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AUGUST 1956

NORTH ATLANTIC TREATY ORGANIZATION
PALAIS DE CHAILLOT, PARIS 16
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This Report was presented at the Fourth Meeting of the Structures and Materials Panel, held from 27th to 31st August 1956, in Brussels, Belgium.
SUMMARY

An account is given of the structural problems associated with heating and large temperature gradients. The thermal stresses due to a non-stationary distribution of temperature are calculated for a plate cooled on one surface and heated on the other by a gas moving at high speed. The cases where the plate is completely free or completely restrained in one direction are discussed. Formulae are obtained which, under these conditions of restraint, make a quick comparison of different materials possible. Possible lines for the extension of theoretical and experimental work are suggested.

SOMMAIRE

Exposition des problèmes relatifs à la structure posés par l'échauffement et par les gradients de température élevés. Les tensions thermiques dues à une distribution inconstante de la température sont calculées pour une plaque dont une surface est refroidie et l'autre est chauffée par un gaz se déplaçant à haute vitesse. Examen des cas où la plaque est complètement libre ou complètement retenue dans une direction. Formulation de formules qui, dans ces conditions de contrainte permettent de comparer des matériaux différents. L'auteur propose des méthodes pour faire avancer les travaux théoriques et expérimentaux dans ce domaine.

533.6.011.6 : 624.073

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NOTATION

$x, y, z$ local coordinates

$x$ coordinate perpendicular to surface

$y$ coordinate in axial direction

$z$ coordinate perpendicular to the $xy$-plane

$2s$ thickness of plate, mm

$x = -s$ heated surface

$x = +s$ cooled surface

$t$ time (seconds)

$\theta$ temperature, °C

$\theta_g$ temperature of gases, °C

$\theta_1$ temperature of the heated surface, °C

$\theta_2$ temperature of the cooled surface, °C

$\theta_m$ mean temperature of plate, °C

$a$ coefficient of heat transfer, cal.cm$^{-2}$.s$^{-1}$.°C$^{-1}$

$\beta$ linear coefficient of thermal expansion, °C$^{-1}$

$\lambda$ heat conductivity, cal.cm$^{-1}$s$^{-1}$°C$^{-1}$

$a$ thermal diffusivity ($\lambda/c\cdot\gamma$), cm$^2$s$^{-1}$

$c$ specific heat, cal g$^{-1}$°C$^{-1}$

$\gamma$ specific weight, g.cm$^{-3}$

$E$ modulus of elasticity, kg.mm$^{-2}$

$\epsilon$ strain, %

$\sigma$ normal stress, kg.mm$^{-2}$

$\sigma_\epsilon$ normal stress, causing $\epsilon$ % permanent strain, kg.mm$^{-2}$
\( \sigma_m \)  
mean stress, kg.mm\(^{-2}\)

\( \sigma_{x,y,z} \)  
normal stress in the x-, y-, z-direction, kg.mm\(^{-2}\)

\( \mu \)  
Poisson's ratio

\( \tilde{\sigma} \)  
0.25 x a.ts\(^{-2}\)

\( \eta \)  
\( |\sigma| E^{-1} \beta^{-1} g^{-1} \)

\( \phi \)  
2 a.s.\( \lambda^{-1}\)
SOME REMARKS UPON THE PROBLEM OF TEMPERATURE SHOCK IN AIRCRAFT

F. Bollenrath*

1. INTRODUCTION

The behaviour of various aircraft materials under varying temperatures has been treated in numerous theoretical and experimental works. The influence of temperature changes on the mechanical, physical and chemical properties, especially on mechanical strength and the changes in shape of structural parts, is of the greatest importance. The economic efficiency and the safety are determined by these factors.

In many branches of engineering, the numerous problems connected with temperature changes have been dealt with for a long time. The problems arise partly from the methods of production e.g. in welding, during formation at elevated temperatures, and with heat treatment during formation when the normal stresses and strains of structural members - both metallic and ceramic - must be controlled. These problems also arise during service e.g. in steam plants, hot rolling mills, oil and fuel establishments and in plants which produce synthetic fuels and plastics. During the last few years, when pressures have risen beyond 170 kg/cm², and temperatures beyond 600-650°C, considerable attention has been given to problems caused by high temperature gradients.

