Vehicle Tracking using a Network of Small Acoustic Arrays

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ABSTRACT

Major advances in base technologies of computer processors and low cost communications have paved the way for a resurgence of interest in unattended ground sensors. Networks of sensors offer the potential of low cost persistent surveillance capability in any area that the sensor network can be placed. Key to this is the choice of sensor on each node. If the system is to be randomly deployed then non line of sight sensor become a necessity. Acoustic sensors potentially offer the greatest level of capability and will be considered here. As a passive sensor, only time of arrival or bearing information can be obtained from an acoustic array, thus the tracking of targets must be done in this domain. This paper explores the critical step between array processing and implementation of the tracking algorithm. Specifically, unlike previous implementations of such a system, the bearings from each frequency interval of interest are not averaged but are used as data points within a Kalman filter. Thus data is not averaged and then filtered but all data is put into the tracking filter.

1.0 INTRODUCTION

In recent years advances in computer processing power and wireless communications has led to a revival of acoustic sensors as an inexpensive yet powerful surveillance system. As a result, a network of acoustic sensors is capable of providing a novel and valuable ISTAR capability over an area of interest. This sensor network offers persistent, covert coverage and is capable of providing a range of intelligence, surveillance and/or target acquisition functions. However, critical technology areas such as ad-hoc communications networks, deployment mechanisms and robust acoustic tracking still need development to validate this concept.

A critical capability of the acoustic sensor network is the tracking of vehicles. Which is necessary, firstly to establish an unambiguous lock on a vehicle over time and secondly to determine future vehicle positioning to enable successful engagement. Vehicle tracking is typically achieved using array signal processing techniques to generate a sequence of bearings from a single network node (consisting of a small array of microphones) to the vehicle. When bearings are brought together from multiple nodes the vehicle position can be determined. There are several possible ways of producing vehicle coordinate data from acoustic signatures, this paper describes the principals of one such method.

2.0 CONCEPT

The concept of vehicle tracking relies on several sensor nodes detecting an acoustic source simultaneously. Where the node comprises of a small array of microphones the bearing to the source can be determined. Multiple bearings produced by the distributed nodes are then communicated to a central command post where the source location coordinate is triangulated.

Paper presented at the RTO SET Symposium on "Capabilities of Acoustics in Air-Ground and Maritime Reconnaissance, Target Classification and Identification", held in Lerici, Italy, 26-28 April 2004, and published in RTO-MP-SET-079.

The method for calculating the bearing on the node, in this paper, is based on the Multiple SIgnal Classification (MUSIC) algorithm. The multispectral nature of acoustic signals emanating form a vehicle requires the MUSIC algorithm to run on each frequency of interest. The result is a set of MUSIC power-spectra for each frequency. Unlike previous methods that tend to integrate the power-spectra to form a single beam pattern (such as the Incoherent MUSIC (IMUSIC)) this study generates a bearing from each frequency and applies Kalman filters to select the vehicle bearings.

This paper outlines the algorithm and shows how it performs when implemented. The results presented in this paper are taken from results of the algorithms running in realtime and in a typical military battlefield environment.

3.0 THE MUSIC ALGORITHM

3.1 The Data Model [1]

The MUSIC algorithm is a method of calculating the bearings from distributed sources whose emissions are detected by an array of sensors. The model for array recordings is given as:

$$\underline{y}(t) = A(\underline{\theta})\underline{s}(t) + \underline{n}(t)$$

Where $\underline{y}(t)$ is the vector of recordings on each channel, $\underline{s}(t)$ is the vector of source signals, $\underline{n}(t)$ is a vector of white Gaussian noise, and $A(\underline{\theta})$ is the array response matrix where $\underline{\theta}$ is a vector of the direction of arrival (DOA) of the wavefronts from each source. The array response matrix characterizes the way in which the signals from the distributed sources are combined to form the recorded signals on each channel. The n^{th} column of $A(\underline{\theta})$ is given by:

$$a(\theta_n) = \left[e^{i2\pi f\tau_{1n}} \quad e^{i2\pi f\tau_{2n}} \quad \cdots \quad e^{i2\pi f\tau_{mn}} \right]^T \tag{1}$$

Where $\tau_{mn}(\theta)$ is the time difference of arrival (TDOA) of the wavefront from the n^{th} source between sensor 1 and sensor m and f is the frequency of the wavefront. The TDOA can be expressed in terms of the sensor positions and the source bearings to give:

$$\tau_{mn} = \frac{(x_1 - x_m)\cos(\theta_n) + (y_1 - y_m)\sin(\theta_n)}{c} \tag{2}$$

Where (x_m, y_m) is the position of the m^{th} sensor and c is the speed of sound. The geometry of the wavefront propagation over the array is illustrated in Figure 1.

