A Strain Based Design Criterion for Solid Propellant Rocket Motors

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1.0 INTRODUCTION

Due to a quite complex molecular structure, the mechanical behaviour of solid propellant grains may hardly be theoretically formalized in a constitutive law and then implemented for a throughout numerical description. It is thus believed that a decisive improvement of mechanical integrity assessment of propellant structures may be expected from any description that would avoid the exact calculation of the stress fields. With this standpoint a method is proposed to assess the service life of solid propellant grains which is based on the analysis of the strain fields calculated during the different specific loading cases of the service life. The center point of the method is a generalized strain criterion which is capable to reduce the three dimensional outputs of a finite elements calculation (strains tensors) into an uneasily equivalent deformation. This uneasily equivalent deformation is then a consistent quantity which may be compared to experimental results which are obviously more easily gathered for this common and convenient strain field of uniaxial tension.

2.0 INDUSTRIAL BACKGROUND

The mechanical integrity assessment of propellant grains is built up on a numerical calculation in which the essential and legitimate assumption refers to the incompressible behaviour of the modelized solid. For solid propellants, this assumption is justified firstly by the experimental evidence of such a behaviour (at least up to limited strain amplitudes) and also by the fact that these calculations are the majorant reference to analyse the simulations of the calculated structures. A second specific feature of the propellant grain numerical simulations lies in the boundaries conditions specified in terms of imposed displacements both during cooling after casting and during the operational mission (internal pressure of combustion). From the combination of these boundaries conditions and of an incompressible material behaviour, it results that the displacement field of this axisymmetric mechanical problem is only at the second order dependant on the model implemented (in fact the solution depends mainly on the volumetric behaviour assumption). The proposed description is thus entirely based on a strain analysis constructed on the assumption of incompressibility with the objective to allow the structure numerical calculations results to be compared by a simple method to uniaxial tensile tests experimental information.

3.0 ULTIMATE STRAINS OF PROPELLANTS

A typical experimental response of a HTPB solid propellant to multiaxial strain loadings is depicted schematically by figure 1. It may be deduced from these responses to uniaxial tension, equibiaxial tension and simple shear, that the propellant mechanical behaviour is strongly strain field dependant and
especially in terms of ultimate strains amplitude. Since the propellant material is strongly viscoelastic, these fracture strains are rate and temperature dependant but it will be considered throughout this paper as an experimental evidence that the ratio between these multiaxial fracture strains is independent of the experimental conditions.

![Figure 1: Typical Response of Solid Propellant to Three Different Strain Fields.](image)

From this experimental start point, the objective is to describe the strain field dependence of the material ultimate strains in a generalized approach keeping in mind the constraint of a simple industrial usable method. The basis of this method refers to the incompressible behaviour assumption for finite deformation amplitudes which may be written as the volume conservation:

\[ \lambda_1 \lambda_2 \lambda_3 = 1 \]

with \( \lambda_1, \lambda_2, \lambda_3 \) the principal components of the displacement gradient matrix, \( \mathbf{F} \), chosen as the deformation measure. This arbitrary choice of the strain measure is justified by the experimental convenience of the Piola-Lagrange continuum mechanics description (the laboratory results are given as \( \Delta \ell/\ell_0 \) and \( T/A_0 \) where \( \ell_0 \) and \( A_0 \) are the unstretched dimensions of the sample and \( T \) the current force). With these definitions, for the different strain fields depicted in fig.1, the characteristic strain matrix, \( \mathbf{F} \), for each field is:

For uniaxial tension:

\[
\mathbf{F} = \begin{pmatrix}
\lambda_1 = \lambda & 0 & 0 \\
0 & \lambda_2 = \lambda^{-1/2} & 0 \\
0 & 0 & \lambda_3 = \lambda^{-1/2}
\end{pmatrix}
\]

For equibiaxial tension:

\[
\mathbf{F} = \begin{pmatrix}
\lambda_1 = \lambda & 0 & 0 \\
0 & \lambda_2 = \lambda & 0 \\
0 & 0 & \lambda_3 = \lambda^{-2}
\end{pmatrix}
\]

