A geometric approach to passive target localization

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Drone surveillance can pose a threat

- A popular use of drones is in surveillance
- Some of the surveillance activities may present security concerns in a number of scenarios
- A situational awareness capability of the drone’s presence is desirable
Drones are getting harder to detect

- Small size, low thermal signature, non-reflective radar materials
- Can be camouflaged and capable of hiding in a non-conspicuous location (e.g., perching on tree branches)
- These drones may present a challenge to EO, IR and radar detection and target localization
Radio-Frequency (RF) emission for detection and target localization

- As an alternative, RF signals emitted by the drones can be exploited for detection and localization.
- RF: remote piloting (First Person View) image transmission (HD/UHD videos and pictures).
- Time-Difference-Of-Arrival (TDOA) method can be used to process the detected RF signals and to find the target location.
- The TDOA method is also capable of detecting and locating multiple moving targets simultaneously.
Estimating target location by **Time-Difference-Of-Arrival (TDOA)**

- A geometric approach to solving the TDOA problem will be presented
- It offers a simpler and more intuitive way to solve the problem
  - as an alternative to the conventional iterative numerical methods
- It may offer a means to provide real-time multi-target localization
The TDOA problem:

\[ \begin{align*}
d_{12} &= c\tau_{12} = r_1 - r_2 \\
d_{34} &= c\tau_{34} = r_3 - r_4 \\
d_{14} &= c\tau_{14} = r_1 - r_4
\end{align*} \]

- **Equations** express the time difference of a signal arriving at a pair of receivers.
- **4 receivers** needed to obtain 3 independent TDOA measurements, \( d_{ij} \)
- **3 equations** to compute the target location \((x,y,z)\)

\( d_{ij} \) is the TDOA measurement (range difference)
\( \tau_{ij} = \tau_i - \tau_j \) is the TDOA; \( d_{ij} = c \tau_{ij} \)

\( r_i \) is the distance between the target and receiver \( i \)

\[ r_i(x, y, z) = \sqrt{(x - X_i)^2 + (y - Y_i)^2 + (z - Z_i)^2} \]
TDOA measurements $d_{ij}$

- The $d_{ij}$ measurements are made by cross-correlating the signals detected by a pair of receivers
  - $d_{12}$ (receiver-pair S1-S2), $d_{34}$ (S3-S4), $d_{14}$ (S1-S4)

- The cross-correlation is obtained using a matched filter

\[
\chi(\tau_{ij}, f_{D,ij}) = \int \mu_i(t - t'(\tau_i, f_{D,i})) \mu^*_j(t - t'(\tau_j, f_{D,j})) dt
\]

\[
\tau_{ij} = \tau_i - \tau_j \quad \text{(TDOA, time-difference-of-arrival)}
\]

\[
f_{D,ij} = f_{D,i} - f_{D,j} \quad \text{(FDOA, frequency-difference-of-arrival)}
\]

The peak of the cross-correlation gives the $d_{ij} (=c\tau_{ij})$ value
Solving the TDOA Equations for the target location \((x,y,z)\)

\[
d_{12} = r_1 - r_2 \\
d_{34} = r_3 - r_4 \\
d_{14} = r_1 - r_4
\]

\[
r_i = \sqrt{(x - X_i)^2 + (y - Y_i)^2 + (z - Z_i)^2}; \quad i = 1, 2, 3, 4
\]

- A set of 3 non-linear equations
- Conventionally solved by iterative numerical methods (e.g., Least Square)
- Complex algorithms and require an initial value; bad guess means slower convergence, hence long computation time
Geometric approach to solving the TDOA equations

TDOA equations:

\[ d_{ij} = r_i - r_j \]

\[ = \sqrt{(x - X_i)^2 + (y - Y_i)^2 + (z - Z_i)^2} - \sqrt{(x - X_j)^2 + (y - Y_j)^2 + (z - Z_j)^2} \]

- Using geometry, each TDOA equation can be solved individually.
- The solution is given by a hyperboloid,

\[ \frac{x' \,^2}{(d_{ij}^2 / 4)} - \left( \frac{y' \,^2}{(d^2 / 4) - (d_{ij}^2 / 4)} + \frac{z' \,^2}{(d^2 / 4) - (d_{ij}^2 / 4)} \right) = 1 \]

in a local coordinate system \((x', y', z')\) (target)

where \((X'_1, Y'_1, Z'_1) = (-d/2, 0, 0), \) (S1)
\((X'_2, Y'_2, Z'_2) = (+d/2, 0, 0), \) (S2)

\(d_{ij}\) = TDOA measurement

\(d\) = distance between the two receivers
TDOA solution = hyperboloid

\[ d_{ij} = r_i - r_j \quad \text{TDOA equation} \]

Solution:

\[
\frac{x'^2}{(d_{ij}^2 / 4)} - \left( \frac{y'^2}{(d_{ij}^2 / 4)} + \frac{z'^2}{(d_{ij}^2 / 4)} \right) = 1
\]

- Positive \( d_{ij} \), right hand side surface \((r_i > r_j)\);
  negative \( d_{ij} \), left hand side surface \((r_i < r_j)\).