In aircraft production, these problems first became very interesting in internal combustion engines e.g. in exhaust-gas and fresh-gas turbines. They made the design and the choice and fabrication of materials difficult. During the last few years, when speed have risen to close to that of sound, the problem of kinetic heating has become more and more important for the aeroplane structure.

2. CLASSIFICATION OF THE VARIOUS PROBLEMS

2.1 General

Whilst the influence of a constant temperature on the various properties of material can be determined in relatively simple, although often lengthy, experiments with a great number of samples the problems connected with non-stationary service conditions are mostly very complicated. This is true for single structural parts of very simple forms, and more than ever for a group of single parts connected one with each other by any method.

The local and time-gradient of temperature as primary factors of influence are difficult to determine, and the relation of these gradients to strain and stress is also complicated. The larger the time gradient of temperature the greater the likelihood of thermal shock.

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The rate of change of temperature in a structural member is limited, and given, by the difference between the initial temperature in the body and the temperature of the heating or cooling medium, the velocity, coefficient of heat transfer, heat conductivity and thermal diffusivity. The changes of temperature are mostly caused by gaseous or liquid mediums e.g. in rockets.

Although the distribution of temperature at any time may be known, nevertheless the calculation of strain and stress is rendered more difficult by the problem of restraint. In most cases stresses due to external forces are superimposed on the thermal stresses, which become very high in turbine blades and discs of compressors and gas-turbines.

A theoretical treatment of the relations between temperature, form of body, strain and stress, developed during heating or cooling can be made in only a few cases, if the form of the body and streamconditions are simple. Whilst simplifications of this grade are not realized in aircraft, nevertheless investigations of this kind give useful information on the choice and treatment of material and on the design.

To characterize the series of problems it seems to be advisable to enumerate the circumstances and factors which are important and connected with material, design and service conditions.

2.2 Materials

Beside the strain and stress due to distribution of temperature and first-order restraints, the secondary stresses caused by anisotropy and heterogeneous grain structure must be considered. Since, under repeated stress - especially under three-axial and inhomogeneous strain-failure of material begins in a very restricted area of local stress concentration, the secondary stresses superimposed on the stresses of the first kind are very important.

Due to temperature change in the material phase transformation or segregation can take place and, depending upon the duration of a temperature change, precipitation or solution of any component may occur. In this way remarkable changes in physical and mechanical properties are produced and the conditions for calculation are completely altered. On the other hand change of grain structure is influenced by stresses and plastic deformation e.g. recovery and recrystallisation. These circumstances are largely responsible for creep and for the transition from secondary to tertiary creep, as well as for the fatigue strength.

2.3 Design, Shape and Dimensions of Structural Members

The shape and dimensions of structural members influence the local and time-gradients of temperature. The distribution of the velocity and direction of the coolant, heat transfer and therefore the heat-flow in any body depends upon the shape. The calculation of the direction of heat-flow in a solid part, due to a temperature gradient along the surface in any direction and due to change of dimensions, becomes exceedingly difficult if the heat does not flow perpendicular to the surface. At present the author does not know a method of determining the distribution of temperature and strain under these circumstances. In the investigations already published examples are given in which the heat enters the body and flows in a direction perpendicular to the surface. If that is not the fact the best way to obtain information is by experiment.
2.4 Groups of Inter-Connected Structural Parts

As soon as several structural parts are connected, by any method such as riveting, welding, screwing, gluing etc. the heat flow from one part to the next one, and the distribution of temperature can be followed only by experiment. That is also true for the strain. The derivation of stress from strain, as well as the determination of restraint requires new methods at elevated temperatures. As soon as the strain becomes partially plastic, internal or inherent stresses are produced, and can be of primary importance in the failure of a structural part, especially under repeated stresses and in the case of instability. High bending or torsional stresses arise between the interconnections, buckling may occur and the structural part no longer carries compression or shear. These deformations will alter the kind of flow of the cooling or heating medium, and finally alter the heat-transfer.