For simple shear the measured properties are not in the principal axis:

\[
\mathbf{F} = \begin{pmatrix}
1 & \gamma & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} \rightarrow \begin{pmatrix}
\lambda_1 = \lambda & 0 & 0 \\
0 & \lambda_2 = \lambda^{-1} & 0 \\
0 & 0 & \lambda_3 = 1
\end{pmatrix}
\]

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with \( \gamma = \frac{U}{l} \) the recorded displacement divided by the sample thickness from which the first principal strain may be deduced by:

\[
\lambda = \frac{1}{2} \left( \gamma + \sqrt{\gamma^2 + 4} \right).
\]

When each of these equations are represented and projected upon the plane \((\lambda_i, \lambda_j)\), appears a system of curves which describes a solid concave surface of revolution centered on the point \((1,1,1)\) defining the free strain state as shown on figure 2. To give a more visual picture of the situation depicted it may be compared to an open umbrella which vertex is merged to the point \((1,1,1)\), the shaft goes along the trissectrix \((\lambda_1 = \lambda_2 = \lambda_3)\) and each of the ribs describes a particular strain path.

![Figure 2: Deformation Paths and Fracture Points.](image)

When each multiaxial fracture strain is placed in this plane, appears clearly the regular figure depicted on figure 2 which is obviously describing an ellipse. This plot may be guessed to be the result of the intersection of a plane perpendicular to the space trissectrix \((\lambda_1 = \lambda_2 = \lambda_3)\) with the ‘umbrella’ solid surface.

To demonstrate the existence of this feature it is proposed to perform a change in the reference axis. Choosing the first principal axis of this new reference merged with the trissectrix, the choice for the two remaining axis is arbitrary but must form a cartesian co-ordinates system. The effective system adopted thereafter is shown on figure 3.
This new set of axis is of course associated with a rotation matrix each component of which being the direction coefficients of the new axis system. If the classical operations of co-ordinates rotation are then applied to the results of figure 2 and if the projections of the transformed results are performed upon planes both parallel and normal to the trissectrix, one obtains respectively figures 4a and 4b.

The analysis of these projected figures confirms the remarkable following evidences:

- the state equation of incompressibility describes a revolution surface centered on the trissectrix of the principal strains space ($\lambda_1=\lambda_2=\lambda_3$)
- the fracture points associated to different strains fields describes the intersection of this surface with a plane perpendicular to the trissectrix which is completely determined by the height, $t$, of the location intersection point along that trissectrix.
As a consequence any three-dimensional strain field fitting the incompressibility requirement equation will fracture when the intensity, \( t \), measured in this same manner along the trissectrix will reach this critical height, \( t_r \).

If it is noticed that the fracture criterion is completely determined by this single value of the height, \( t_r \), which defines the intersection plane position along the trissectrix, the description of fracture may be generalized to any multiaxial strain field which is not involved in the criterion construction. In this way, evaluating \( t \) is a measure of the ‘intensity’ of any multiaxial strain fields encountered in numeric calculations. For convenience, the strain field intensity is converted into an uniaxial equivalent strain of equal intensity. This is done by the simply solving:

\[
    t = P_{31} \lambda_1 + P_{32} \lambda_2 + P_{33} \lambda_3 = P_{31} \lambda_{UT} + \left( P_{32} + P_{33} \right) \sqrt{\lambda_{UT}}
\]

With \( P_{ij} \) the rotation matrix components, \((\lambda_1, \lambda_2, \lambda_3)\) the calculated components of the strain tensor and \( \lambda_{UT} \) the equivalent uniaxial strain researched.