- The target is somewhere on the surface of the hyperboloid
- Since the +/− sign of $d_{ij}$ is known from the cross-correlator, and knowing the target is above ground, we can further narrow down the target’s location.

- Do the same for the other 2 equations (i.e., receiver pairs S3-S4 and S1-S4)

- Hence obtain 3 hyperboloids as solutions for the 3 TDOA equations

- The 3 hyperboloids are then used to pinpoint the target’s location.
Target localization from intersection of 3 hyperboloids

4 receivers in a “forward-looking” system configuration, with 3 receiver-pairs: S1-S2, S3-S4, S1-S4

- Place the 3 hyperboloids in the same orientations as the receiver pairs in the system configuration
- The 3 hyperboloids will intersect with one another
- The target location is where the 3 hyperboloids intersect at one point \((x,y,z)\)
The intersection point is searched by scanning the intersecting hyperboloids layer by layer along $z$.

This intersection point is found at $z$ where the 3 intersecting hyperbolic curves form the smallest area (i.e., the sharpest point).
The positioning precision of the hyperboloid depends on the accuracy of $d_{ij}$.

If TDOA measurements ($d_{ij}$) have very small error, then target localization would be very accurate because the hyperboloids can be placed precisely.

Using error-free $d_{ij}$ measurements

<table>
<thead>
<tr>
<th>Time (arb.unit)</th>
<th>Target ground truths (m)</th>
<th>Computed target locations (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$X_{Tg}$</td>
<td>$Y_{Tg}$</td>
</tr>
<tr>
<td>1</td>
<td>-660.00</td>
<td>9998.50</td>
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<tr>
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<td>-299.11</td>
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</tr>
<tr>
<td>10</td>
<td>-196.00</td>
<td>3968.32</td>
</tr>
</tbody>
</table>
TDOA measurements deviated from the error-free values

- Real TDOA measurements \( (d_{ij}) \) have errors
- The errors are characterized by the Cramer-Rao Lower Bound variance \( \sigma^2 \)
- The standard deviation ("root mean square error"),
  \[
  \sigma \geq \frac{1}{\beta \sqrt{6.5 \text{SNR}}}
  \]
- \( \sigma \) is dependent on signal bandwidth \( \beta \) and SNR
- Drone’s emitting signal bandwidths:
  - 1-3 MHz (telemetry data)
  - 15 MHz (first person view)
  - 20 MHz (UHD videos)
- \( \text{SNR} = 16 \) (12 dB) “detection threshold” of signals
- \( \sigma \approx 10^{-8} – 10^{-7} \) s
- Error for \( d_{ij} \):
  \[
  \varepsilon = c \sigma \approx 3 – 30 \text{ m} \quad (c = \text{speed of light})
  \]
TDOA measurements \( (d_{ij}) \) with large deviations from the error-free values

- \( \epsilon = c\sigma = 30 \, \text{m} \)
- \( \sigma \) parameters: \( \beta = 1 \, \text{MHz}, \, \text{SNR} = 12 \, \text{dB} \)

<table>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( X_{T_g} )</td>
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<td>3968.32</td>
</tr>
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</table>
TDOA measurements \( (d_{ij}) \) with a smaller error

- \( \epsilon = 1.5 \text{ m} \)
- \( \sigma \) parameters: \( \beta = 20 \text{ MHz} \), \( \text{SNR} = 12 \text{ dB} \)

<table>
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<tr>
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<th>( Z_{Tg} )</th>
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</table>
Summarize briefly:

- The target localization accuracy is fundamentally linked to the signal’s bandwidth and the SNR via the Cramer-Rao Lower Bound relation that characterizes the error in the TDOA measurements ($d_{ij}$)
Multi-targets detection and localization

- Drones are becoming cheaper and more accessible
- Use of multiple drones in surveillance will become more likely and may even be a norm
- An effective drone detection system must be able to detect and localize multiple targets simultaneously and in real-time in order to deal with the threats
- There has not been much work published on multi-target localization
- Applying the geometric method to multi-target localization
A 7-target scenario

Open circles:
TDOA measurements made by the receiver pairs
S1-S2
S3-S4
S1-S4
at 10 time instants

target altitude = 1000 m
Multiple Target Localization Scenario

- Each receiver-pair detects 7 targets and generates 7 TDOA $d_{ij}$; i.e.,
  - S1-S2: 7 $d_{12}$ values
  - S3-S4: 7 $d_{34}$ values
  - S1-S4: 7 $d_{14}$ values

- 3 sets of 7 TDOA measurements ($d_{ij}$) feeding the TDOA equations

- Need to search a $n^3$ permutation ($7^3 = 343$ sets) of TDOA ($3-d_{ij}$) combinations to determine the locations of the 7 targets

