Fundamentals of Signal Processing for Phased Array Radar

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ABSTRACT

This section gives a short survey of the principles and the terminology of phased array radar. Beamforming, radar detection and parameter estimation are described. The concept of subarrays and monopulse estimation with arbitrary subarrays is developed. As a preparation to adaptive beam forming, which is treated in several other sections, the topic of pattern shaping by deterministic weighting is presented in more detail.

1.0 INTRODUCTION

Arrays are today used for many applications and the view and terminology is quite different. We give here an introduction to the specific features of radar phased array antennas and the associated signal processing following the description of [1]. First the radar principle and the terminology is explained. Beamforming with a large number of array elements is the typical radar feature and the problems with such antennas are in other applications not known. We discuss therefore the special problems of fully filled arrays, large apertures and bandwidth. To reduce cost and space the antenna outputs are usually summed up into subarrays. Digital processing is done only with the subarray outputs. The problems of such partial analogue and digital beamforming, in particular the grating problems are discussed. This topic will be reconsidered for adaptive beamforming, space-time adaptive processing (STAP), and SAR.

Radar detection, range and direction estimation is derived from statistical hypotheses testing and parameter estimation theory. The main application of this theory is the derivation of adaptive beamforming to be considered in the following lectures. In this lecture we present as an application the derivation of the monopulse estimator which is in the following lectures extended to monopulse estimators for adaptive arrays or STAP.

As beamforming plays a central role in phased arrays and as a preparation to all kinds of adaptive beamforming, a detailed presentation of deterministic antenna pattern shaping and the associated channel accuracy requirements is given.

2.0 FUNDAMENTALS OF RADAR AND ARRAYS

2.1 Nomenclature

The radar principle is sketched in Figure 1. A pulse of length τ is transmitted, is reflected at the target and is received again at time t_0 at the radar. From this signal travelling time the range is calculated $R_0 = ct_0/2$. The process is repeated at the *pulse repetition interval* (PRI) T. The maximum unambiguous range is

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therefore $R_{max} = cT/2$. The ratio $\eta = \tau/T$ is called the *duty factor*.

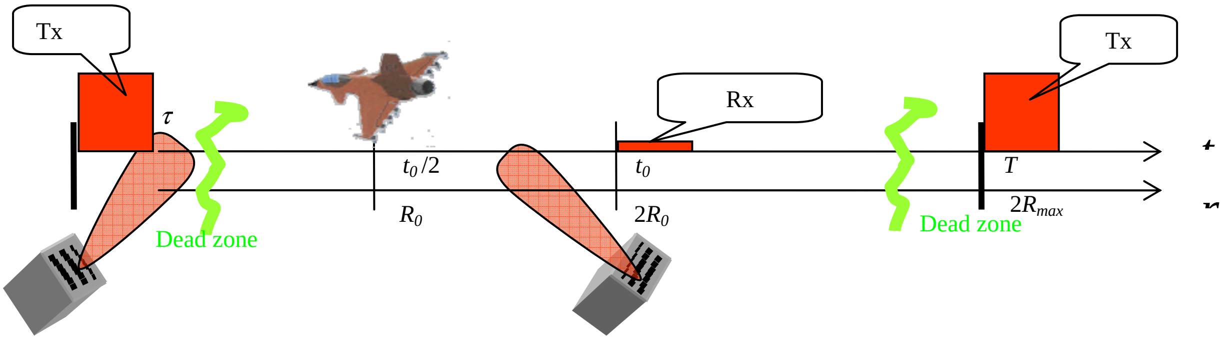


Figure 1: The principle of radar reception

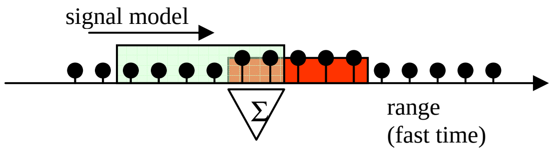
The received signal-to-noise power ratio (SNR) is described by the *radar equation*

$$SNR = \frac{1}{(4\pi R^2)^2} \cdot \frac{P_{Signal}}{P_{noise}} = \frac{1}{(4\pi)^2 R^4} \cdot \frac{P_m G_t \cdot \sigma_0 \cdot G_r \lambda^2}{(kT_0 FB) \cdot L}$$

R	target range	(km)
P_m	mean power of transmit pulse (W)	
G_t, G_r	gain of transmit/ receive antenna	
σ_0	radar cross section of target (m²)	
λ	wavelength	(cm)
kT_0	$=4*10^{-21}$ Ws	(W/Hz)
F	noise figure	(dim-less)
B	badwidth	(Hz)
L	losses	(dim-less)

It is the $1/R^4$ law that forces the radar designer to increase transmit or receive energy as much as possible.

<u>Fast time processing</u>: The received pulse is filtered such that the signal energy is maximally extracted (*matched filtering, pulse compression*). This is achieved by convolving the received data samples z_k with the transmit wave form s_k, k=1..L, $y_k = \sum_{r=1}^{L} \bar{s}_r z_{k+r}$. The *range resolution* after pulse compression is given by $\Delta R \geq c\tau/2$, where τ is the effective pulse length after compression. By suitable coding long transmit pulses with short length after compression and thus high range resolution can be constructed. This requires a larger bandwidth. The ratio of the pulse length before and after compression is called the *compression ratio* K which is the same as the *time-bandwidth product*, $K = \tau_{before}/\tau_{after} = B\,\tau_{before}$. Analogue waveforms like linear frequency modulation (or chirp) are used for pulse compression, or discrete codes which are switched at certain subpulses, e.g. binary codes or polyphase codes. For radar the sidelobes after pulse compression are important to avoid false targets. Also, the compressed pulse must be tolerant against Doppler frequency shifts which a typical echo of a moving target has.



<u>Slow time processing</u>: The receiver signal energy can be increased by integrating the power from pulse to pulse. The echo of a target with certain radial velocity v_R undergoes a frequency shift $f_D = 2v_R/\lambda$ due to the Doppler effect. From pulse to pulse at PRI T we observe thus a phase shift $\phi_D = 2\pi f_D T$. Maximum energy is collected if this shift is compensated. The summation with correct phase compensation is called *coherent integration*, $\langle y \rangle = \sum_{k=1}^{K} e^{-j2\pi f_D kT} y_k$. Of course, the radial velocity and hence the Doppler frequency f_D is unknown and must be estimated. The integration time KT is called *coherent processing interval*, CPI.

Alternatively one may sum up only the magnitude, called *incoherent integration*, $\left\langle |y|^2 \right\rangle = \sum_{k=1}^{K} |y_k|^2$. The time of all processing in a fixed look direction of the radar (e.g. with several CPIs) is called the *dwell time*.

2.2 The phased array principle

The principle of phased arrays is to generate a plane wavefront from a large number of elementary spherical waves as sketched in Figure 2. Some technical realisations of array antennas are also shown in this figure. The spherical waves are approximately realised by elementary antenna elements with near omni-directional characteristic. The application of suitable excitations at the elementary antennas and the summation of all signals on receive is called beamforming.

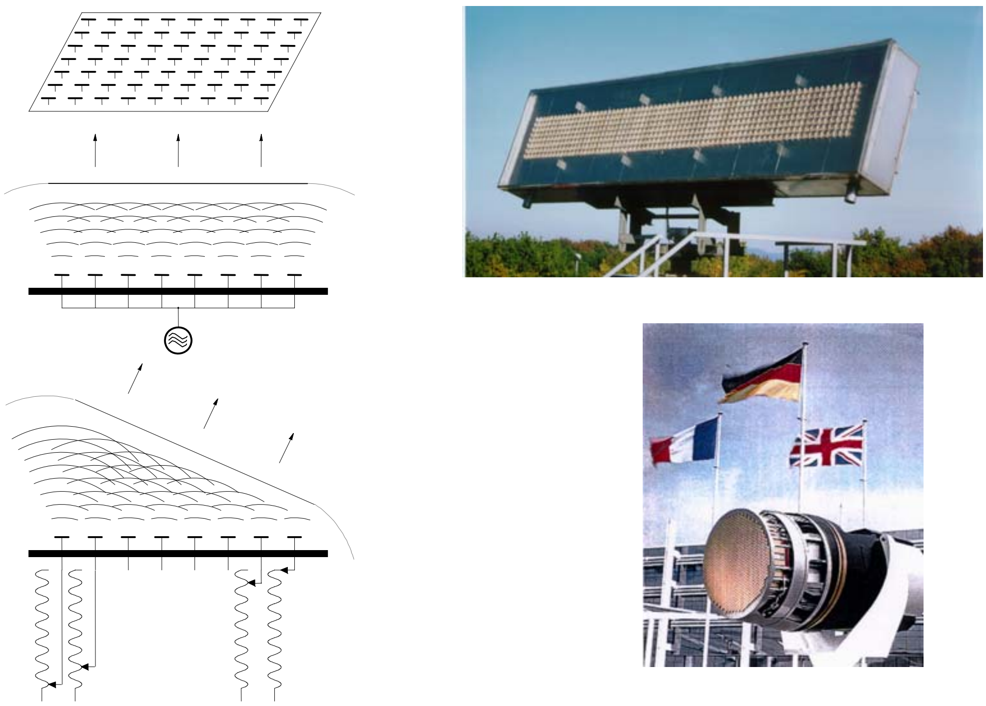


Figure 2: Principle of array antenna and some realisations of phased arrays

Why are phased array antennas of interest? The main advantage is the nearly infinitely fast switching of the look direction of the array. This allows to illuminate the search space according to some optimality criterion instead of according to a continuous mechanical movement. Recall that the $1/R^4$ law forces us to concentrate transmit energy. All aspects of optimising target illumination the received energy are denoted by the key word *energy management*, which is the principle advantage of phased arrays. Particular components of energy management are

- Coherent integration can be made nearly arbitrarily long. This allows better clutter suppression (Doppler discrimination), target classification by extracting spectral features, and finally SAR and ISAR processing.