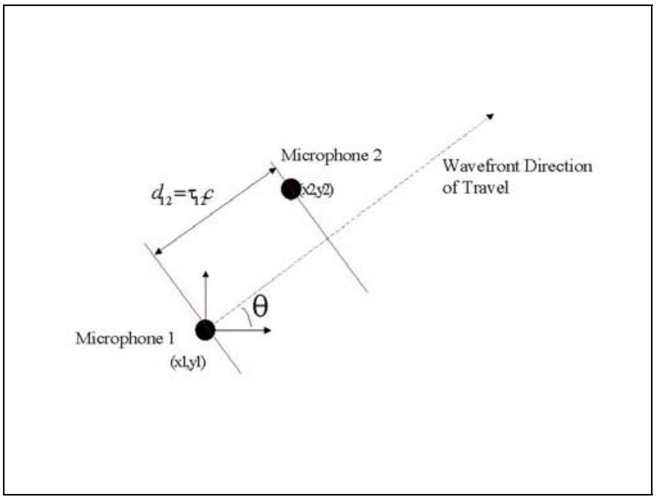

Figure 1 Geometry of wavefront passing over two sensors.

3.2 Source Signal and Noise Subspaces

Using this signal model the bearings can be calculated by applying Eigen decomposition to the signal covariance matrix. This process allows the signal space to be split into the source-signal subspace and the noise subspace. As the subspaces are orthogonal any vector that belongs in the signal subspace projects onto noise subspace as a null-vector (i.e. a vector that has and absolute value of zero). It is this characteristic that is exploited to calculate the source DOA.

The signal covariance matrix is the covariance of the data matrix $\underline{y}(t)$, and is given by:

$$R = E\left[\underline{y}(t)\underline{y}^{H}(t)\right] \tag{3}$$

Eigen decomposition of *R* produces a set of eigen-vectors spanning the signal subspace, S, of dimension $N \times M$, and noise subspace, U, of dimension $(M-N)\times M$. The eigen-vectors are characterized as noise or signal vectors by their eigen-values. In particular the signal eigen-values are typically larger than the noise eigen-values and the noise eigen-values are equal to the variance of the noise in equation (1). So in the case where the eigen-values are $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N \geq \sigma^2{}_{(N+1)} \cdots \geq \sigma^2{}_M$, the first *N* values are the signal eigen-values and the remaining *M-N* are the noise eigen-values.

3.3 Calculating the DOA

It is known that the columns of the array response matrix, $A(\underline{\theta})$, are spanned by the signal subspace. Therefore they will project into the null space of the noise subspace. The only parameter in the array response matrix that is unknown is the source bearings. For this reason a generic array response vector, known as the steering vector, can be constructed in exactly the same way as $a(\theta_n)$ in equation (2) except now using a generic θ. In this way the steering vector can be projected onto the noise subspace for a range of test values θ. When the projection is a null vector it indicates that the guessed θ matches a signal bearing. This process is represented by forming the MUSIC pseudo power spectrum:

$$P_{MUSIC}(\theta) = \frac{1}{|a(\theta) \cdot U|} \tag{4}$$

Where this time

$$a(\theta) = \left[e^{i2\pi f \tau_1(\theta)} \quad e^{i2\pi f \tau_2(\theta)} \quad \cdots \quad e^{i2\pi f \tau_m(\theta)} \right]^T \tag{5}$$

The inversion is applied so that nulls become peaks in $P_{MUSIC}(\theta)$.

3.4 From Theory to Real Data

When applying the MUSIC algorithm to real data the data is in the form of multi-channel recordings over the *M* element array. Typically it will be discretely sampled over a fixed period of time, and it is this data that the *M by M* signal covariance matrix is calculated from. As the recorded data is sampled over a finite period of time, the resulting covariance matrix is only an approximation to the covariance matrix in the signal model. This covariance matrix is called the sample covariance matrix.

Another feature of the acoustic recorded data that does not fully match the MUSIC algorithm's signal model is the narrow-band assumption. In the expression for the TDOA in equation (3) the frequency term implies that the MUSIC algorithm applies only to single frequency signals. However the acoustic signatures from vehicles such as tanks or automobiles are rarely single frequency. For this reason some additional per-processing is required before application of the MUSIC algorithm. The nature of this additional processing is discussed in the next section.

4.0 WIDEBAND MUSIC

4.1 From Wideband to Narrow Band

In order to use the MUSIC algorithm the recorded data has to be arranged in a way that matches the narrow band restrictions of the algorithm. A simple method for doing this is to split the wideband acoustic signal into a set of narrowband components. This is easily achieved by using the fast fourier transform (FFT). By doing this the signal frequencies of interest can be isolated and the MUSIC algorithm applied.

It would be desirable to apply MUSIC to every bin of the FFT to produce a set of power spectra for each frequency. However, in practice, the computational demands for such coverage would be too high. Therefore a subset of bins containing the most likely frequencies of interest need to be carefully selected. This can be done in a number of ways, from simple thresholding to harmonic line analysis, and is typically applied within a pre-selected bandwidth. This limits the large number of frequencies to be processed to a manageable number. One example of a wideband MUSIC algorithm is Incoherent MUSIC (IMUSIC) presented by Pham, Manfai and Sadler in reference [2]. Much of their approach forms the foundation for our technique.

4.2 IMUSIC

The IMUSIC algorithm provides a technique for splitting a wideband signal into narrowband components suitable for use with the original MUSIC algorithm. Once the MUSIC power spectrum is calculated for each selected frequency bin, the power spectra are summed to produce a power single spectrum where the peaks indicate the bearings of sources. The algorithm is set out in Figure 2.