2.5 Service Conditions

The service conditions responsible for the change and distribution of temperature are as manifold and changing as the conditions mentioned above. Reference will be made only to the heating or cooling medium in respect to temperature, velocity and direction of flow, turbulence and thickness of the boundary layer. In engines and aircraft these factors often change between varying limits. During service the nature of surfaces also changes due to deposits or corrosion and the coefficient of heat-transfer changes; this is especially true for engines, because the components operate at high temperatures in corrosive gases. Initially the various parts have smooth and clean metallic surfaces, but these soon show oxydation or scaling. This lowers the heat transfer and the stresses caused by non-stationary distribution of temperature are also reduced.

3. Calculation of Thermal Stresses Due to Non-Stationary Distribution of Temperature

3.1 General

From a consideration of the great number of factors influencing the distribution of temperature, one may understand readily that the theoretical treatment of thermal stresses due to instationary heat flow can be carried through only in a few very simple cases. That is because the dependence of the important mechanical and physical properties upon the temperature is mainly non-linear. Exact solutions have been obtained for a space limited by a plane and for a plane plate cooled or heated on both surfaces under the same conditions, assuming the plate to be made from a material perfectly elastic, homogeneous and isotropic material. It is also assumed that the mechanical and physical properties remain constant. For smaller ranges of temperature this simplification gives solutions to quite a good approximation. On the other hand, if greater ranges of temperature must be considered, a coherent representation of the relations between temperature, time, strain and stress cannot be given. In this case graphical methods and finite difference calculations must be applied.
3.2 The Plane Plate Cooled on Both Surfaces

A full description of the state of theoretical treatment of these problems has been given by S.S. Manson. The author extends the determination of the thermal stresses in a plane plate cooled on both surfaces by the same medium to the case where strains are perfectly unrestrained. He develops an approximation which gives a measure of the resistance of a fully elastic material until failure or, in the case of a ductile material until the elastic limit is reached. The measure is the temperature which a plate is allowed to have when it is exposed to a cooling medium, so that a given stress \( \sigma \), the ultimate strength for instance or a stress for a limited permanent strain is not exceeded.

\[
\theta = \frac{\lambda \sigma}{E \beta} \cdot \frac{3.25 (1 - \mu)}{s \alpha}.
\]

If the value \( \lambda/(s \alpha) \) becomes very small this means that the conductivity is no longer of great influence compared with the heat conductivity in very thick plates. In this case the formula is simplified to:

\[
\theta = \frac{\sigma}{E \beta} (1 - \mu).
\]

3.3 The Plane Plate Cooled on One, and Heated on the Other Surface by Gases Moving with High Velocity

In many cases we will have the problem in which a body is cooled or heated on opposite surfaces by gases, e.g. the walls of a combustion chamber, or internally cooled thin walled turbine blades or nozzle blades. In this case the thermal stresses will become greater than in plates dipped into the same medium on both sides.

In the following the simplified case of a plane plate or slab of constant thickness \( 2s \), heated on one side \( (x = -s) \) and cooled on the opposite surface \( (x = +S) \) by gases, having a very high velocity \( (300 \text{ m/s}) \) will be treated. The temperature of the heating gas is 1000°C, whilst the plate and the cooling gas have the initial temperature of 0°C. The breadth of the plate may be limited to some centimeters, so that bending may take place in the \( x \)-direction. Since the thermal stresses are highly dependent upon the rate of restraint, the thermal stresses will be calculated for the following cases:

(a) Deformation when completely restrained in the \( z \)-direction (as it may be caused by adjacent slabs);

(b) Deformation when completely free in all directions.

The distribution of temperature in the \( x \)-direction may be the same for all \( z-x \)-planes at a given time.

As the results will show, the strain and stress which may be allowed in the case of repeated temperature-shocks of such a strength that the safety and efficiency are not reduced, will be developed in a relatively short time and be produced by such small changes of temperature in the plate that a calculation with average values of the
properties of material is justified. This has been done with the materials chosen as examples: Nimonic 90, a Cr-Steel and copper, the properties of which cover a wide range. (See Table I).