- Performance of different radar task in time multiplex, like search and tracking of multiple targets. This allows to use one phased array radar as *multi-function radar*.

- Optimisation of individual radar tasks: optimised waveforms for search, acquisition and track, high precision measurements when required, variable beam shapes, optimised algorithms for tracking (the radar is "steered" by the tracking algorithm and a-priori information)

- Lower prime energy consumption (only for active array, saving of about a factor of 2)
- High mean time between failure (MTBF) due to graceful degradation (only active array)
- If spatial samples over the antenna aperture are available: adaptive beam forming (ABF), space-time adaptive processing (STAP), super-resolution

2.3 Beamforming

One of the key technical problems of phased arrays is the beamforming operation. To sum all signals coherently the time delay of the signal received at antenna element at position $\mathbf{r}=(x,y,z)^T$ has to be compensated. We denote the angle of incidence of an incoming plane wave by the unit direction vector in the antenna coordinate system $\mathbf{u}$ as indicated in Figure 3 (sometimes called the "direction cosines"). The plane in green may represent the aperture of a planar antenna. The formulas are also valid for 3-dimensional arrays. The path length difference between the element at position $\mathbf{r}$ and the origin is then just $\mathbf{r}^T\mathbf{u}=xu+yv+zw=|\mathbf{r}|\cdot\underbrace{|\mathbf{u}|}_{=1}\cdot\cos\measuredangle(\mathbf{r},\mathbf{u})$. For a linear antenna as indicated in the left hand subplot of Figure 3 this is equal to $x\sin\theta$.

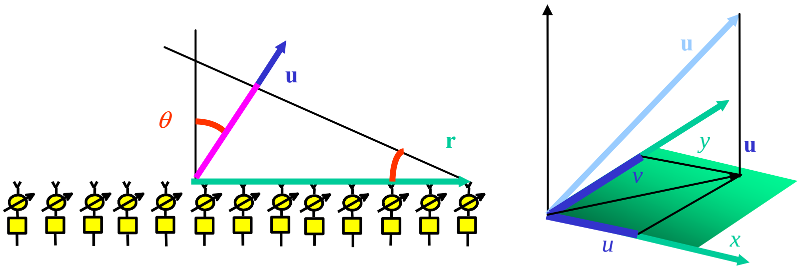


Figure 3: Delays and direction cosines

The signal at element $\mathbf{r}$ is written as $s_{\mathbf{r}}(t,\mathbf{u})=b\cdot e^{j2\pi ft}e^{j2\pi f\mathbf{r}^T\mathbf{u}/c}$, where f is the transmit frequency and c the velocity of light. Correspondingly, we form a beam into a direction $\mathbf{u}_0$ with N antenna elements by compensating these delays

$$S_t(\mathbf{u},\mathbf{u}_0)=\sum_{k=1}^{N}\underbrace{e^{-j2\pi f\mathbf{r}_k^T\mathbf{u}_0/c}}_{\bar{a}_k(\mathbf{u}_0)}\cdot s_{\mathbf{r}_k}(t,\mathbf{u})=\mathbf{a}^H(\mathbf{u}_0)\mathbf{s}(t,\mathbf{u})\,. \tag{1}$$

The superscript H denotes conjugate transpose. We call $\mathbf{a}(\mathbf{u}_0)$ the *steering vector*. For the special case of a linear antenna with equidistant elements at $x_k=kd\lambda/2$ (the elements are separated by $d\lambda/2$) this results in the well known function

$$
\begin{aligned}
S_t(u,u_0) &= \sum_{k=1}^{N} s_k(t,u)\, e^{-j2\pi f\, x_k u_0/c} = \sum_{k=1}^{N} b\cdot e^{j2\pi f t} e^{j\frac{2\pi}{\lambda}x_k(u-u_0)} \\
&= b\cdot e^{j2\pi f t}\sum_{k=1}^{N} e^{j\frac{2\pi}{\lambda}x_k(u-u_0)} \qquad (2)\\
&\xrightarrow[\text{demodulation}]{} b\sum_{k=1}^{N} e^{j\frac{2\pi}{\lambda}x_k(u-u_0)} = b\cdot \underbrace{\frac{\sin N\pi d\,(u-u_0)}{\sin \pi d\,(u-u_0)}}_{f(u-u_0)}
\end{aligned}
$$

The function $f(\mathbf{u}) = \mathbf{a}(\mathbf{0})^H\mathbf{a}(\mathbf{u})$ is called the *antenna* (directivity) *pattern*. From (2) one can see that for a uniform linear antenna this is obviously a periodic function, as for any integer p we have an u_p such that $\pi Nd\ u+2\pi p=\pi Nd\ u_p$. This means for element distances $d>1$ we have angles in the visible region $|u_p|<1$ where the main beam is repeated, called the *grating lobes*. Such grating lobes can be avoided if irregular distances are chosen. Array with irregular spacings are often chosen to achieve a narrow beamwidth with few antenna elements (e.g. minimum redundancy arrays, thinned arrays).

The (directivity) *gain* in a direction $\mathbf{u}_0$ of the antenna is defined as

$$
G = \left| f(\mathbf{u}_0)\right|^2 \Big/ \tfrac{1}{|\Omega|}\int_\Omega \left| f(\mathbf{u}-\mathbf{u}_0)\right|^2 d\mathbf{u}\,, \qquad (3)
$$

where Ω denotes the visible region, i.e. $\Omega=\{(u,v)\mid u^2+v^2<1\}$. With phase-only beam steering, the gain is approximately equal to the number of elements, $G\approx N$. Thus to achieve high gain a large number of elements is needed.

The 3 dB *beamwidth* BW is defined as $\left| f(u_0 \pm \frac{\mathrm{BW}}{2})\right|^2 = 1/2\cdot\left| f(u_0)\right|^2$. For uniform linear antennas we have BW= $0.887/A$, or $\theta_{BW}= 51/A[°]$, where A denotes the diameter (length) of the antenna aperture measured in units of the wavelength λ. For planar arrays we have approximately BW$\approx 1/A$, or $\theta_{BW}\approx 60/A[°]$.

Optimum beamforming depends obviously on the signal frequency. The beamforming operation by multiplying the signal with a suitable phase shift is therefore only correct for very narrowband signals. For broadband signals we have to apply at each antenna element the true time delays (TTD) $\tau_k = \mathbf{r}_k^T\mathbf{u}_0/c$. This is called broadband beamforming indicating that a "phased" array is always narrowband.

What does narrowband mean in this context? First, note that for planar phased arrays we have a frequency-angle ambiguity $\exp[j2\pi(f+\Delta f)\mathbf{r}^T\mathbf{u}/c] = \exp[j2\pi f\,\mathbf{r}^T(\mathbf{u}+\underbrace{\frac{\Delta f}{f}\mathbf{u}}_{\Delta\mathbf{u}})/c]$, such that $\frac{\Delta u}{u}=\frac{\Delta f}{f}$ and $\frac{\Delta v}{v}=\frac{\Delta f}{f}$. From this one can derive a rule of thumb for the angle error induced by a frequency deviation of Δf: $\Delta\theta[°]\approx\frac{\Delta f}{f}[\%]$ at $\theta=60°$, which is usually the maximum scan angle. One can now set an admissible beam pointing error at the maximum scan angle to obtain the admissible bandwidth. For a typical definition of narrowband one sets $\Delta\theta=\theta_{BW}$. This mean e.g. that for a pointing error less than 3° we have an admissible relative bandwidth below 3%.

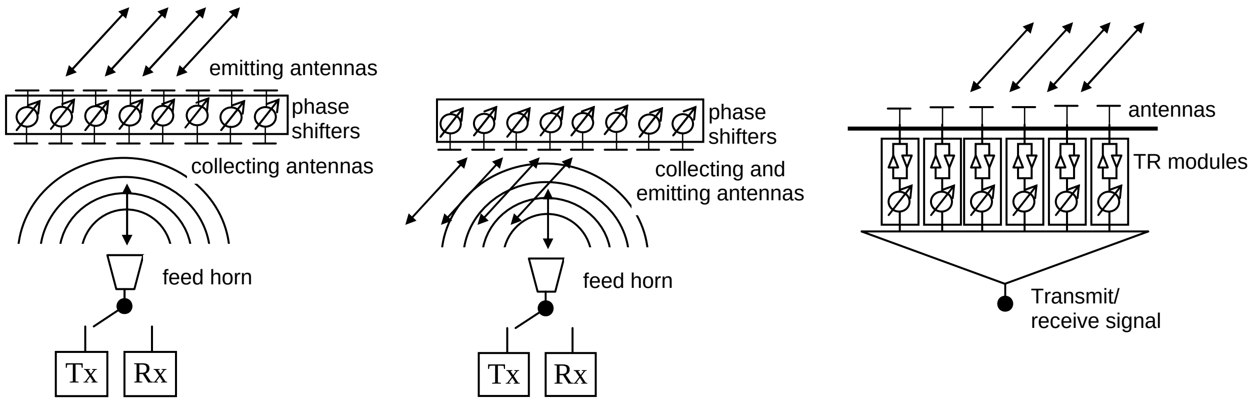
A broadband source can be written as a sum (integral) of independent sources over all frequencies of the band. For a narrowband phased array such a source looks like an extended source in angle

$$\int_{-B/2}^{B/2} \exp\left[j2\pi(f_0+f)\mathbf{r}^T\mathbf{u}/c\right] p(f)\, n_f\, df = \int_{-Bu_0/2f_0}^{Bu_0/2f_0}\int_{-Bv_0/2f_0}^{Bv_0/2f_0} \exp\left[j2\pi f_0\, \mathbf{r}^T(\mathbf{u}_0+\mathbf{u})/c\right] \tilde{p}(\mathbf{u})\, n_{\mathbf{u}}\, d\mathbf{u}\,,$$

where n_f, $n_{\mathbf{u}}$ denote white noise processes. This is important feature for suppressing broadband interference. Note that this frequency-angle ambiguity is only valid for linear and planar arrays, but not for true 3-dimensional arrays. For a 3D array $\mathbf{u}$ is always a vector of length 1, no ambiguous $\Delta\mathbf{u}$ exists. For too large offset frequencies the gain of this type of antenna simply breaks down. This means that 3D phased arrays have an inherent narrowband filtering property.