In reference [2] the frequency band is set to run between 20Hz and 200Hz and a range of values for the number of frequency has been investigated. A value regularly settled on is 20. The selection method used was a simple adaptive threshold technique. Using this system published results have a typical accuracy of approximately 1.5 degrees for a tracked vehicle.

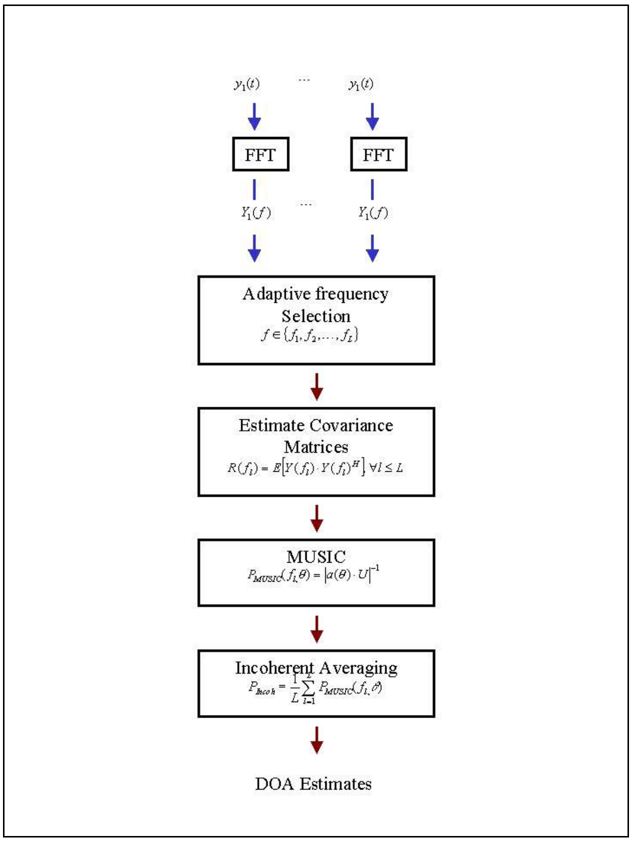

Figure 2 Incoherent MUSIC algorithm.

4.3 Observations on IMUSIC

When applying IMUSIC there were some circumstances under which bearing information was potentially lost. In particular it is apparent that the methods for frequency selection and the integration of the MUSIC power spectra occasionally caused degradation of the bearing accuracy.

Frequency Selection—The method for selection presented in [2] firstly restricts the frequencies to a very small range. There are many frequencies beyond the 200Hz limit suggested that emanate from the vehicles. Although this paper cites the fact that high frequencies attenuate beyond audible range very quickly, it does not account for the fact that on many occasions the vehicles maneuovre close to the sensors, especially with the advent of unattended ground sensors (UGS). At such ranges the high frequencies are audible and can be utilized. In addition when an acoustic source is tracked away from the sensors the bearing track tends to a single stationary angle. That is, the bearing varies most as a vehicle drives past the sensor and varies very little the further it moves from the sensor. Therefore at long ranges, where low frequencies are the only audible components, the bearing track will yield very little additional information about the vehicle location.

In addition, using low frequencies to extend the range of the bearing track could have the detrimental effect of increasing the level of clutter. As low frequencies do not attenuate as rapidly as high frequencies, low frequency noise sources, other than the vehicle's, will also be present in the signal recordings. In the low frequency band there is more clutter and therefore the signals being used in the MUSIC algorithm will potentially have lower signal to noise ratios (SNR). In addition wind noise is not limited to bandwidths below the 20Hz lower limit set in this example of the IMUSIC algorithm.

Integration of MUSIC Spectra—Due to some of problems with the frequency selection process it is possible for some of the peak frequencies selected to operate MUSIC on are not from a vehicle of interest. In many cases they are wind noise or from a powerful unwanted clutter source. Under these circumstances the MUSIC power spectra are poorly defined and of a high amplitude. The effect this has on the integration process is to swamp the power spectra generated by the frequencies with target signal components and thus the integrated power spectrum peaks no longer represent the bearings of the source vehicles.

4.4 Alterations to IMUSIC

To overcome some the problems outlined there are several simple alterations to IMUSIC that can be made.

Alternative Frequency Selection— In order to prevent all the peak frequencies being selected from the low frequency regions (as these are typically highest) the algorithm can be forced into selecting a frequency from a number of bandwidths ranging from the low frequencies up to the higher ranges. This is achieved by predetermining a range of frequencies and also predetermining the number of frequencies selected for the MUSIC process. Using this technique a number of evenly spaced sub-bands are defined and the peak frequency is taken from each. An example where the frequency band runs from 20Hz to 1kHz with ten sub-bands is illustrated in Figure 3.

