Coherent simulation of sea-clutter for a scanning radar

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Airborne maritime surveillance

- Detection of small targets on the sea surface
  - Currently operate airborne platforms at low altitude.
  - Low grazing angles have reduced clutter backscatter.
  - Non-coherent processing.
  - Single aperture systems.
- Desire to operate with platforms at higher altitudes.
  - UAVs flying at > 10,000 ft.
- Future radars will likely be multi-aperture and use coherent processing.
- Most clutter simulation techniques only consider a single look direction.
- Need scanning simulation method to help with design and testing of new single and multiple aperture detection schemes.
Background

- Significant work over past years to improve sea-clutter modelling techniques.
- Time and spatially varying Doppler spectra model developed by Watts for low grazing angle clutter (2010).
- Found to be consistent with the Ingara medium grazing angle dataset and extended to a bi-modal model for better representing sea-spikes (2014).
- Adapted to work with the Pareto and K+Rayleigh distributions (2014).
- Model was also extended to include multiple phase centres with a single look direction (2014).
- Parameter models developed to enable generic simulation of original and bi-modal Doppler spectrum models (2015).
- Spectrum model is now used to model the sea-clutter observed from a multi-phase centre scanning radar (2016).
Scanning radar scenario

- Scanning radar platform travelling with velocity, $v_p$.
- Radar look angle is $\theta_{\text{plat}}$, angle relative to wind direction is $\psi$.
- Assume medium grazing angle scenario to utilise existing Doppler parameter models.
Evolving Doppler spectrum model

• Spectrum model is a single Gaussian with normalised underlying mean (texture), \( \tau \):

\[
G_0(f, \tau, s, \Omega) = \frac{\tau}{\sqrt{2\pi} s(\Omega)} \exp \left[ -\frac{(f - m_f(\tau, \Omega))^2}{2s(\Omega)^2} \right]
\]

• Mean Doppler is related to the normalised texture: \( m_f(\tau, \Omega) = A(\Omega) + B(\Omega) \tau + f_{\text{plat}} \)
• Doppler shift due to moving platform: \( f_{\text{plat}} = \frac{2v_p}{\lambda} \cos\theta_{\text{plat}} \)
• The spectral width, \( s \), is modelled as a Gaussian RV with mean, \( m_s \) and std. deviation, \( \sigma_s \).
• Parameter models required for: \( A, B, m_s \) and \( \sigma_s \).
• Modelled with parameters contained in the set \( \Omega = \{\theta_{\text{plat}}, \psi, \theta_{\text{swell}}, \eta, U, H_{1/3}\} \).
• Azimuth two-way beampattern, \( A(\theta) \) related to \( A(f) \) by \( f = \frac{2v_p}{\lambda} \sin\theta \), width is a function of look angle, \( \sin(\theta_{\text{plat}}) \) – maximum when side looking.
• Can model spreading due to azimuth two-way beampattern through the convolution:

\[
G(f, \tau, s, \Omega) = A(f) * G_0(f, \tau, s, \Omega)
\]
Simulation method

- Simulation method based on the convolution of the desired clutter impulse response $g$ (order N) and the compound Gaussian model $x\sqrt{\tau}$:

$$y(m, \tau) = \frac{1}{\sqrt{2}} \sum_{n=-N/2}^{N/2} g(n, \tau, s, \Omega)x(m + n)\sqrt{\tau(m + n)}$$  \hspace{1cm} (1)

- Texture, $\tau$, modelled with a Gamma distribution with Gaussian (spatial) correlation – produces $K$-distributed clutter model for $y(m, \tau)$.

- Speckle, $x$, modelled as complex normal RV.

- Discretised impulse response, $g$ found by taking Fourier transform of spectral model where $f_r$ is the PRF:

$$g(n, \tau, s, \Omega) = A\left(\frac{n}{f_r}\right)\exp\left[-j\frac{m_f(\tau, \Omega)2\pi n}{f_r}\right]\exp[-(2\pi s(\Omega)n f_r)^2]$$

- Can be implemented as a sliding window by repeating (1) for each pulse.
Simulation steps

1. Simulate the normalised texture over the entire scan.
   - Only generate realisations for a small number of positions ~ 10 (need approx. constant texture over filter integration time).
   - Distribution shape and spatial decorrelation length determined by user defined inputs in \( \Omega \) (sea-state, wind and swell direction, grazing angle).
   - Interpolate to the correct time scale (pulse repetition interval).