2.4 Realisations of phased arrays

The summation of the possibly large number of array channels can be done either by free space (*space feed array*) or a network (*constraint feed array*). Most of the before mentioned advantages of a phased array are offered by the so called *active array*, where each antenna element has its own transmitter and receiver, highly integrated in a TR module. Figure 4 shows sketches of these types of arrays. The highly integrated TR modules of an active array are a significant cost factor.



(a) Space feed array (b) reflect array (c) active array

Figure 4: Realisations of phased arrays

A planar array on a conducting plane has only a limited field of view, typically the sector of ±60°. To overcome this limitation several approaches have been taken. The simplest solution is to rotate the array mechanically. This does of course not allow the full energy management over 360°. Next, four array faces may be combined, but this implies the fourfold cost. Instead of four planar faces one may arrange the antenna elements on a curved surface. This is called conformal array. However, the problem is the same as with the four face array. The effective aperture uses only a small part of the antenna elements because the other elements are shadowed. In addition, for a fully coherent integration in beamforming the different polarisations measured at the differently oriented antenna elements have to be compensated. This means full polarimetric reception, i.e. two receivers per channel, and this means roughly a doubling of the cost. Figure 5 shows the power distribution in both polarisations of a spherical conformal array.

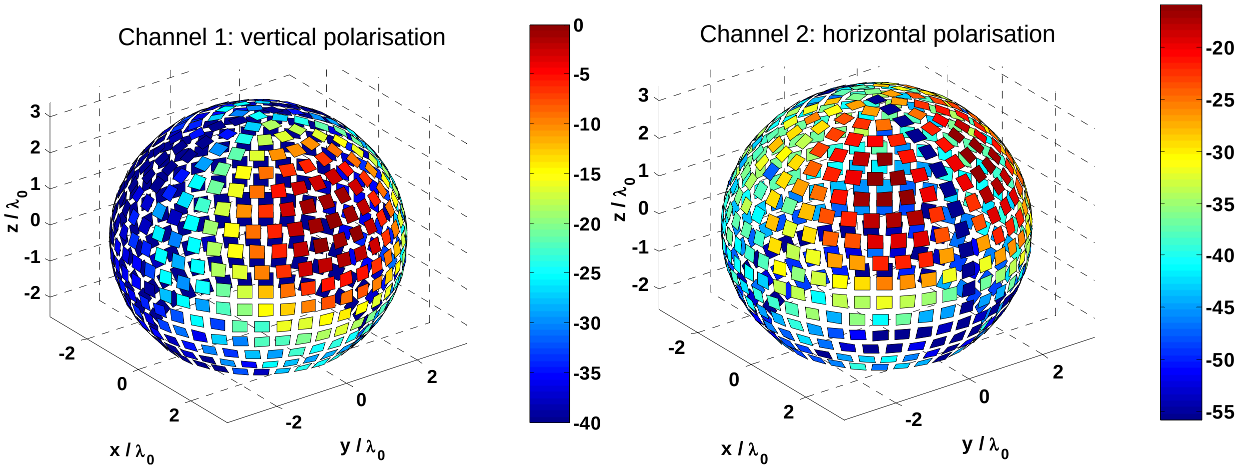
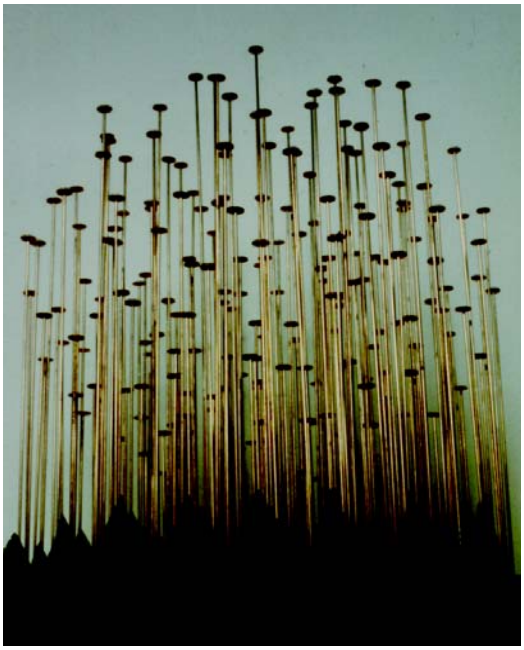
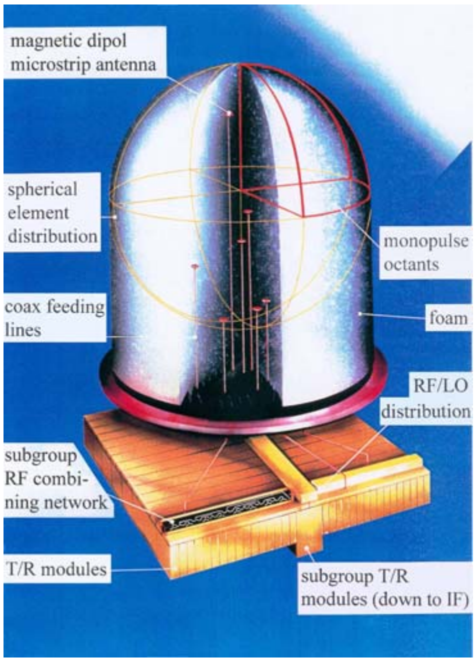


Figure 5: Power distribution resulting from illumination of a conformal array on a sphere by plane wave

Finally, the array elements may be arranged in a volume. To avoid interaction with the feeding lines the polarisation of the antenna elements has to be chosen orthogonal to these. This requires a careful design of the antenna elements. With this constraint and some loss due to the feeding lines given, the volume array is the only configuration that has full hemispherical coverage with full use of all TR modules. Figure 6 sketches the experimental array called Crow's Nest antenna realised at FGAN-FHR.



(a) Crow's Nest antenna with 512 elements randomly distributed



(b) Concept of radar with Crow's Nest antenna

Figure 6: 3D volume array

2.5 Subarrays

Beamforming can be performed in an analogue manner by a combiner network or digitally after analogue-to-digital (AD) conversion of the signals. For stability and accuracy we prefer digital beamforming. For a high gain antenna with thousands of elements AD conversion of all element outputs may be prohibitive with respect to cost. Cost effective solutions can be obtained by subarraying. First analogue summation of the element outputs is performed after phase shifting, typically at RF, then the reduced number of channels is AD converted as indicated in Figure 7.

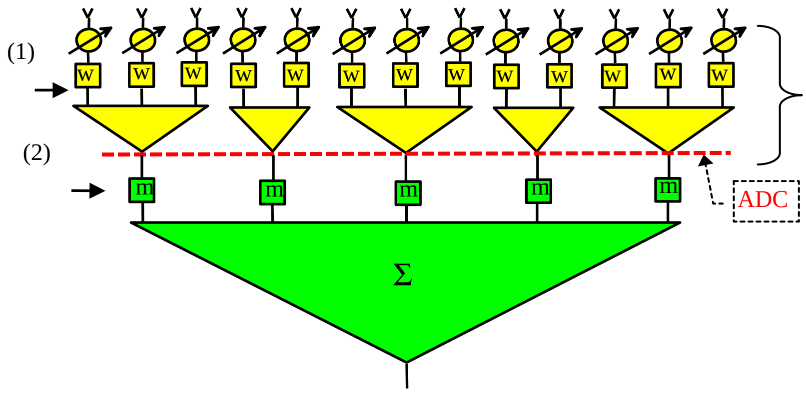


Figure 7: Beamforming with subarrays

Mathematically subarray forming can be described by a transformation matrix $\mathbf{T}$. Suppose the received complex data at the elements are described by the vector $\mathbf{z} \in \mathbb{C}^N$, let phase shifting for steering the beam and a possible amplitude weighting be described by a diagonal matrix $\mathbf{D}$, then we can describe the summation of the data into subarrays by $\tilde{\mathbf{z}} = \mathbf{T}^H \mathbf{D}\mathbf{z}$, where $\mathbf{T}$ is of size $N \times L$, if we have L subarrays. $\mathbf{T}$ contains in the i-th column ones for the elements belonging to subarray i, and zero else. The phase shifting and amplitude weighting operation can be incorporated into the matrix $\mathbf{T}_0 = \mathbf{D}^H\mathbf{T}$. The index 0 indicates the dependence on the chosen look direction $\mathbf{u}_0$. If the subarrays are disjoint (non-overlapping), then $\mathbf{T}_0^H \mathbf{T}_0$ is a diagonal matrix and the receiver noise of the TR modules remains uncorrelated at the subarray outputs, because $\mathrm{E}\left\{\tilde{\mathbf{n}}\tilde{\mathbf{n}}^H\right\} = \mathrm{E}\left\{\mathbf{T}_0^H \mathbf{n}\mathbf{n}^H \mathbf{T}_0\right\} = \mathbf{T}_0^H \underbrace{\mathrm{E}\left\{\mathbf{n}\mathbf{n}^H\right\}}_{\sigma^2 \mathbf{I}} \mathbf{T}_0 = \sigma^2 \mathbf{T}_0^H \mathbf{T}_0$ is diagonal. A plane wave at the elements transforms into $\tilde{\mathbf{a}}(\mathbf{u}) = \mathbf{T}_0^H \mathbf{a}(\mathbf{u})$. The elements of $\tilde{\mathbf{a}}(\mathbf{u})$ contain just the antenna patterns of the subarrays formed into direction $\mathbf{u}_0$.