4.0 SOLID PROPELLANT GRAIN INTEGRITY ASSESSMENT

To give an overview of the potential applications of this criterion to solid propellant grains mechanical design, an integrity assessment evaluation is performed for a ‘finocyl’ propellant structure which is the grain of a tactical rocket motor. The geometry of such a structure is depicted on figure 5. One will assume the case of a relatively cold temperature operational condition (let say -40°C or more precisely \( \Delta \theta = -100°C \)) and a quite high internal pressure of 12 MPa. The calculation solves the two loading cases of thermal cooling and pressurisation in the same problem and takes into account for the structure stiffness. The propellant is assumed to be incompressible and is reproduced in the calculation by a simple neo-Hookean behaviour with a modulus \( C_1 \) (the shear modulus) fixed at 1 MPa. The value of this modulus has no important influence on the result in term of displacement field (as mentioned §1) since only the strain field is required for the mechanical integrity assessment. The thermal and pressure induced strains are extracted from the numerical simulation process in a series of points expected to be critical towards fracture. Generally these points are located in center of the internal bore and at the feet and head of the slot.

![Figure 5: Typical Geometry of a Tactical Missile Propellant Grain.](image)

The results of this calculation in terms of principal strains during this simplified service life are shown for example in the middle of the central bore on figure 6.
If attention is paid to these results, one may observe that the strain fields in these points are far away from uniaxial tension and it is thus difficult to compare the principal values of this tensor directly to an experimental observation. The presented strain criterion introduces the lacking tools to overpass this problem and to continue the mechanical design of the structure. Though the comparison to isothermal fracture data is therefore accessible since the uniaxial equivalent strain has been evaluated, it is preferred to take into account for the viscoelastic nature of the fracture behaviour at this stage. This specific feature of the mechanical behaviour of propellant leads to a continuously varying value of the fracture strain as the temperature is decreased. The figure 7 shows, as an example, the master curve of the fracture strain for a HTPB solid propellant as a function of the reduce inverse strain rate $\frac{1}{\varepsilon a_T}$.

![Master Curve of the Fracture Strain of an HTPB Propellant.](image)
An original way to treat this difficulty is to cumulate a service life variable defined as the ratio of each increment of deformation applied at the current temperature and strain rate to the corresponding fracture value for this temperature and strain rate [1]. The resulting definition of the service life variable is thus:

\[ D = \int_{0}^{\varepsilon} \frac{d\varepsilon}{\varepsilon_{f}(\theta, \dot{\varepsilon}, P, ...)} \]

To check the pertinence of this approach for the thermal loading, a series of uniaxial tension tests to which a continuous cooling is superposed are performed. The material response to this particular test is shown on figure 8 for different couples of strain rate and cooling rates. Also shown on this figure is the corresponding set of isothermal responses.

These experimental results demonstrate firstly the very particular feature of the material response to these loading conditions and secondly the sensible alteration of the ultimate properties in term of strain which obviously should be accounted for in the structure design towards fracture. The validation of the strain variable concept is the result of the confrontation of the cumulated variable with the ratio of the tensile rate to the cooling rate since these two properties are constant during the tests.

\[ \begin{align*}
\frac{d\varepsilon}{d\theta} &= \dot{\varepsilon} \cdot dt \\
\frac{d\theta}{d\varepsilon} &= \dot{\theta} \cdot dt
\end{align*} \]

\[ D = \int \frac{d\varepsilon}{\varepsilon_{f}(\theta)} = \frac{\dot{\varepsilon}}{\dot{\theta}} \cdot \int \frac{d\theta}{\varepsilon_{f}(\theta)} \quad \text{and} \quad D = 1 \leftrightarrow \int \frac{d\theta}{\varepsilon_{f}(\theta)} = \frac{\dot{\theta}}{\dot{\varepsilon}} \]

These relationships provide a fracture temperature prediction which is compared for constant rates \( \frac{\dot{\varepsilon}}{\dot{\theta}} \) ratios to the experimental results as shown on figure 9.
For the pressure loading case, the same approach is used but, this time, it should be taken into account for the pressure sensitive material strain capability. Still as an example, the fracture strain for the specific strain rate and temperature of this particular grain firing condition is shown on figure 10 as a function of pressure. The benefit effect of the pressure on material properties have been the object of an extended work in propellant industry [2,3] and no details will be given here, but it is pointed out that a convenient way to describe the pressure influence is to introduce a ‘pressure effect coefficient’. This coefficient is defined as the ratio of the strain (or stress) capability at pressure \( P \) to its corresponding value at ambient pressure (this value is strain rate and temperature dependent).