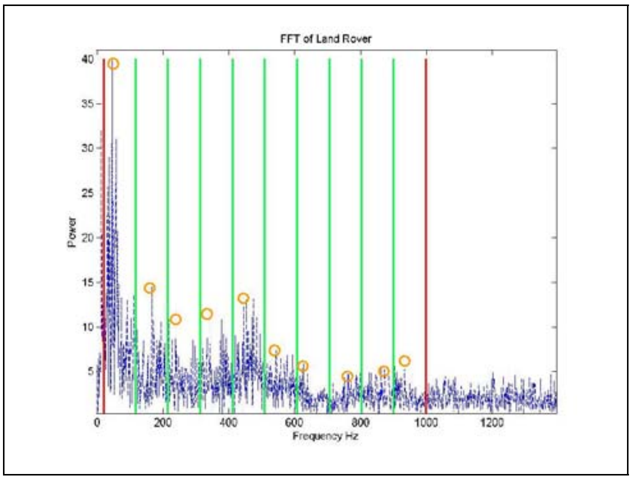

Figure 3 FFT of landrover showing the banwidth divisions used in the alteration to IMUSIC.

In the figure the peak frequencies are circled. Using the technique of selecting one peak frequency per sub-band an even selection of bearings across the full bandwidth is guaranteed.

Alternative to Integration of MUSIC Spectra—The problem with integrating the spectra is that high powered unwanted frequencies may dominate. The FFT of the Land Rover illustrated in Figure 3 shows a typical acoustic signature where the peak frequency in the first bin is significantly higher than the peak frequency in the later bins. This is not a significant problem if the low frequency peak is a component of the vehicle being tracked. However it is a problem if it is unwanted noise. In this case the low frequency

MUSIC power spectra may be poorly defined and degrade good quality MUSIC spectra from the higher frequencies. An alternative approach is to calculate the bearing for each frequency bin individually. In this way several bearings are generated at each instant. A typical plot of a tracked vehicle driving by is shown Figure 4.

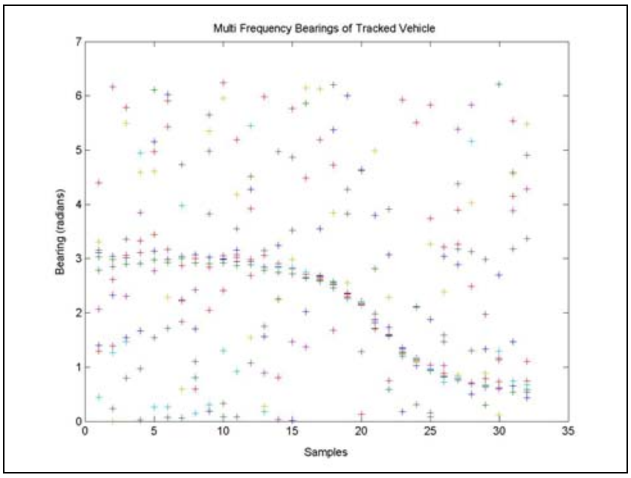

Figure 4 Bearing plots from 10 frequencies of a tracked vehicle driving past an array of acoustic sensors, here each sample = 0.5 seconds.

In this case the problem of selecting meaningful bearing information from the many instantaneous bearings has to be addressed. To reduce the number of bearings clustering algorithms are applied. After clustering Kalman Filtering techniques are used to establish the consistency in the tracks. These processes are described below.

5.0 CLUSTERING

The clustering techniques are designed to disregard the spurious bearing plots generated in some of the frequency bins. This is done using nearest neighbor techniques. If three or more bearings point in the same direction, they are retained. Any bearings from frequencies that are isolated are disregarded. The result of applying the clustering algorithms to the raw bearings displayed in Figure 4 is shown in Figure 5.

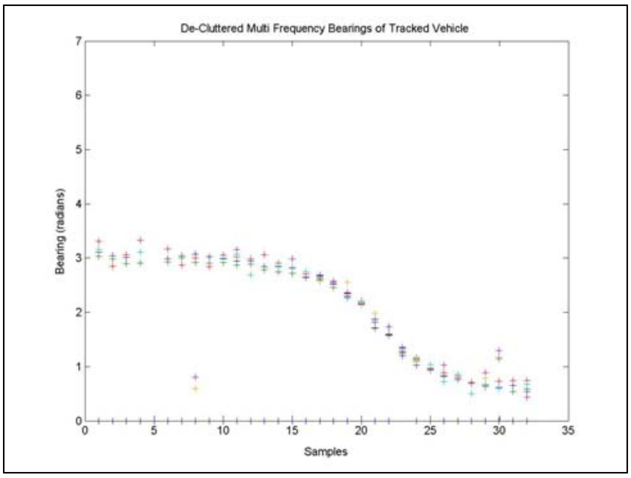

Figure 5 De-cluttered bearing plots of a tracked vehicle driving past an array of acoustic sensors, here each sample = 0.5 seconds.

6.0 KALMAN FILTERING

In Kalman filtering terms the bearing data calculated by the acoustic node is known a plot or measurement and the path traveled by the acoustic source (or target) is known as the track. There are three main groups of tracking algorithm. There is the single plot single track, the multiple plot single track (this is the case where there is more than one plot at each moment in time), and the multiple plot multiple track case. The later case is the one that represents the data output by our multi-frequency clustered MUSIC algorithm. Here the clustering algorithm outputs at multiple plots for each time instance where each plot represents either a target or clutter. The Kalman filtering process required for this case is the Probabilistic Data Association Filter (PDAF). This algorithm was designed for use in the multiple plot single track case (the second case listed in the above list). It can be applied to the multiple track case by running it separately assuming only one track exists at a time. It is then repeated until all the tracks have been processed.