An array with subarrays can be viewed as a *superarray* having elements at the centres of the subarrays with antenna element patterns $\tilde{a}_i(\mathbf{u})$, i=1..L. The centres of the subarrays $\rho_{x,i}$, $\rho_{y,i}$, can be calculated from the identity of the patterns $\tilde{a}_i(\mathbf{u}) = f_i(\mathbf{u}) \exp\left[j2\pi f\,(\rho_{x,i} u + \rho_{y,i} v)/c \right]$, and one obtains for a given amplitude weighting at the elements w_k

$$\rho_{x,i} = \frac{\sum_{k \in U_i} w_k g_k}{\sum_{k \in U_i} w_k}, \tag{4}$$

where we insert x or y for g, and U_i denotes the set of indices of the i-th subarray.

Note that the spacing of the superarray elements may be greater than $\lambda/2$, such that grating lobes may occur. This is not a problem if we consider the beamforming operation performed only by the phase shifters, because the full array does not have grating lobes. The problem is only apparent if beamforming operations are performed at the subarray outputs, like steering a beam by subarray weighting or with adaptive beamforming at the subarray outputs. As we have noted before, grating lobes can be avoided by irregular spacing. For a fully filled array (i.e. with elements at $\lambda/2$ spacing) this can only be achieved by irregular subarray shapes. This is illustrated in the following figures. Figure 8 shows a generic array with 902 elements on a triangular grid at $\lambda/2$ spacing configured into 32 irregular subarrays. We will make frequent use of this array later.

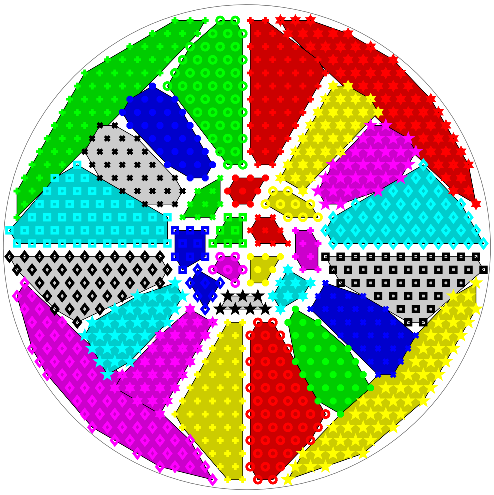
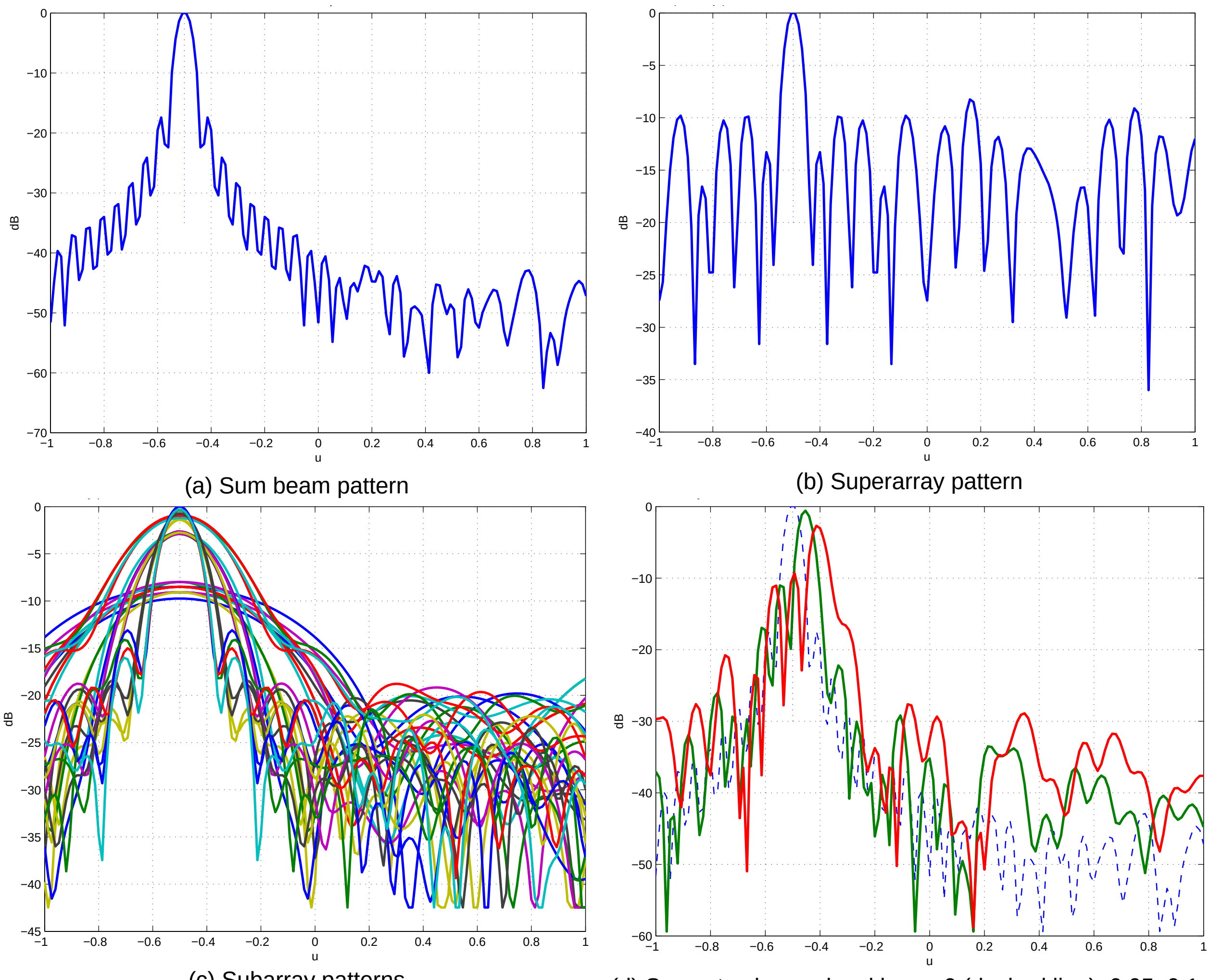


Figure 8: 2D generic array with 902 elements grouped into 32 subarrays

The Figure 9 shows the resulting antenna patterns, in subplot (a) the corresponding sum beam pattern with typical highest sidelobe at –17 dB, in subplot (b) the superarray pattern due to the subarray centres (without the influence of the subarray patterns), in subplot (c) the 32 subarray patterns, and the sum patterns for some small scan at subarray level in subplot (d). One can see that grating lobes in the superarray pattern are avoided due to careful irregular subarray shaping, but scanning at subarray level still leads to increased sidelobes and some loss in gain.



(a) Sum beam pattern

(b) Superarray pattern

(c) Subarray patterns

(d) Scan at subarray level by u= 0 (dashed line), 0.05, 0.1

Figure 9: Antenna patterns with generic array

3.0 RADAR DETECTION AND PARAMETER ESTIMATION

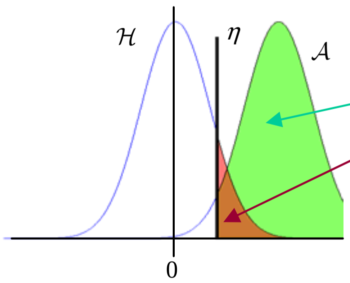
The first operation in classical radar processing is the detection of possible targets. This reduces considerably the data rate. Only for those range cells where a target has been detected an accurate estimation of the direction, range, radial velocity etc. is performed. Possibly a second look is taken with a transmit waveform that allows better parameter estimation (range, Doppler etc.). Such a second look is called acquisition or confirmation. The initialisation of a target track may be performed after confirmation using possibly again a different waveform optimised for the estimated target parameters.