The calculation of thermal stresses with increasing time has been carried out in the following way.

**3.3.1 Deformations in the Z-Direction Completely Restrained**

First the temperature distribution in the x-direction (plate-thickness) is determined graphically by the Schmidt-method. From the distribution of temperature the thermal stresses are derived by the equilibrium of stresses and moments. The stresses always have the greatest values at the heated surface.

The results of these investigations are represented in the following diagrams plotted to a logarithmic scale. From the values for the mechanical and physical properties of three characteristic materials mentioned above, the universal relations for the different factors of influence have been derived.

In Figure 1 the temperature $\theta_1$ at the heated surface, divided by the temperature $\theta_g$ of the heating gas, is shown as it is changed during the (non-dimensional) time $\xi$. The parameter is the non-dimensional plate thickness

$$\phi = \frac{2sa}{\lambda}.$$ 

For the plate with deformations in the z-direction restrained, the maximum stress ($\sigma_2$) increases during the heating time as shown in Figure 2 as the non-dimensional stress

$$\eta = \frac{\sigma_2}{E_0 \beta_0 \theta_g}.$$ 

The parameter is again the non-dimensional plate thickness.

To a very good approximation the relation $\eta(\phi, \xi)$ can be written

$$\eta = 0.9\phi^{-0.82}\xi^{0.38}\phi^{0.18}.$$ 

This diagram shows the curves for various thicknesses of plates made from Nimonic 90 plate thickness 1 mm, 2.2 mm and 10 mm; a curve for a 2 mm - Cr-steelplate is also outlined.

The heating times which cause a special amount of stress in a plate of a given thickness are of especial interest when comparing plates of different materials. Figure 3 shows the time to reach a stress $\sigma_{0.2}$ (which causes a permanent deformation of 0.2%) in plates made of the three materials, e.g.

$$\begin{align*}
\sigma_{0.2} &= 80 \text{ kg/mm}^2, \text{ Nimonic 90}, \\
&= 60 \text{ kg/mm}^2, \text{ Cr-steel}, \\
&= 33 \text{ kg/mm}^2, \text{ Cu (hard)}. 
\end{align*}$$
These are for materials with very different values of heat conductivity and 
thermal diffusivity.

It can be seen from this diagram that for every material there exists an optimum 
plate thickness, i.e. the thickness of plate which can be exposed to the hot gases 
for the longest period without exceeding a limited stress. If a longer time of 
exposure cannot be avoided the temperature of heating gas or the velocity must be 
diminished.

For the given conditions the longest heating time is 4.4 secs for the 0.7 mm 
Nimonic-plate, 12 secs for the 3 mm Cr-steel-plate and 35 secs for the 15 mm 
Cu-Plate.

If we choose the non-dimensional thickness \( \phi = 2 \frac{s \alpha}{\lambda} \) and the time to reaching 
a certain stress as \( t_{0.2} \), then the importance of \( \phi \) is shown in Figure 4.

The best times are found in the range between \( \phi = 0.33 \) and \( \phi = 0.5 \). The parameter 
for the material is \( (\sigma_{0.2} \alpha/E/\beta) \).

9.7 for Nimonic 90

32 Cr-Steel

168 Cu

This parameter gives a measure of the lack of sensitivity to thermal shock. The 
results dealt with here are valid also for the case of strain hindered in z-direction, 
e.g. for a slab clamped at one end; the stresses are the greatest at the clamped 
end and decrease with distance from this end.

3.3.2 Deformations in the Z-Direction Unrestrained

An opposite case is obtained, when the deformation in all directions is unrestrained. 
Bending will be possible in the x- and z-direction. The relations for this case are 
shown in Figure 5.

The non-dimensional stress \((\gamma)\) is given for any value of non-dimensional time \((\xi)\). 
The parameter is the non-dimensional plate-thickness. The stress-curves have a 
maximum value for a definite thickness of the plate. The maximum stresses remain 
far beyond the \( \sigma_{0.2} \)-stress, as is shown in Table II in the upper right hand 
corner and are 4.5 up to 29 kg/mm\(^2\) for Nimonic plates, 3 to 7.5 for Cr-steel and 2.5 
to 5.9 for Cu-plates.