2. Generate realisations of the speckle RV, \( x \) and the spread RV, \( s \) (but no scaling or offset).
   - For each pulse, extract correct texture samples from the previous step.
   - Determine model parameters: \( A, B, m_s \) and \( \sigma_s \).
   - Scale and offset the RV \( s \).
   - Form the impulse response \( g \) and perform convolution.
   - Add thermal noise with the correct CNR (determined by radar range equation and IRSG-linear mean backscatter model).
Parameter modelling

- Parameter model provides a basis for modelling the statistical parameters in the simulation:
- Used for each polarisation channel independently and contains distinct models for **geometry and sea-state**.
- Geometry variation modelled with Fourier series:
  - \( X(\eta, \psi) = a_0 \eta \gamma [1 + a_1 \cos \psi + a_2 \cos(2\psi) + a_3 \cos(\psi - \phi) + a_4 \cos(2(\psi - \phi))] \)
  - \( \eta \) is the grazing angle, \( \psi \) is the azimuth angle,
  - \( \phi \) is the wind swell direction,
  - \( \gamma, a_0, ..., a_4 \) are the model coefficients.
- Sea-state variation modelled with:
  - \( Y = b_0 + b_1 \log_{10}(U) + b_2 H_{1/3}, \quad (2) \)
  - \( U \) is the wind speed, \( H_{1/3} \) is the significant wave height,
  - \( b_0, b_1, b_2 \) are the model coefficients.
Parameter modelling

• To relate these two models, coefficients are altered to be independent of grazing angle by introducing a normalisation factor, $\eta_0$ and then redefining previous equation:

$$X(\theta, \psi) = \left(\frac{\eta}{\eta_0}\right)^\gamma [\alpha_0 + \alpha_1 \cos \psi + \alpha_2 \cos(2\psi) + \alpha_3 \cos(\psi - \phi) + \alpha_4 \cos(2(\psi - \phi))]$$

where the new coefficients are related by

$$\alpha_0 = a_0 \eta_0^\gamma, \quad \alpha_1 = a_0 a_1 \eta_0^\gamma, \quad \ldots, \quad \alpha_4 = a_0 a_4 \eta_0^\gamma.$$ 

• The model is then implemented by equating each coefficient $\gamma, \alpha_0, \ldots, \alpha_4$ to the model Y in (2).

• Results in 18 coefficients per polarisation.

• Fixed the normalisation factor to $30^\circ$.

• Model is used for the shape, $\nu$, spatial decorrelation length, Doppler spectrum parameters $A, B, m_s$ and $\sigma_s$. 
Example parameter model

- Results show measured Pareto shape / model.
- Some regions where shape is less than 1 – model is not valid.
Modelling sea-spikes

- To improve sea-spike modelling, use bi-modal spectrum model:

\[ G_{bi}(f, \tau, s, \Omega) = (1 - \beta)G_0(f, \tau, s_1, \Omega) + \beta G_0(f, \tau, s_2, \Omega) \]

- Gaussian mixture model with ratio \( \beta \) for each component.
- Discrete scatterers due to targets or sea-spikes will cause an extra Doppler spread due to scanning motion of radar:

\[ s_{\text{scan}} = \frac{\sqrt{2 \ln 2} \omega_{\text{scan}}}{2\pi} \frac{\phi_{3\text{dB}}}{\phi_{3\text{dB}}} \]

where \( \phi_{3\text{dB}} \) is the azimuth 3dB beamwidth, \( \omega_{\text{scan}} \) is the scan rate.

- Set Doppler spread, \( s_1(\Omega) = s(\Omega) \), second spread component:

\[ s_2(\Omega) = \sqrt{s^2(\Omega) + s_{\text{scan}}^2} \]
Multiple phase centres

- Require representation of $K$ clutter sub-patches where $D$ is the spacing between phase centres.
- See example for 2 apertures:
- Texture is broken up into $K$ parts:
  \[ \sum_{k=1}^{K} \tau_k = \tau \]

where $\tau_k = A(f_k)\tau$ models the return from each part of the beam.
- Can now remove beampattern convolution – i.e just use $G_0$ or $G_{bi}$. 