3.1 Statistical test theory

In a statistical setting we consider the radar measurements (beam outputs) $z_1,..z_K$ as random numbers consisting of an expectation value $s_k(\boldsymbol{\theta})$, which is the signal, plus noise, $z_k = s_k(\boldsymbol{\theta}) + n_k$. $\boldsymbol{\theta}$ is a general parameter vector. The whole output data vector $\mathbf{z}$ is characterised by a probability density $p(\mathbf{z} ; \boldsymbol{\theta})$, e.g by the normal (Gaussian) density N($\mathbf{s}(\boldsymbol{\theta})$, $\sigma^2\mathbf{I}$) over complex vectors. We use the notation $\mathbf{z} \sim N(\mathbf{0}, \mathbf{R})$ for "$\mathbf{z}$ is normal distributed with mean $\mathbf{0}$ and covariance matrix $\mathbf{R}$". Detecting the presence of a signal is a statistical hypothesis test between two underlying distributions, e.g.

$$\begin{aligned} &\text{hypothesis } \mathcal{H} \quad \mathbf{z} \sim N(\mathbf{0},\sigma^2\mathbf{I}) \\ &\text{alternative } \mathcal{A} \quad \mathbf{z} \sim N(\mathbf{s}(\theta),\sigma^2\mathbf{I}) \end{aligned} \tag{5}$$

Such a decision is made by calculating from the data a test statistic $T(\mathbf{z})$ which is compared with a threshold η, and one decides for $\mathcal{H}$ if $T(\mathbf{z})<\eta$, and for $\mathcal{A}$ otherwise. This is illustrated in Figure 10.



$$P_D = \mathrm{Prob}\{T \geq \eta \,|\, \mathcal{A}\} \quad \text{detection probability}$$

$$P_{F1} = \mathrm{Prob}\{T \geq \eta \,|\, \mathcal{H}\} \quad \text{false alarm probability}$$

$$P_{F2} = \mathrm{Prob}\{T < \eta \,|\, \mathcal{A}\} = 1 - P_D$$

Figure 10: Distribution of test statistic and detection and false alarm probabilities

The question is: how do we get a good test statistic ? One criterion for the choice of the test statistic is that it should give maximum P_D while the false alarm probability is below a given value α which we want to control, and this criterion should be fulfilled uniformly for all parameters. Such a test is called uniformly most powerful test (UMP). It turns out that the ratio of the densities under hypothesis and alternative with the measured data inserted (which is called the likelihood) has some optimality properties. If the hypothesis and alternative consist of a set of densities, the maximum likelihood has to be taken. This is called the generalised likelihood ratio test (GLRT).

$$\text{Generalised likelihood ratio } T(\mathbf{z}) = \frac{\max\limits_{\boldsymbol{\theta}\in\mathcal{A}}\{p(\mathbf{z};\boldsymbol{\theta})\}}{\max\limits_{\boldsymbol{\theta}\in\mathcal{H}}\{p(\mathbf{z};\boldsymbol{\theta})\}} \tag{6}$$

The theory of likelihood ratio tests now gives us the following results:

- If $\mathbf{s}(\boldsymbol{\theta})$ is a linear subspace (in particular if it is only a point) then the GLRT is UMP. The bad message is that for non-linear alternatives and hypotheses in general no UMP test exist. In this case we have only the following weaker result.

- The GLRT is asymptotically UMP for $K\to\infty$ (under some general assumptions).

The parameter that maximises the alternative is called the maximum likelihood (ML) estimate of the signal parameters.

For radar applications with z_k being the sum beam output for successive pulses we have the following results, see [3] Sect. 2, 3:

- If the system noise is Gaussian distributed with known σ^2 and if the signal has deterministic but unknown, amplitude, phase, Doppler and direction, then the GLRT yields the coherently integrated sum beam output as test statistic.

- If the system noise is Gaussian distributed with known σ^2 and if the signal has random phase and

Doppler, then the GLRT yields the incoherently integrated sum beam output as an approximation of the optimum test statistic.

- The detection probability is in general a monotone function of the signal-to-noise ratio (SNR).

These results tell us when coherent and incoherent integration is optimum, and that for the detection problem it is sufficient to consider as optimisation criterion the maximisation of the SNR.

3.2 Practical problems in radar detection

The measured noise power is only a constant value σ^2 if only receiver noise is present. In reality we have all kinds of clutter which (although it is often filtered out before detection) constitute a varying unknown (residual) noise contribution. The noise power σ^2 used in the threshold η is therefore often estimated from neighbouring range or Doppler cells of the cell under test. The aim is to maintain a constant false alarm probability even in a nonhomogeneous background interference situation. This is called a *CFAR* (constant false alarm rate) *detector*. The problem is how to estimate the interference power, in particular at the clutter boundaries (e.g. from land to see clutter) and for multiple targets, such that no target is suppressed. The selection of the cells for CFAR threshold calculation and the type of averaging is important here. Mean and median value, greatest of, and order statistics estimation procedures have been proposed. All of these have advantages and disadvantages, none of these is globally optimum.

A second problem is impulsive interference (e.g. the pulses of a neighbouring radar). Only if such pulses are received through the main beam they can represent targets. A detector to discriminate between mainlobe and sidelobe echoes is therefore implemented, called *sidelobe blanker* (SLB). This is achieved by adding a single omni-directional element to the antenna, called the guard channel, whose output is compared with the sum channel. If $|z_{main}|^2 < \eta |z_{guard}|^2$ then it is decided that the echoes come from the sidelobes and the echo is "blanked". The principle is illustrated in Figure 11.

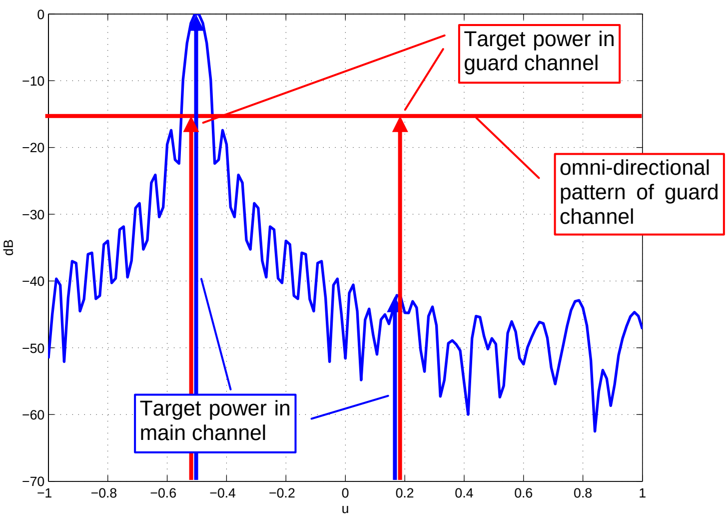


Figure 11: Sidelobe blanking with main beam and guard channel

3.3 Maximum Likelihood estimation

In statistical estimation theory we consider the problem of measured data $z_1,..z_K$ for which we have a parameterised model given by a parameterised distribution $p(z_1,..z_K\,;\,\theta)$. We are looking for a parameter that "fits best" to the assumed model. What does best fit mean? Any estimation $\hat{\boldsymbol{\theta}}$ of the desired parameter $\boldsymbol{\theta}$ is a function of the random data, $\hat{\boldsymbol{\theta}}_K(\mathbf{z})$, i.e. a random quantity. We have two desirable properties that should be fulfilled, unbiasedness and efficiency.

The estimator $\hat{\boldsymbol{\theta}}$ is called unbiased, if $\mathrm{E}\left\{\hat{\boldsymbol{\theta}}\right\}=\boldsymbol{\theta}_0$, where $\boldsymbol{\theta}_0$ denoted the true parameter vector.

For unbiased estimators the following bound is valid (*Cramér-Rao bound*, CRB):

$$\mathrm{E}\left\{(\hat{\boldsymbol{\theta}}_K-\boldsymbol{\theta}_0)(\hat{\boldsymbol{\theta}}_K-\boldsymbol{\theta}_0)^T\right\}\geq \tfrac{1}{K}\mathbf{F}^{-1}(\boldsymbol{\theta}_0)\,, \tag{7}$$

where the matrix $\mathbf{F}$ is called the *Fisher information matrix* and has elements

$$F_{i,k}(\boldsymbol{\theta}_0)=\int \frac{\partial \ln p(\mathbf{z};\boldsymbol{\theta})}{\partial\theta_i}\frac{\partial \ln p(\mathbf{z};\boldsymbol{\theta})}{\partial\theta_k}\,p(\mathbf{z};\boldsymbol{\theta})d\mathbf{z}\bigg|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}\,. \tag{8}$$

An estimator is now called *efficient*, if it has minimum variance, i.e. if it attains the CRB, $\mathrm{E}\left\{(\hat{\boldsymbol{\theta}}_K-\boldsymbol{\theta}_0)(\hat{\boldsymbol{\theta}}_K-\boldsymbol{\theta}_0)^T\right\}=\tfrac{1}{K}\mathbf{F}^{-1}(\boldsymbol{\theta}_0)$.

In Sect. 3.1 we have introduced the Maximum Likelihood (ML) estimator $\hat{\boldsymbol{\theta}}$ with $\max\limits_{\boldsymbol{\theta}\in\mathcal{H}}\left\{p(\mathbf{z};\boldsymbol{\theta})\right\}=p(\mathbf{z};\hat{\boldsymbol{\theta}})$. For non-linear parameter spaces unbiased estimators rarely exist. The ML estimator possesses this property at least approximately. We have

- For non-linear hypotheses the ML-estimator is asymptotically unbiased, for $K\to\infty$ (sometimes this property is also called consistency).
- The ML estimator is asymptotically efficient, for $K\to\infty$ or SNR$\to\infty$.

ML estimators and LR tests are often quite convenient to calculate. Because they have only asymptotically optimum properties one has to prove by simulation that e.g. the bias is for the considered sample size really small and that the estimator variance is close to the CRB.

We remark that an unbiased estimator is not always a good estimator. Forcing an estimator to be unbiased may increase the variance, so there is in general a trade off between both. The total mean squared error can also be a good criterion. Other more general optimality criteria exist, like Bayesian estimators, robust estimators, non-parametric estimators. For iterative processes like tracking unbiasedness is in general a requirement.