More detailed information is given in Table II.

The highest stresses occur in the y-direction and these are verified in the heating 
times (also shown in the table) required to reach the surface temperatures.

The thicknesses of plates (2 s mm), for equal temperatures in the heated surface 
and non-dimensional plate thicknesses, \( \phi \), are also given. Since the curves have very 
flat maxima, the times during which the maximum stresses are present are relatively 
long. The time for the maximum stress of 2.3 kg/mm\(^2\) in a 7mm Cu-plate lies between 
2.2 and 17.6 sec.
It seems to be remarkable that, in none of the plates mentioned in the table, is the \( \sigma_{0.2} \) stress verified: for the 10 mm Nimonic-plate only 40\% of this stress is obtained.

Further on the temperatures at which the maximum stresses are reached, are relatively low: only 167\(^\circ\) for the 10 mm Nimonic-plate, whereas the temperature for the constant temperature distribution is about four times as high.

In spite of the fact, that the two cases treated here do not occur in practice - usually lower or higher factors of restraint are present - it can be seen from these examples that it is possible to get a good idea of the development of the distribution of non-stationary stresses under severe service-conditions when the Schmidt-method is used to find the non-stationary distributions of temperature. This method is also most valuable when with non-linear relations exist between temperature and the properties of materials. In most cases calculations with average-values will give results with good or even sufficient approximation, since the important ranges of temperatures can be very small.

Once again, very clear relations exist between the non-dimensional

\[
\eta = \frac{\sigma}{E(R \sqrt{\frac{\beta}{g}})}
\]

\[
\xi = 0.25 \alpha t s^{-2}
\]

\[
\phi = 2 s a / \lambda
\]

These values make a quick evaluation and comparison of materials with very different properties possible.

The influence of anisotropy and complex structures, which cause stresses of second kind, cannot be taken into account in the calculation.

4. CONCLUSIONS

Assuming first constant values for the physical and mechanical properties in the interesting temperature-range, calculations of the kind explained enable us whether to decide these assumptions are justified. But if the changes of temperatures in the critical stress-strain region are too large, the calculation must be carried out in several stages covering only those ranges in which the application of mean-values gives results to a satisfactory degree of accuracy. But this can be avoided in most cases.

Since, in actual service, temperature shocks cannot be controlled in such a way as to avoid plastic deformations, the calculation-methods should be extended to mixed elastic-plastic deformations. This calculation will not provide serious difficulties as long as the plastic components of strain are of the same order of magnitude as the plastic strain. Another question related to the number of applicable cycles of thermal shocks. Since the failure of any material under repeated loading has its origin in events restricted to exceedingly small areas in the material, smaller than a grain, the following experimental methods must be applied:
(a) Fatigue tests at only those temperatures, in the critical range of strains or deformation, which can be allowed from the points of view of safety and efficiency. The frequency should be chosen so that the calculated rate of deformation is obtained.

(b) Application of temperature cycles at an intermediate temperature and at a temperature rate corresponding to the initial heating or cooling temperature.

Since it is difficult in an experiment with a gaseous or a liquid medium to realise the desired temperature-gradient in a given time, the application of induction heating to experiments with metallic materials would be advisable.

By means of the frequency of the eddy current and of the design of the induction-coil, the local and time-gradient of temperature distribution may be influenced in the desired manner during the heating period. If the thermal stress is superimposed on a high stress, caused by external forces or gyrating masses, it must be kept in mind, that the creep-resistance can be lowered very seriously by a small increment of temperature and that the creep rate increases quickly.

In design the restraint must be made as small as possible. It has been shown that for every grade of hindrance of deformation strain there is an optimum wall-thickness. The heating time, or the temperature and velocity of the heating (or cooling) medium, should be adapted to the grade of restraint. These factors are just as important in the determination of admissible stresses and strains as are bending or buckling.