The Kalman filters operate by comparing the measured data, such as position or bearing, to a predicted position or bearing. If the measurement is similar to the prediction for a particular track then that plot is associated with that track, if it is not similar it will not be associated to the current track. The PDAF algorithm is described in the Bar-Shalom and Fortmann book [3] the details of which can be found in the appendix.

6.1 Clutter Rejection

An important capability of the multiple plot multiple track algorithms is clutter rejection. That is the ability to disregard plots that are not associated to any tracks. This is achieved using a gating process. At any moment in time the tracker will compare the current predicted track position, with the current measured track position. The difference between the two values is known as the innovation. The Mahalonobis distance is the metric used to determine the size of the innovation, and is given by:

$$d^2(\underline{z}_{kj}) = (\underline{z}_{kj} - H\underline{\bar{x}}_k)^T S_k^{-1}(\underline{z}_{kj} - H\underline{\bar{x}}_k) > T \tag{6}$$

where $\underline{\bar{x}}_k$ is the predicted state, $\underline{z}_{kj}$ is measured state (i.e. the plot), H is a matrix the defines which characteristics of the measurements and predictions are to be compared (in this study only the bearing is compared, in other applications it may include velocity and acceleration) and S_k is the covariance of the innovation and calculated at each time step as part of the Kalman filtering process. (Incidentally $(\underline{z}_{kj} - H\underline{\bar{x}}_k)$ is the innovation). T is the threshold that determines whether the measurement is clutter to be ignored or a possible part of the track.

7.0 TRIANGULATION

Triangulation can be used to convert the bearing information from multiple nodes into coordinate information. The formulae combining bearing information from two sensor nodes to give xy-coordinates are given by:

$$x = \frac{y_2 - x_2\tan(\theta_2) - y_1 + x_1\tan(\theta_1)}{\tan(\theta_1) - \tan(\theta_2)} \tag{7}$$

$$y = \frac{[y_2 - x_2\tan(\theta_2)]\tan(\theta_1) - [y_1 - x_1\tan(\theta_1)]\tan(\theta_2)}{\tan(\theta_1) - \tan(\theta_2)} \tag{8}$$

Where x_i, y_i are the coordinates of node i and θ_i is the bearing from the i^{th} sensor node to the target. As in the case of the bearing calculation Kalman filtering can be applied to enhance the accuracy of the sequential coordinate plots. Figure 6 illustrates the triangulation process.

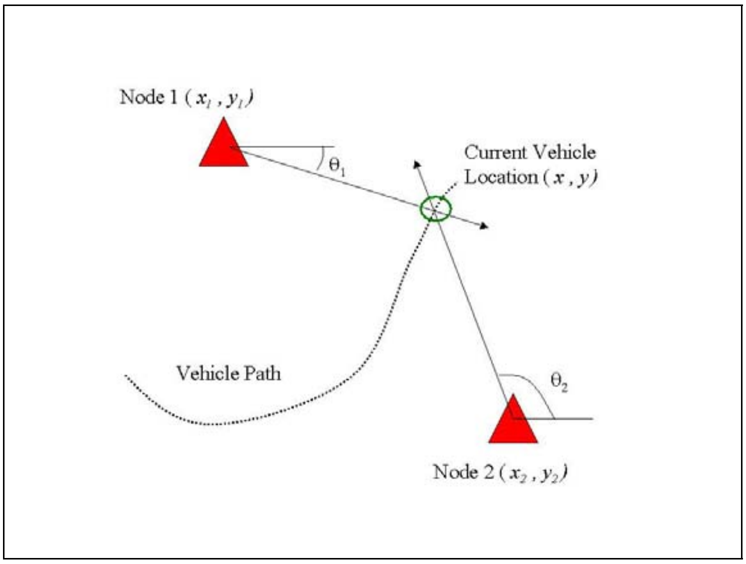

Figure 6 Triangulation using node location and bearing data

8.0 ALGORITHM SUMMARY

The sequence of the algorithm is outlined in Figure 7. Here the updated IMUSIC algorithm, along with the tracking and clustering elements, are shown as the processing applied at each sensor node. In addition to this the triangulation and tracking processing, that generates the XY-track at the central node, is also shown.

9.0 EXPERIMENTAL RESULTS

The primary drive of the work outlined in this paper is to produce a sensor node capable of generating a sequence of bearings. The next step is to translate those bearings into vehicle tracks. The emphasis of the group within QinetiQ has been on producing a real time system capable of achieving this goal. To that end the system described above has been developed and constructed. All the processing upto the point before clustering is done in real time, from the clustering onwards the processing is done off-line. It is the intention to have a complete real-time system developed by the end of the next phase of the work.

Ultimately the goal is to produce a network of low cost rugged sensor nodes that form an unattended ground sensor (UGS) network. In order to maintain realism in the results that reflect the quality of bearing likely to be attained in the final system, low cost components, such as the microphones, were used. Whilst in the development phases of the work laptop computers are used for the processing. In the final solution dedicated processors will replace the laptops.