3.4 Monopulse estimation

In this section we will derive the ML angle estimator for phased arrays. It turns out that this is just a well established estimator introduced earlier by ad hoc engineering arguments, the monopulse estimator.

Suppose the model for the data received at the array elements is $\mathbf{z}\sim N(\mathbf{a}(\mathbf{u})b,\sigma^2\mathbf{I})$. For ML estimation we have to maximise the complex normal density

$$p(\mathbf{z};\mathbf{u},b) = \frac{1}{(\pi\sigma^2)^N} \exp\left[-\left(\mathbf{z} - \mathbf{a}(\mathbf{u})b\right)^H \left(\mathbf{z} - \mathbf{a}(\mathbf{u})b\right) / \sigma^2 \right]. \tag{9}$$

This is equivalent to minimising $\left\|\mathbf{z} - \mathbf{a}(\mathbf{u})b\right\|^2$. The minimum over b is a least squares problem, it ca be written for all $\mathbf{u}$ as $\hat{b} = \left(\mathbf{a}(\mathbf{u})^H \mathbf{a}(\mathbf{u})\right)^{-1} \mathbf{a}(\mathbf{u})^H \mathbf{z}$. Inserting this into the mean squared expression, using $\mathbf{a}^H\mathbf{a}$= const and dropping constant terms, we are left with the maximisation of the sum beam power $\hat{\mathbf{u}} = \arg\max \left|\mathbf{a}(\mathbf{u})\mathbf{z}\right|^2$, also called the scan pattern. Note that this is different from the antenna pattern, which is the response to source surrounding the antenna having a fixed look direction, while for the scan pattern we steer the beam electronically over the scene.

This scanning of the beam would mean to transmit pulses in a fine grid of directions, which is not desirable for transmit power economy. We would like to estimate the direction from a measurement which is already close to the true direction. For detection, which is the process before angle estimation, we will scan the observation space in a grid of the beamwidth BW. The 3 dB beamwidth is just the resolution limit where the scan pattern over two targets of equal power produces a pattern with two distinct maxima. In the following estimation process we then want to estimate the target direction within BW accurately. The estimation can be done by approximating the scan pattern main beam by a parabola. We need two measurements to calculate the parameters of a parabola, three measurements to calculate the parameters of a 2D paraboloid for azimuth and elevation estimation. For monopulse estimation one uses measurements of the function itself (the beam output) plus its derivatives, called the difference beams, using a Taylor expansion of the function. The trick in monopulse estimation is that one applies the Taylor expansion to the function $G(\mathbf{u}) = \ln\left|\mathbf{a}(\mathbf{u})^H \mathbf{z}\right|^2$. This results in a numerically more stable linear extrapolation formula. The first order Taylor expansion of the derivative of G at the position of the maximum $\hat{\mathbf{u}}$ is

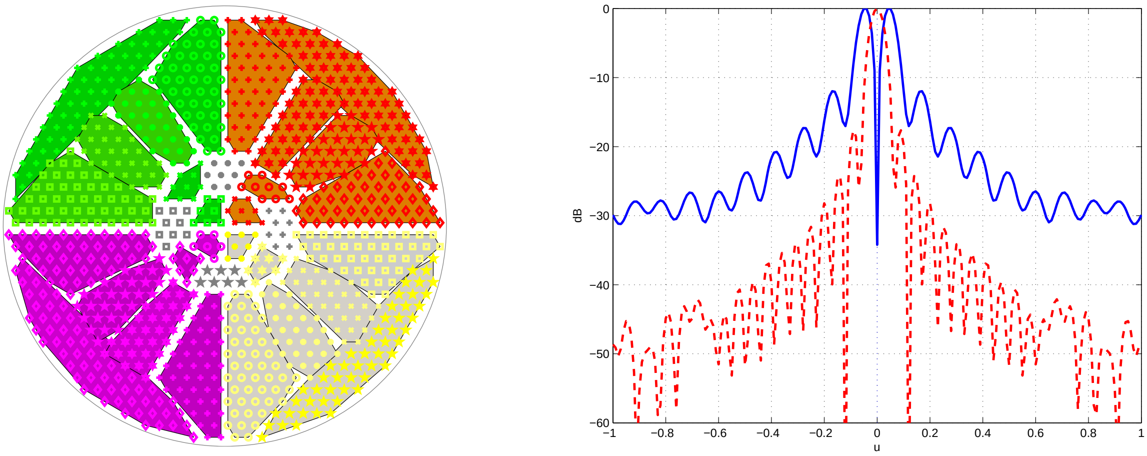
$$G'(\mathbf{u}) \approx \underbrace{G'(\hat{\mathbf{u}})}_{=0} + G''(\hat{\mathbf{u}})(\mathbf{u} - \hat{\mathbf{u}}) \text{ from which follows } \hat{\mathbf{u}} \approx \mathbf{u} - G'(\mathbf{u}) / G''(\hat{\mathbf{u}}).$$

One can now calculate that $G'(\mathbf{u}) = 2\,\mathrm{Re}\left\{\dfrac{\mathbf{a}_h^H(\mathbf{u})\mathbf{z}}{\mathbf{a}^H(\mathbf{u})\mathbf{z}}\right\}$ with the notation $\mathbf{a}_h = \dfrac{\partial \mathbf{a}}{\partial h}$ for h=u or v. This is just the ratio of the measured sum and difference beam output. The elements of the difference beam are $d_{x,i} = \partial a_i / \partial u = 2\pi j x_i a_i(\mathbf{u})$, i= 1..N (if we measure x_i in units of the wavelength λ) and the analogous weighting for the derivative with respect to elevation. This is a beam with an additional amplitude weighting according to the element position. The second derivative is approximated by its expectation value, which is $\mathrm{E}\left\{G''(\hat{\mathbf{u}})\right\} \approx 2\,\mathrm{Re}\left\{\dfrac{\mathbf{a}_{ht}^H \mathbf{a}}{\mathbf{a}^H \mathbf{a}}\right\} = \gamma$ (h,t = u or v). This is a constant value depending on the antenna element distribution, $\gamma_x = -N / \sum_{i=1}^{N} x_i^2$. We have thus obtained the classical monopulse formulas ([3] Sect. 3, [4])

$$\begin{aligned} \hat{u} &= u_0 - \gamma_x R_x(\mathbf{u}_0) \\ \hat{v} &= v_0 - \gamma_y R_y(\mathbf{u}_0) \end{aligned} \quad \text{where } R_h = \mathrm{Re}\left\{\frac{\mathbf{d}_h^H \mathbf{z}}{\mathbf{a}^H \mathbf{z}}\right\} \text{ for } h{=}x, y. \tag{10}$$

All beams are formed in a look direction $\mathbf{u}_0$, R is called the monopulse ratio.

For a phased array three different weightings have to be applied at the elements for the sum and the two difference beams which is difficult to realise. Therefore these different weightings are applied as an approximation at subarray level. With the digital subarray outputs we can form arbitrarily many beams with the stored data. The smallest number of subarrays would the be four, and the weighting with the subarray centres then corresponds essentially to a weighting with ±1 (subtract left from right aperture half for azimuth difference beam, lower from upper half for elevation difference beam). This subtraction of the two halves of is the reason for the name difference beam for the derivative beams. For the generic array considered before this is equivalent to replacing the subarray amplitude weighting x_i by sign(x_i). Subarrays with centre zero are not used for difference beam forming, see Figure 12.



(a) Generic array as 4 quadrant monopulse antenna

(b) Azimuth cuts through sum and difference beams

Figure 12: Four quadrant monopulse weighting and antenna patterns

4.0 DETERMINISTIC BEAM SHAPING

In an active phased array we have in general TR modules that can not only control the phase, but also the amplitude by digitally controlled attenuators. This can be used to reduce the highest sidelobes, e.g. below the –17 dB level, which is the highest sidelobe of the untapered planar array, or to reduce the sidelobes in certain angular sectors where interference is expected, or to broaden the transmit beam to illuminate a larger sector to save search time for targets at close ranges. This is the topic of this section.

Let us first derive some general rules for arrays with arbitrary beamforming weights $\mathbf{w} \in \mathbb{C}^N$ at the elements. The antenna pattern is

$$f(\mathbf{u}) = \left| \mathbf{w}^H \mathbf{a}(\mathbf{u}) \right|^2 \tag{11}$$

which we assume to be normalised to a maximum value of 1, i.e. $\mathbf{w}^H\mathbf{a}(\mathbf{0}) = 1$. The average sidelobe level then is

$$\overline{SL} = \frac{1}{|\Omega|} \int_{\Omega} |f(\mathbf{u})|^2 \, d\mathbf{u} = \frac{1}{|\Omega|} \int_{\Omega} \mathbf{w}^H \mathbf{a}\mathbf{a}^H \mathbf{w} \, d\mathbf{u} = \mathbf{w}^H \mathbf{C} \mathbf{w}$$

with $\mathbf{C} = \frac{1}{|\Omega|} \int_{\Omega} \mathbf{a}(\mathbf{u})\mathbf{a}^H(\mathbf{u}) \, d\mathbf{u}$. Note that we have averaged also over the main lobe. If the integration is extended to $\Omega = \mathbb{R}^2$, then $\mathbf{C} \to \mathbf{I}$, and we obtain (under the above normalisation) a general rule of thumb for the sidelobe level

$$\overline{SL} \approx \mathbf{w}^H \mathbf{w} \tag{12}$$

In particular we have for conventional beamforming ($\mathbf{w}$ equal to $\mathbf{a}_0/N$ with steering vector $\mathbf{a}_0$) that $\overline{SL} = \sum_{k=1}^{N} 1/N^2 = 1/N$.