REFERENCES

### TABLE I

Heating-times (t sec) for reaching maximum stresses $\sigma_{\text{max}}$ in plates with various thicknesses (strains free)

<table>
<thead>
<tr>
<th>Surface-temp. $^\circ$C</th>
<th>Thickness $\varphi$ 2sa/$\lambda$</th>
<th>Material</th>
<th>$\sigma_{\text{max}}$ kg/mm$^2$</th>
<th>$2s$ mm</th>
<th>$t$ sec.</th>
<th>$\sigma_{\text{max}}$ kg/mm$^2$</th>
<th>$2s$ mm</th>
<th>$t$ sec.</th>
<th>$\sigma_{\text{max}}$ kg/mm$^2$</th>
<th>$2s$ mm</th>
<th>$t$ sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>76</td>
<td>0.0297</td>
<td>Nimonic 90</td>
<td>3.0</td>
<td>2</td>
<td>0.35</td>
<td>2.8</td>
<td>2.3</td>
<td>7</td>
<td>2.2</td>
<td>17.6</td>
<td></td>
</tr>
<tr>
<td>0.0665</td>
<td>4.5</td>
<td>Cr-steel</td>
<td>4.3</td>
<td>4.5</td>
<td>1</td>
<td>3.5</td>
<td>30</td>
<td>8.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.148</td>
<td>7.8</td>
<td></td>
<td>7.5</td>
<td>10</td>
<td>2.1</td>
<td>4.2</td>
<td>5.9</td>
<td>68</td>
<td>13.5</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>0.665</td>
<td>29.0</td>
<td>Copper</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

### TABLE II

Properties of the Materials

<table>
<thead>
<tr>
<th></th>
<th>Nimonic 90</th>
<th>Cr-Steel</th>
<th>Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{0.2}$ kg/mm$^2$</td>
<td>80</td>
<td>60</td>
<td>33</td>
</tr>
<tr>
<td>$E, \beta$ kg/mm$^2$ $^\circ$C$^{-1}$</td>
<td>0.28</td>
<td>0.27</td>
<td>0.21</td>
</tr>
<tr>
<td>$\lambda$ cal cm$^{-1}$s$^{-1}$ $^\circ$C$^{-1}$</td>
<td>0.029</td>
<td>0.13</td>
<td>0.89</td>
</tr>
<tr>
<td>$a$ cm$^2$ s$^{-1}$</td>
<td>0.034</td>
<td>0.144</td>
<td>1.07</td>
</tr>
</tbody>
</table>
\( v_0 \) = initial temp. of plate and cooling gas
\( v_\gamma \) = temp. of heating gas
\( v_\eta \) = temp. at surface \((x = -s)\)
2s = thickness of plate
\( \lambda \) = heat conductivity
\( \alpha \) = coeff. of heat transfer
\( t \) = time
\( a \) = therm. diffusivity

Fig. 1 Temperature at heated surface \((x = -s)\) Slab heated and cooled on opposite surfaces

\( \psi = 0.5 \)
0.25
0.075
0.05
0.025

Thickness of slab \( \psi = 2s\alpha/\lambda \)

Fig. 2 Slab heated and cooled on opposite surfaces. Stress \((\sigma_2)\) at heated surface
Fig. 3  Time to reach $\sigma_{0.2}$ for various plate thicknesses

Fig. 4  Time to reach 0.2% strain for various plate thicknesses

- $\sigma_{0.2} = \alpha / E \beta$
- $t_{0.2 \text{max}}$

<table>
<thead>
<tr>
<th>$\sigma_{0.2} \alpha / E \beta$</th>
<th>$t_{0.2 \text{max}}$</th>
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<tbody>
<tr>
<td>$\frac{3300}{21} \cdot 1.07 = 168$</td>
<td>35 sec</td>
</tr>
<tr>
<td>$\frac{6000}{27} \cdot 0.144 = 32$</td>
<td>12 sec</td>
</tr>
<tr>
<td>$\frac{8000}{28} \cdot 0.034 = 9.7$</td>
<td>4.4 sec</td>
</tr>
</tbody>
</table>