9.1 System Setup

The system comprises a five channel microphone array. Four of the microphones are located on the vertices of a 600mm square, with the fifth in the middle. The microphones are Knowles Acoustics WP3502 models. The microphones are connected to a computer with and Intel Celeron processor via a

Daq-System analogue to digital converter card. The data is sampled at a rate of 8192Hz and processed to produce a bearing from 10 frequency bins every half-second. The frequency bins are distributed between 100Hz and 1500Hz. The sensor configuration is shown in Figure 8.

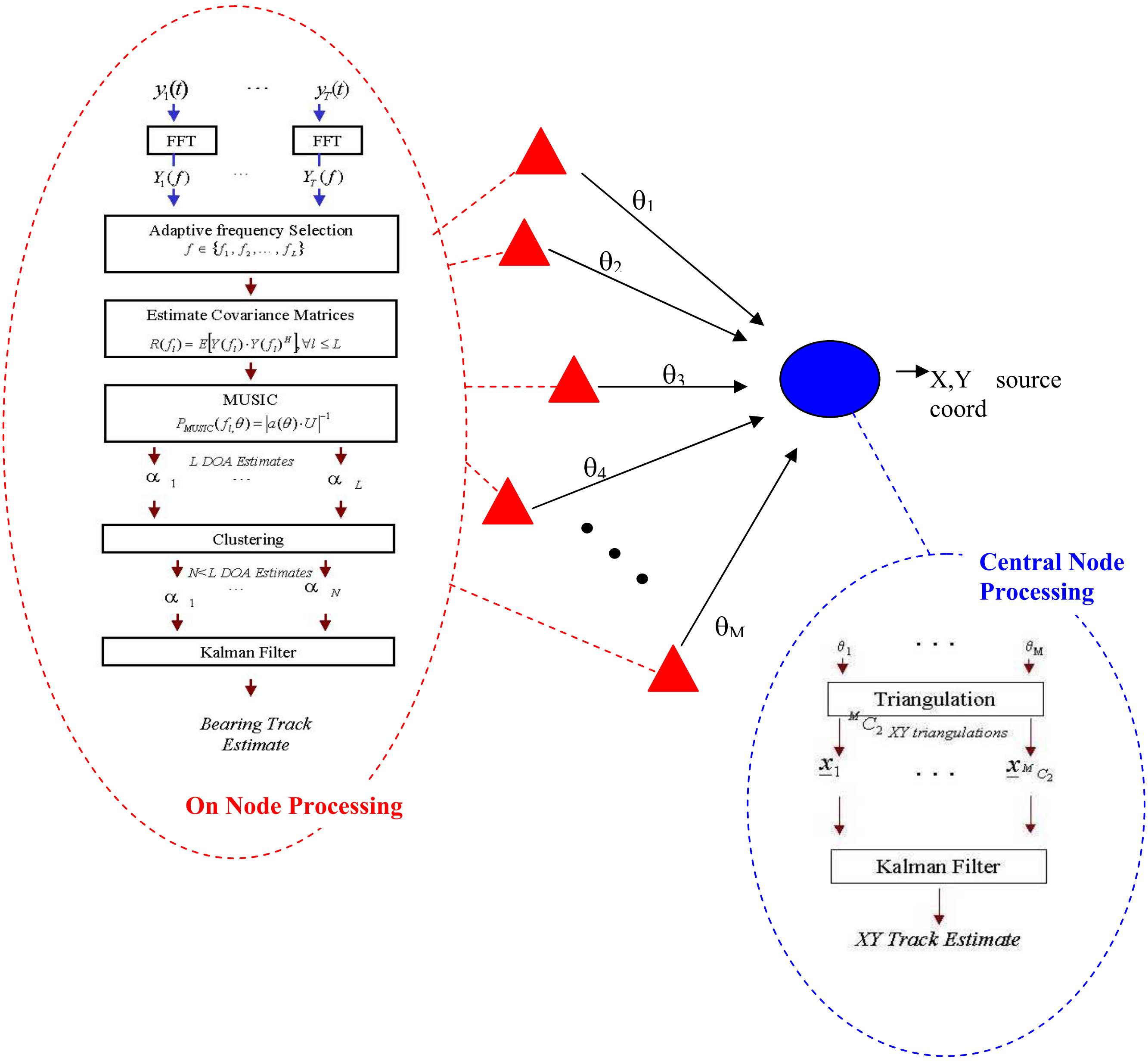

Figure 7 The on-node and the central-node processing for acoustic tracking

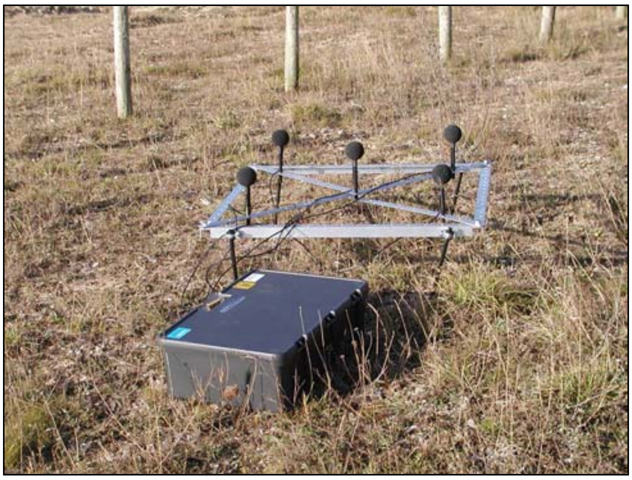

Figure 8 Sensor configuration

9.2 Preliminary Results

The results obtained from a recent trial indicate that good quality bearing information is generated from the sensor node. However the bearing data generated contain a huge number of clutter points that detract from the main bearing plots. The result of applying the clustering algorithms and the Kalman filtering process to the bearing data displayed in Figure 4 and Figure 5 is shown in Figure 9.