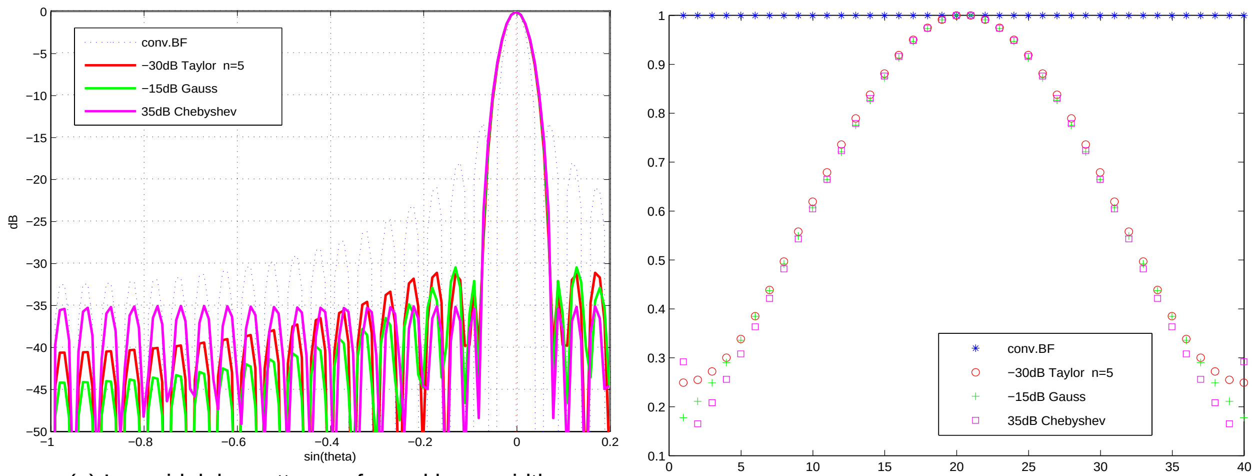
In Section 3.1 we have stated that for optimum detection the SNR should be maximised. Which weightings maximise the SNR? If only white (receiver) noise is present, we have

$$\begin{aligned} SNR &= \mathrm{E}\left\{|\mathbf{w}^H\mathbf{s}|^2\right\} \Big/ \mathrm{E}\left\{|\mathbf{w}^H\mathbf{n}|^2\right\} = \mathbf{w}^H\,\mathrm{E}\left\{\mathbf{s}\mathbf{s}^H\right\}\mathbf{w} \Big/ \mathbf{w}^H\,\mathrm{E}\left\{\mathbf{n}\mathbf{n}^H\right\}\mathbf{w} \\ &= \beta^2\mathbf{w}^H\mathbf{a}_0\mathbf{a}_0^H\mathbf{w} \Big/ \sigma^2\mathbf{w}^H\mathbf{w} \quad \Rightarrow \quad \mathbf{w} = \mathbf{a}_0 \end{aligned}.$$

This says that the conventional (steering vector) weighting produces the maximal SNR with respect to receiver noise, or, in other words, any other weighting producing lower sidelobes has lower SNR against receiver noise.

4.1 Aperture tapering

Now we assume the complex beamforming weights $w_i = g_i e^{j2\pi f \mathbf{r}_i^T \mathbf{u}/c}$, i=1..$N$. The simplest way of pattern shaping is to impose some bell-shaped amplitude weighting over the aperture, e.g. $g_i = \cos^{\nu}(\pi x_i / A) + \alpha$, for suitable constants ν, α, or $g_i = e^{-\nu x_i^2}$. The foundation of these weightings is only heuristical. The Taylor weighting is optimised in the sense that it leaves the conventional (uniformly weighted) pattern undistorted except for a reduction of the first n sidelobes below a level of x dB, [5]. The Dolph-Chebyshev weighting creates a pattern with all sidelobes equal at level of y dB. Figure 13 shows examples of such patterns for a uniform linear array of 40 elements. The taper functions for low sidelobes were selected such that the 3 dB beamwidth of all patterns is equal. The conventional pattern is plotted for reference showing that all tapering increases the beamwidth. It depends on the emphasis on close-in and far-off low sidelobes which tapering is preferred. Another point of interest is a small dynamic range of the weights, because at the array elements only an attenuation can be applied. One can see that the Taylor tapering has the smallest dynamic range. For planar arrays the efficiency of the tapering is slightly different.



(a) Low sidelobe patterns of equal beamwidth

(b) Taper functions for low sidelobes

Figure 13: Low sidelobes by amplitude tapering

4.2 Optimum pattern shaping

From a signal processing point of view it is the interference power coming over the sidelobes that we want to minimise. This can be achieved by solving the following optimisation problem, [6],

$$\min_{\mathbf{w}} \int_{\Omega} \left|\mathbf{w}^H \mathbf{a}(\mathbf{u})\right|^2 p(\mathbf{u})\, d\mathbf{u} \quad \text{subject to} \quad \mathbf{w}^H\mathbf{a}_0 = 1, \text{ or} \tag{13}$$

$$\min_{\mathbf{w}} \mathbf{wCw} \quad s.t. \quad \mathbf{w}^H \mathbf{a}_0 = \mathbf{1}, \text{ with } \mathbf{C} = \int_{\Omega} \mathbf{a}(\mathbf{u})\mathbf{a}(\mathbf{u})^H\, p(\mathbf{u})\, d\mathbf{u}\,.$$

As before, Ω denotes the angular sector where we want to influence the pattern, e.g. the whole visible region $u^2+v^2<1$, and p is a weighting function which allows to put different emphasis on the criterion in different angular regions.

The solution of the above optimisation is

$$\mathbf{w} = \frac{\mathbf{C}^{-1}\mathbf{a}_0}{\mathbf{a}_0^H \mathbf{C}^{-1}\mathbf{a}_0} \tag{14}$$

For the choice of p some remarks are necessary:

1. For a global reduction of the sidelobes, $\Omega = \left\{\mathbf{u} \in \mathbb{R}^2 \,\middle|\, u^2 + v^2 \le 1\right\}$, one should exclude the main beam from the minimisation by setting $p = 0$ on this set of directions. In fact, it is recommended to exclude a slightly larger region (e.g. the null-to-null width) to allow a certain mainbeam broadening.

2. One may form discrete nulls in directions $\mathbf{u}_1, \ldots \mathbf{u}_M$ by setting $p(\mathbf{u}) = \sum_{k=1}^{M} \delta(\mathbf{u} - \mathbf{u}_k)$. The solution then is

$$\mathbf{w} = \frac{\mathbf{Pa_0}}{\mathbf{a}_0^H \mathbf{Pa_0}} \text{ with } \mathbf{P} = \mathbf{I} - \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H \quad \text{with } \mathbf{A} = \left(\mathbf{a}(\mathbf{u}_1), \cdots \mathbf{a}(\mathbf{u}_M)\right) \tag{15}$$

   To avoid insufficient suppression due to channel inaccuracies one may also create small extended nulls using the matrix **C**.

3. One may reduce the sidelobes in selected areas where interference is expected. This is shown in Figure 14, where the sidelobes in the lower elevation space have been reduced for the application in an airborne radar to reduce ground clutter reflections, see [7].