- $\alpha$ = thermal diffusivity
- $E$ = modulus of elasticity
- $\beta$ = coefficient of expansion
- $\alpha$ = coefficient of heat-transfer
- $\lambda$ = conductivity

thickness of plate $\rightarrow \varphi = 2s \alpha / \lambda$
maximum stress ($\sigma_{\text{max}}$)
for thickness $\varphi$

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>Nim.90</th>
<th>Cr-steel</th>
<th>Cu</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0297</td>
<td>-</td>
<td>3.0</td>
<td>2.3</td>
</tr>
<tr>
<td>0.0665</td>
<td>4.5</td>
<td>4.3</td>
<td>3.5</td>
</tr>
<tr>
<td>0.148</td>
<td>7.8</td>
<td>7.5</td>
<td>5.9</td>
</tr>
<tr>
<td>0.665</td>
<td>29.0</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 5  Stress at various times (unrestrained)
Copies of AGARD publications may be obtained in the various countries at the addresses given below.

On peut se procurer des exemplaires des publications de l'AGARD aux adresses suivantes.

**BELGIUM**  
**BELGIQUE**  
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Bruxelles.

**CANADA**  
Director of Scientific Information Services, Defence Research Board  
Department of National Defence  
'A' Building  
Ottawa, Ontario.

**DENMARK**  
**DANEMARK**  
Military Research Board  
Defence Staff  
Kastellet  
Copenhagen Ø.

**FRANCE**  
O.N.E.R.A. (Direction)  
25, avenue de la Division-Leclerc  
Châtillon-sous-Bagneux (Seine)

**GERMANY**  
**ALLEMAGNE**  
Wissenschaftliche Gesellschaft für Luftfahrt  
Zentralstelle der Luftfahrtdokumentation  
Munchen 64, Flughafen  
Attn: Dr. H.J. Rautenberg

**GREECE**  
**GRECE**  
Greek Nat. Def. Gen. Staff  
B. MEO  
Athens.

**ICELAND**  
**ISLANDE**  
Iceland Delegation to NATO  
Palais de Chaillot  
Paris 16.

**ITALY**  
**ITALIE**  
Centro Consultivo Studi e Ricerche  
Ministero Difesa - Aeronautica  
Via Salaria 336  
Rome.
<table>
<thead>
<tr>
<th>Country</th>
<th>Address</th>
</tr>
</thead>
</table>
| Luxemburg    | Luxemburg Delegation to NATO  
Palais de Chaillot  
Paris 16.               |
| Netherlands  | Netherlands Delegation to AGARD  
10 Kanaalstraat  
Delft, Holland.          |
| Norway       | Chief Engineering Division  
Royal Norwegian Air Force  
Deputy Chief of Staff/Material  
Myntgaten 2  
Oslo.  
Attn: Major S. Heglund |
| Portugal     | Subsecretariado da Estado da Aeronautica  
Av. da Liberdade 252  
Lisbon.  
Attn: Lt. Col. Jose Pereira do Nascimento |
| Turkey       | M. M. Vekaleti  
Erkaniharbiyei Umumiye Riyaseti  
İlmi İstisare Kurulu Müdurlüğü  
Ankara.  
Attn: Colonel Puat Ulug |
| United Kingdom | Ministry of Supply  
TIL, Room 009A  
First Avenue House  
High Holborn  
| United States | National Advisory Committee for Aeronautics  
1512 H Street, N.W.  
Washington 25, D.C. |
An account is given of the structural problems associated with heating and large temperature gradients. The thermal stresses due to a non-stationary distribution of temperature are calculated for a plate cooled on one surface and heated on the other by a gas moving at high speed. The cases where the plate is completely free or com-
pletely restrained in one direction are discussed. Formulae are obtained which, under these conditions of restraint, make a quick comparison of different materials possible. Possible lines for the extension of theoretical and experimental work are suggested.

Presented at the Fourth Meeting of the Structures and Materials Panel, held from 27th to 31st August 1956, in Brussels, Belgium.

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