To complete the mechanical integrity assessment of the grain, the pressure effect coefficient has to be fitted with an exponential adjustment:
At the term of these efforts the propellant grain mechanical integrity is accessible since we have:

- the strain tensors calculated along the service life in the critical locations
- an equivalent strain criterion to reduce these strains to an uniaxial equivalent
- a material capability model taking into account for the service life loading conditions
- a cumulative service life variable to quantify the impact of each service life individual loading on the mechanical integrity of the material.

These different tools are gathered to produce the plot of figure 11, on which appears as a function of the loading path in the middle of the bore, the material capability and the equivalent strain in the sense of the proposed criterion. Is also reminded on this figure the conventional equivalent strains in common application for mechanical integrity assessment in SNPE official design method [4]. Namely and supported by a somehow different analysis, a Stassi equivalent strain is used for the thermal loading case and a Von Mises equivalent strain (Von Mises stress divided by the calculation modulus) for the pressure loading case. It should be emphasized that the inconvenience of this double definition of the equivalent strain leads to discontinuous safety coefficient and that, as a strong benefit, this problem is overpassed by the proposed method.

Nota: the octaedric stresses are the first and second invariant respectively of the spheric and deviatoric components of the stress tensor. The octaedric pressure is:

$$\sigma_{oct} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

and the octaedric shear (in term of global tensor terms)

$$\tau_{oct} = \frac{1}{3}\left[2\sigma_1 - (\sigma_2 + \sigma_3)\right]^{1/2}$$

the corresponding strains are the value of these stresses divided by the modulus.

Nota: the drop in the Stassi equivalent strain is due to the presence of the pressure in the equivalent strain definition (see table 1).
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It should be noted on this figure that the strain criterion is very close to Stassi criterion in the thermal loading phase and exhibits a comparable slope to the Von Mises criterion in the pressurisation phase. A convenient manner to quantify the differences introduced in the safety coefficient calculation by this new method is to compare the safety coefficients that would have been calculated by the ‘classical’ Stassi-Von Mises approach, to those issued from an identical definition of the safety coefficient but including the strain criterion and finally to the complete service life variable approach. To allow a clear lecture of the obtained figures in table 1 which compares all the methods, the definitions involved in their calculation are reminded in the following synopsis:

Table 1: Synopsis of the Different Safety Coefficient Calculations

<table>
<thead>
<tr>
<th>Loading case</th>
<th>Thermal</th>
<th>pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stassi éq. strain</td>
<td>Von Mises éq. stress</td>
</tr>
<tr>
<td>load measure</td>
<td>STASSI Strain</td>
<td>Von MISES Stress</td>
</tr>
<tr>
<td>ε_{STASSI}</td>
<td>( \frac{\sigma_{\text{oct}} + \sqrt{\sigma_{\text{oct}}^2 + 12\tau_{\text{oct}}^2}}{2E} )</td>
<td>( \sigma_{\text{MISES}} = \frac{9\tau_{\text{oct}}^2}{2C_\sigma} )</td>
</tr>
<tr>
<td>remarks</td>
<td>thermal stress free</td>
<td></td>
</tr>
<tr>
<td>Capability measure</td>
<td>fracture strain at final θ and overall cooling time: ( \varepsilon_r \left( \frac{t}{a_0} \right) )</td>
<td>fracture strain at firing θ , P and pressurisation time: ( \sigma_r \left( \frac{t}{a_0} , P \right) )</td>
</tr>
<tr>
<td>Safety coefficient definition</td>
<td>( \frac{\varepsilon_r \left( \frac{t}{a_0} \right)}{\varepsilon_{\text{STASSI}}} )</td>
<td>( \frac{\sigma_r \left( \frac{t}{a_0} \right)}{\sigma_{\text{MISES}}} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Strain Criterion Method</th>
<th>Complete service life variable method</th>
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<tr>
<td>Loading case</td>
<td>Thermal</td>
</tr>
<tr>
<td></td>
<td>equivalent strain</td>
</tr>
<tr>
<td>load measure</td>
<td>( \varepsilon_{\text{TH}} )</td>
</tr>
<tr>
<td>Capability measure</td>
<td>fracture strain at final θ and overall cooling time: ( \varepsilon_r \left( \frac{t}{a_0} \right) )</td>
</tr>
<tr>
<td>Safety coefficient definition</td>
<td>( \frac{\varepsilon_r \left( \frac{t}{a_0} \right)}{\varepsilon_{\text{TH}}} )</td>
</tr>
<tr>
<td>thermal strain free</td>
<td>both load cases</td>
</tr>
<tr>
<td>fracture strain evolution as a function of reduced strain rate and pressure:</td>
<td>( \varepsilon_r \left( \frac{t}{a_0} , P \right) )</td>
</tr>
<tr>
<td>thermal strain free</td>
<td>( D = \int_0^\varepsilon \frac{d\varepsilon}{\varepsilon_r (\theta, \varepsilon, P,...)} )</td>
</tr>
</tbody>
</table>
With these different design methods, the following figures for the safety coefficients are gathered in Table 2 together with the values of the loadings and the corresponding capability.

<table>
<thead>
<tr>
<th>Table 2: Safety Coefficient Figures from Different Methods</th>
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<tr>
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<td></td>
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<tr>
<td>Standard</td>
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<tr>
<td>method</td>
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<tr>
<td>standard Sc</td>
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<tr>
<td>Strain</td>
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<tr>
<td>method</td>
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<tr>
<td>standard Sc</td>
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<tr>
<td>Cumulated</td>
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<tr>
<td>service life</td>
</tr>
<tr>
<td>variable</td>
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As could be expected from the different definitions, and from the results of Figure 11, the safety coefficients figures are very close whatever the method used as long as the thermal loading is concerned. The values for the pressurization loading calls for more comments: the first of which being that in the standard method the safety coefficient figure is higher for this loading case than for the thermal one. This is obviously due to the coefficient definition discontinuity between this two loading cases and may lead to incoherence in the interpretation of these figures since one would expect the safety coefficient to be a decreasing function during the service life. When the safety coefficient is evaluated in terms of strains only this problem vanishes but the resulting figures are rather low for the cold temperature pressure loading specially in the feet of the slot (Sc=1.05). These figures are due to once again to the choice in the security coefficient method since the strain capability for this case is the von Mises stress (7.88MPa including the pressure effect) divided by the modulus which for this high strain rate and cold temperature is 150MPa. The last range of safety coefficient in the table is the result of the proposed method which gives continuously decreasing values as the loading case is applied which magnitude are judged as satisfactory since this particular rocket motor never shown fracture for cold temperature firing conditions. It is of course of principal concern to check now on analog motors design towards fracture the predictive potential of the proposed approach.

5.0 CONCLUSIONS

An original method is proposed to assess the mechanical integrity of propellant grain in tactical solid rocket motors applications with a special attention paid to the cold firing loading conditions. The method is entirely built up on a homogeneous analysis of the bore strains which is supported by the introduction of a multiaxial strain failure criteria. This criteria allows the reduction of the multidimensional strain tensor which results of the numerical simulations into an uniaxial equivalent strain comparable to experimental data. Instead of a discrete evaluation at the end of each loading case (thermal and pressure), the safety coefficient definition is a cumulated comparison of the actual strain applied to the instantaneous value of the strain capability of the material. As a result of this definition, the service life integrity assessment is measured by a continuously decreasing value which is expected to give a more convenient interpretation.
It should be mentioned that, though attractive, the method lack of a throughout validation on analog motors designed towards fracture in cold firing conditions.

6.0 ACKNOWLEDGEMENTS

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7.0 REFERENCES