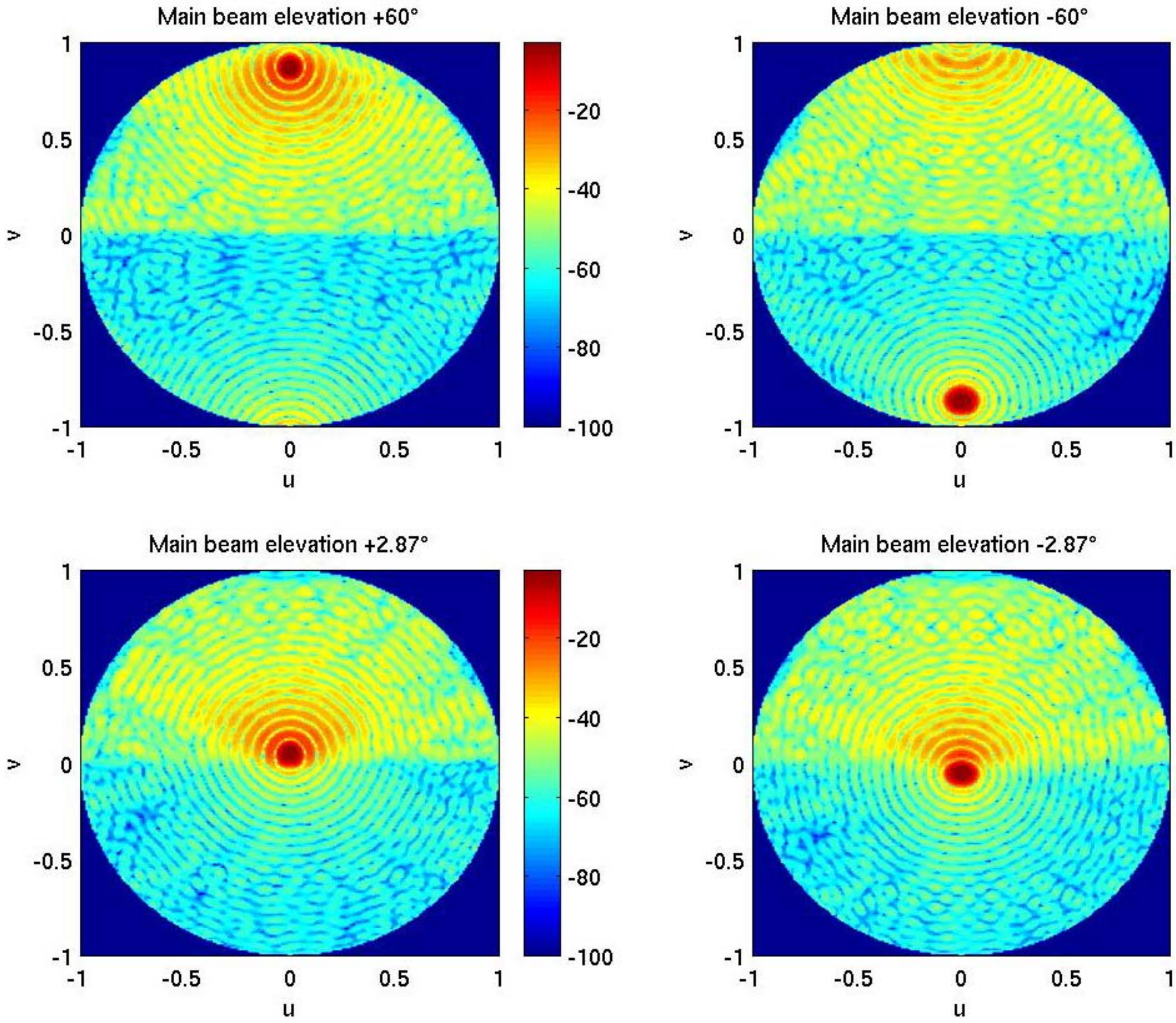


Figure 14: Antenna patterns with reduced sidelobes at lower elevations for different beam pointing directions

4.3 Phase-only pattern shaping

Sometimes the array may not offer the possibility to apply amplitude control, or one does not want to apply any attenuation (typically for transmit pattern shaping). In these cases pattern shaping by phase-only control is of interest.

Beam broadening

To reduce the revisit time in search mode one may broaden the main beam on transmit and apply multiple receive beams formed at subarray level. A simple broadening could be achieved by dividing the aperture into four quadrants and applying in each quadrant a weighting for a slightly squinted beam. The sum of these four beams gives a pattern with broader beam. This would correspond to a sort of triangular phase variation over the aperture. More general, one can apply any bell shaped phase variation similar to the amplitude variations given in Sect. 4.1. It depends on the acceptance of ripple within the main beam and the height of the first sidelobes which of these phase variations is preferred.

Phase-only nulling

Phase-only nulling has been a topic of research since many decades. The problem is that the corresponding optimisation problem (13) is not analytically solvable. It is even not known if a solution exists in general. The parameter space (the phases) is a non-convex space which creates problems for gradient methods. For well separated nulls iterative methods have been suggested, but these converge only under the assumption of small phase variations. It is also known that the creation of two nulls in opposite directions **u** and –**u** creates problems. Some important references here are [8], [9], [11], [12].

The convexity problem can be overcome by formulation of the optimisation problem and the gradient algorithm on the phase manifold. This requires the use of tools of calculus on Riemannian manifolds. This approach and associated Newton iterations have been formulated in [10].

4.4 Stability considerations for low sidelobe antennas

Influence of errors

Suppose we can apply instead of the desired weights $w_{0,i}$ only the weights $w_i = \left(|w_{0i}| + \Delta g_i\right)\exp\left[\, j\Delta\phi_i\,\right]$. In addition, we assume that we have errors in the coordinates of the element positions $x_i = x_{0i} + \Delta x_i, \quad y_i = y_{0i} + \Delta y_i$. The antenna pattern then is

$$f(u,v) = \sum_{i=1}^{N}\left(|w_{0i}| + \Delta g_i\right)\exp\left[\, j2\pi\left((x_i + \Delta x_i)u + (y_i + \Delta y_i)v\right) - j\Delta\phi_i\,\right]$$

Assume that all errors are uniformly distributed in a given interval $|\Delta g_i| < \Delta G, \quad |\Delta\phi_i| < \Delta\Phi, \quad |\Delta x_i| < \Delta X$ (otherwise the elements are declared defective), then one can calculate the expectation and variance over the uniform distribution. For a linear array we obtain

$$E\left\{f(u)\right\} = f_0(u)\frac{\sin\Delta\Phi}{\Delta\Phi}\cdot\frac{\sin 2\pi u\Delta X}{2\pi u\Delta X}\cdot\frac{\sin 2\pi v\Delta Y}{2\pi v\Delta Y}$$
$$\operatorname{var}\left\{f(u)\right\} = \left[\frac{\Delta G^2}{3\frac{1}{N}\sum|w_{0i}|^2} + 1 - \frac{\sin^2\Delta\Phi}{(\Delta\Phi)^2}\cdot\frac{\sin^2 2\pi u\Delta X}{(2\pi u\Delta X)^2}\cdot\frac{\sin^2 2\pi v\Delta Y}{(2\pi v\Delta Y)^2}\right]\sum|w_{0i}|^2 .$$

One may now plot confidence interval patterns $\left|E\left\{f(u)\right\}\right| \pm \operatorname{var}\left\{f(u)\right\}$. By simulation we found that 99% of the erroneous patterns lie within this interval, so this is a good approximation (for a Normal distribution the confidence level would be 68%). Figure 15 shows this bound applied to the generic array of Figure 8 with a 15 dB Gauss taper at the elements resulting in a peak sidelobe level of –30 dB. We have assumed amplitude errors of ±2.5% of the maximum taper value, and phase errors of ±2.5% of 2π. This is a good indication of the sensitivity of low sidelobe antenna patterns.

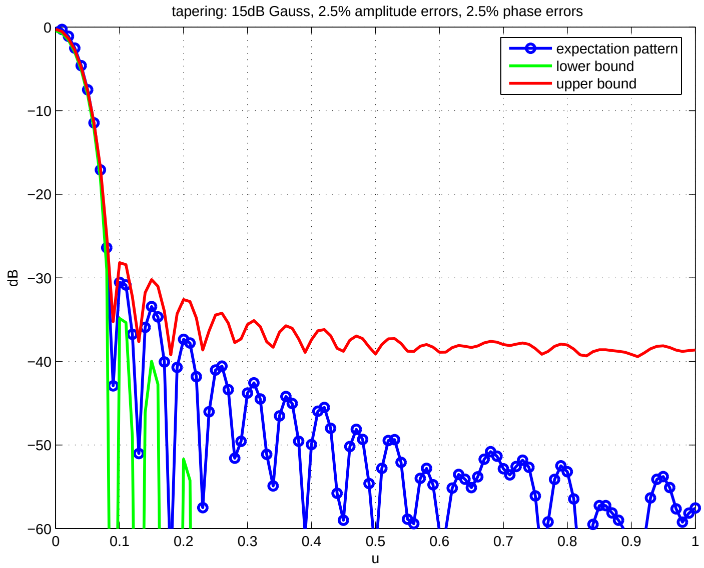


Figure 15: 99% confidence intervals for sidelobes with 15dB Gauss tapering and 2.5% amplitude and phase errors

Influence of element failures

If say M elements fail and are switched off (w_i=0), we have the distorted directivity pattern (11)

$$\tilde{f}(\mathbf{u}) = f(\mathbf{u}) - \sum_{i=1}^{M} e^{j2\pi f \mathbf{x}_i^T \mathbf{u}/c} w_i \,.$$

The M elements are a random subset of all N elements. The power density then results in

$$\left|\tilde{f}(\mathbf{u})\right|^2 = \left|f(\mathbf{u})\right|^2 - 2\,\mathrm{Re}\left\{ f(\mathbf{u}) \sum_{i=1}^{M} e^{j2\pi f \mathbf{x}_i^T \mathbf{u}/c} w_i \right\} + \sum_{k=1}^{M}\sum_{i=1}^{M} e^{j2\pi f (\mathbf{x}_i - \mathbf{x}_k)^T \mathbf{u}/c} w_i w_k^* \,.$$

Now assume that we have low sidelobes, i.e. that $|f(\mathbf{u})| \ll 1$ in the sidelobe region. Then the following approximation holds in the sidelobe region

$$\left|\tilde{f}(\mathbf{u})\right|^2 \approx \sum_{k=1}^{M}\sum_{i=1}^{M} e^{j2\pi f (\mathbf{x}_i - \mathbf{x}_k)^T \mathbf{u}/c} w_i w_k^* \,.$$

Because of the assumed randomness of the defective elements the contributions for $i \neq k$ will cancel. This means the average sidelobe level is determined by the power of the defective elements

$$\left|\tilde{f}(\mathbf{u})\right|^2 \approx \sum_{i=1}^{M} |w_i|^2 \,.$$

In fact, it is very important whether an element with high or low taper weight fails. We may further approximate each $|w_i|^2$ by the average taper power $\|\mathbf{w}\|^2 / N$ and obtain the rough approximation for the sidelobe level

$$\left|\tilde{f}(\mathbf{u})\right|^2 \approx M \|\mathbf{w}\|^2 / N \,.$$

For example, for conventional beamforming with w_i= 1 /N failures of few elements will have nearly no effect, as the sidelobe level is already high. For the 40 elements antenna of Figure 13 we have

10log10(1/N^2)= 32 dB and sidelobes only outside ±0.6 will be affected. For the 30 dB Taylor pattern we have $10 \cdot \log 10(\|\mathbf{w}\|^2 / N) = 31.3\,\mathrm{dB}$ and nearly all sidelobes may rise up to this level.

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