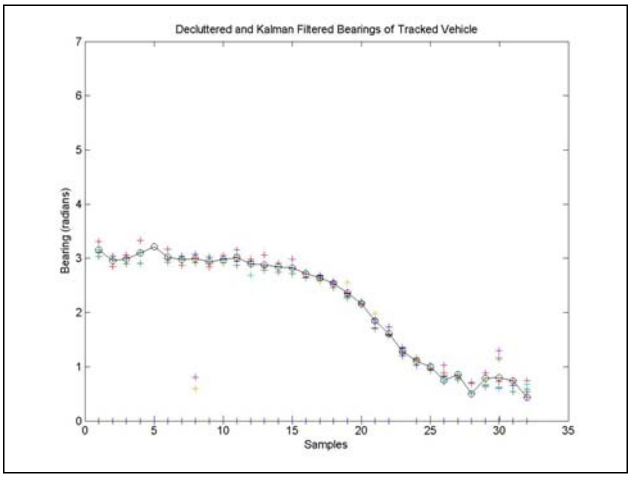

Figure 9 Kalman filtered bearing plots of a tracked vehicle driving past an array of acoustic sensors, here each sample = 0.5 seconds.

Triangulation is then applied to bearing data from at least two nodes to produce the XY coordinates. Figure 10 shows some preliminary results of sequential coordinate plots from real data enhanced using tracking techniques. Here the tracking algorithm removes some of the uncertainty in the raw data plots and bridges gaps where the sequential triangulated target position has been lost.

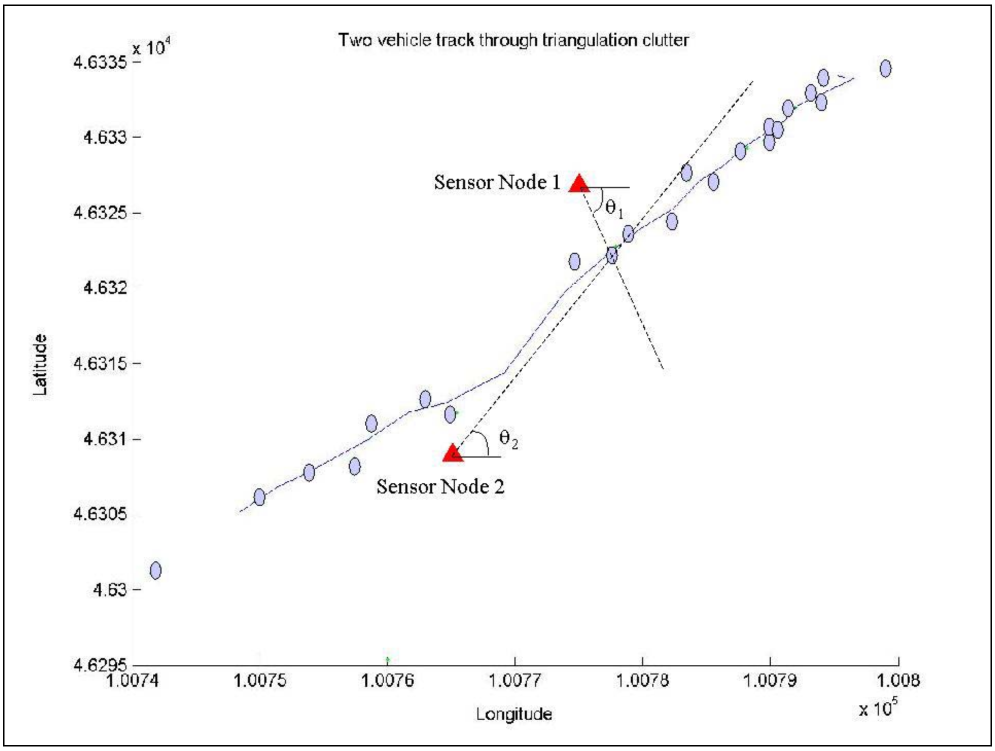

Figure 10 Triangulation of bearing data to form xy coordinate plots (blue circles) enhanced by tracking algorithm to improve accuracy and robustness (solid line). The effect of triangulating, for a single instance in time, is illustrated by the crossing of the nodes' calculated bearings (dashed lines).