\[ \tau_k = \tau \]

\[ K \]

\[ k = 1 \]

\[ D \]

\[ \theta_k \]

\[ R_k,1 \]

\[ R_k,2 \]

\[ R_0 \]
Multiple phase centres

- Need to adjust platform Doppler shift:
  \[ f_{\text{plat}} = \frac{2v_p}{\lambda} \cos(\theta_{\text{plat}} - \theta_k). \]
- Overall return from \( l^{th} \) sub-aperture:
  \[
  \tilde{y}_l(m) = \sum_{k=1}^{K} y(m, \tau_k) \exp\left(\frac{j2\pi}{\lambda} (R_{k,l} - R_{k,1})\right)
  \approx \sum_{k=1}^{K} y(m, \tau_k) \exp\left(\frac{j2\pi}{\lambda} D (l - 1) \sin \theta_k\right)
  \]
- Implementation more computationally intensive due to extra summation.
- Can reduce this by only including those sub-patches where \( A(f_k) > \epsilon \), where \( \epsilon \approx 10^{-3}. \)
Simulation examples

- Consider the following example with the radar scanning clockwise with the radar platform heading North.
- Wind and swell are both coming from the West.
- Filter order N=64.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radar scan rate</td>
<td>5 rpm</td>
</tr>
<tr>
<td>Aircraft speed</td>
<td>80 m/s</td>
</tr>
<tr>
<td>Azimuth 3 dB beamwidth (two-way)</td>
<td>3°</td>
</tr>
<tr>
<td>Centre frequency</td>
<td>10 GHz</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>200 MHz</td>
</tr>
<tr>
<td>Pulse repetition frequency</td>
<td>3 KHz</td>
</tr>
<tr>
<td>Grazing angle</td>
<td>30°</td>
</tr>
<tr>
<td>Sea state</td>
<td>3</td>
</tr>
<tr>
<td>No. of pulses a target will be in the beam</td>
<td>300</td>
</tr>
<tr>
<td>Amplitude PDF model</td>
<td>K distribution</td>
</tr>
<tr>
<td>Spectral model</td>
<td>Bimodal</td>
</tr>
</tbody>
</table>
Single aperture results

- Range / time result shown on left for the forward looking (crosswind) and side looking (upwind) cases.
- Doppler spectrum shown on right (after Doppler centroid correction) with greater correlation in the forward looking case (narrower spectrum).
Multi-aperture results

- Multi-aperture model has \( D = \frac{\lambda}{2} \) and \( L = 10 \) spatial channels.
- Forward looking spectrum appears narrower, but on average (over range) the width is correct.
Simulation verification

- Simulation is verified by comparing the estimated spatial correlation, shape and spectral width with the desired / specified values around the orbit.
- All estimated results show a close match with the desired ones.
- Left - single aperture, right – multi-aperture.
Multi-aperture power spectra

- Fourier / optimum power spectrum used to verify the multi-aperture simulation.

\[ P_F = \mathbf{v}^H(\theta, f) \hat{\mathbf{R}} \mathbf{v}(\theta, f) \]

\[ P_{opt} = \frac{1}{\mathbf{v}^H(\theta, f) \hat{\mathbf{R}}^{-1} \mathbf{v}(\theta, f)} \]

with steering vectors:

\[ \mathbf{v}(\theta, f) = \mathbf{v}_{\text{spat}}(\theta) \otimes \mathbf{v}_{\text{temp}}(f) \in \mathbb{C}^{LC \times 1} \]

containing elements:

\[ v_{\text{spat},l}(\theta) = \exp \left[ j2\pi(l - 1) \frac{d}{\lambda} \sin(\theta) \right] \]

\[ v_{\text{temp},c}(f) = \exp \left[ j2\pi(c - 1) \frac{f}{f_r} \right] \]

Covariance estimated with sample covariance matrix: \( \hat{\mathbf{R}} = \sum_{q=1}^{2LC} \mathbf{y}_q \mathbf{y}_q^H. \)
Conclusions and future work

- Sea clutter simulation method presented for both single and multi-aperture scanning radars.
- Simulation verified by comparing the estimated shape, spatial decorrelation length and spectral widths with the desired values.
- Multi-aperture simulation also verified by forming the Fourier and optimal spectra.
- Future work will investigate the extension of the parameter models to include:
  - variation of the CNR with range,
  - the impact of varying the coherent processing interval when characterising the model parameters,
  - how to include other range resolutions (beyond 0.75 m).