10.0 CONCLUSIONS

This work has outlined some potential problems with one version of the IMUSIC algorithm, and presented techniques that can be employed to overcome them. By allowing each frequency bin to return a bearing, no one frequency dominates the integration. A further benefit of using the higher frequencies is that the resolution of the MUSIC power spectra is improved and enables the acoustic array size to be reduced without loss of accuracy. However the added number of data points requires extra processing in the form of the clustering algorithm. Once the clustering process has been applied the data emerges in a state similar to that of the IMUSIC output. As recommended by the authors of the IMUSIC algorithm tracking algorithms are then be used to produce single-track outputs for each vehicle present. The results from recent trials indicate that the bearings generated by the nodes are indicative of the events being monitored. However much testing and evaluation of the system for range capability and bearing accuracy needs to be done.

The focus of the sensors group within QinetiQ has been to produce a real-time working system. In the current state the wideband DOA calculation is real-time. The future focus is to include the clustering and tracking algorithms so each node can autonomously process bearing-track information. Following this we will communicate bearing-track data from a network of nodes to a central command post where it will be combined to produce grid coordintes of vehicles passing through the sensor network.

ACKNOWLEDGMENTS

This paper was produced under funding from UK MoD ARP 8 in up to March 2003 and internal QinetiQ funding.

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APPENDIX

A1. The Probabilistic Data Association Filte (PDAF)

The algorithm used here is as presented in [4] and is presented in two parts. The first outlines the prediction and the second outlines the update phase of the algorithm. It is the values calculated in the prediction part of the algorithm that are used to generate the updated elements. It is these update values that represent the current position and speed of the vehicle being tracked. In the following expressions k indicates the time index. The time between each event (i.e. between sequential k) is *dt* (set to 0.5 seconds in the examples presented in the main text).

Prediction Stage (k>0)—The predicted state vector is given by:

$$\underline{\bar{x}}_k = \Phi_{k-1}\underline{\hat{x}}_{k-1}$$

and the predicted covariance matrix by:

$$M_k = \Phi_{k-1}P_{k-1}\Phi_{k-1}^T + \Gamma_{k-1}Q_{k-1}\Gamma_{k-1}^T$$

where Φ_{k-1} is the transition matrix and relates to the position and velocity components of the model:

$$\Phi_{k-1} = \begin{bmatrix} 1 & dt \\ 0 & 1 \end{bmatrix}$$

and Γ_{k-1} relates to the acceleration components of the motion model such that:

$$\Gamma_{k-1} = \begin{bmatrix} \frac{dt^2}{2} & dt \end{bmatrix}$$

and Q is the process noise covariance. In the example presented in the main body of the paper it is set to 2 for all time intervals.

Update Stage (k>0)— The innovation represents the distance between the measured data, $\underline{z}_{ki}$, and the predicted data, $\underline{\bar{x}}_k$. In this case there is more than one measurement at each time interval. Therefore the innovation must be calculated for each plot at each time. Hence innovation of the k^{th} time interval for the i^{th} measurement is:

$$\underline{v}_{ki} = \underline{z}_{ki} - H\underline{\bar{x}}_k$$

For the PDAF the innovations are combined to form a single innovation thus:

$$\underline{v}'_k = \sum_{i=1}^{N_k} \beta_{ki}\underline{v}_{ki}$$

H is called the measurement matrix and defines the elements of the state vectors used to calculate the innovation. In the examples in the main body of the paper $H=[1\ 0]$ because our measurement is positional (i.e. a bearing) and does not include a velocity measurement. The β weights needed to calculate the combined innovation are given by:

$$\beta_{ki} = \begin{cases} e_{ki} \Big/ \left[b + \sum_{j=1}^{N_k} e_{kj} \right] & for\ \ i \neq 0 \\ b \Big/ \left[b + \sum_{j=1}^{N_k} e_{kj} \right] & for\ \ i = 0 \end{cases}$$

where

$$e_{ki} = \exp\left(-\frac{1}{2}\underline{v}_{ki}^T S_k^{-1} \underline{v}_{ki} \right) \quad for\ \ i \neq 0$$

and

$$b = \rho\,(1-P_D)\sqrt{\left|2\pi\ S_k\right|} \Big/ P_D$$

ρ is the clutter density and P_D is the probability of a track being detected, in our example set to 0.2 and 0.9 respectively.

The Kalman gain matrix is:

$$K_k = M_k H_k^T S_k^{-1} \quad where\ \ S_k = H_k M_k H_k^T + R_k$$

S is called the innovation covariance and R is set to 0.1.

All the expressions above are combined to give the state estimate update:

$$\boxed{\underline{\hat{x}}_k = \underline{\bar{x}}_k + K_k \underline{v}'_k}$$

and the error covariance update is:

$$\boxed{P_k = \beta_{k0} M_k + (1-\beta_{k0}) P^* + K_k \left[\left(\sum_{i=1}^{N_k} \beta_{ki} \underline{v}_{ki} \underline{v}_{ki}^T \right) - \underline{v}_{ki} \underline{v}_{ki}^T \right] K_k^T}$$

where:

$$P^* = [1 - K_k H_k] M_k$$

The expressions in the boxes are the final solutions for each time instant. Therefore the tracked position and velocity is given by $\hat{\underline{x}}_k$.