- TDOA equations have to be solved 343 times; this requires a bit of computing time

TDOA equations:

\[
\begin{align*}
  d_{12} &= r_1 - r_2 \\
  d_{34} &= r_3 - r_4 \\
  d_{14} &= r_1 - r_4
\end{align*}
\]
Target localization results for the case, $\varepsilon = 1.5$ m (TDOA measurement error)

- 5 target mis-locations occur
- They are due to combinations of $d_{ij}$ values in the permutation that are not all from the same target, but have nonetheless generated the sharpest intersection point from the 3 intersecting hyperboloids
- Mis-locations are due to the TDOA measurements $(d_{ij})$ having too large an error $\varepsilon$

- target ground truth $(x,y)$
- computed location $(x,y)$
Reducing TDOA error to $\varepsilon = 0.15 \text{ m}$ (from 1.5)

$\beta = 20 \text{ MHz, SNR} = 32 \text{ dB}$

For multi-target localization, the TDOA error $\varepsilon$ should be kept small to minimize mis-locations

<table>
<thead>
<tr>
<th>time</th>
<th>T#5</th>
<th>T#3</th>
<th>T#7</th>
<th>T#1</th>
<th>T#6</th>
<th>T#2</th>
<th>T#4</th>
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</tbody>
</table>

actual target altitude = 1000 m

= ground truth

= computed target location (x,y)
Geometric approach to achieve real-time multi-target localization?

- The geometry-based solution is not real-time
- Most of the computing time is spent on the hyperboloids

<table>
<thead>
<tr>
<th>$n$ (no. of targets detected)</th>
<th>$t$ (non-coplanar receiver configuration) per time instant of sampling</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18.6s</td>
</tr>
<tr>
<td>3</td>
<td>56.1</td>
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<tr>
<td>7</td>
<td>147.1</td>
</tr>
<tr>
<td>10</td>
<td>236.0</td>
</tr>
</tbody>
</table>

- Real-time via the geometric method: each of the 3 TDOA Equations is solved individually (i.e., computing a hyperboloid)

$$d_{12} = r_1 - r_2 \quad \Rightarrow \quad \text{hyperboloid1}$$
$$d_{34} = r_3 - r_4 \quad \Rightarrow \quad \text{hyperboloid2}$$
$$d_{14} = r_1 - r_4 \quad \Rightarrow \quad \text{hyperboloid3}$$

- The hyperboloids can be pre-computed for a range of different $d_{ij}$ values for each of the 3 TDOA equations and stored as look-up tables to save considerable computing time
Approach to real-time multi-target processing

\[ d_{ij} = r_i - r_j \]

\[
\frac{x'\,^2}{(d_{ij}^2 / 4)} - \left( \frac{y'\,^2}{(d^2 / 4) - (d_{ij}^2 / 4)} + \frac{z'\,^2}{(d^2 / 4) - (d_{ij}^2 / 4)} \right) = 1
\]

- Each single TDOA equation has a hyperboloid as solution
- \( d \) is the known separation distance between a pair of receivers
- \(-d < d_{ij} < d\)
- For a given TDOA error \( \varepsilon \), there are \((2d/\varepsilon + 1)\) possible \( d_{ij} \) values
- \((2d/\varepsilon + 1)\) hyperboloids can be pre-computed and stored as look-up tables
Number of look-up tables for the hyperboloids

TDOA error: \( \epsilon = 1.5 \text{m} \)

(\( \sigma \) parameters: \( \beta = 20 \) MHz, SNR=12dB)

+ detection system size with \( d \) as shown on the left

# of hyperboloids = \((2d/\epsilon + 1)\)

13334 \((S1-S4)\)

6417 \((S1-S2)\)

6417 \((S3-S4)\)

= 26168 (total)

hyperboloids to be pre-computed and stored as look-up tables; each corresponds to a specific \( d_{ij} \) value.

This total is applicable to any \( n \)-target scenarios, as long as the correlator can resolve 2 targets to within \( \epsilon \).

Assume FPV transmitter power = 500 mW
How multi-target localization in real-time could be achieved

- Use look-up tables
  - Large data storage capacity and fast data retrieval algorithms make this viable

- Apply parallel computing algorithms
  - The $n^3$ permutation is highly parallel in computing structure

- Using both look-up tables and multi-core parallel computing, real-time (~ 1s) multi-target localization may be realizable
Thank you
Computing time: coplanar vs non-coplanar

Table 5.10: Computation time consumed in target localization processing for different number of targets detected using sequential processing.

<table>
<thead>
<tr>
<th>n (no. of targets detected)</th>
<th>$t$ (coplanar configuration)</th>
<th>$t$ (non-coplanar configuration)</th>
</tr>
</thead>
<tbody>
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<td>0.4s</td>
<td>18.6s</td>
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<tr>
<td>3</td>
<td>2.8</td>
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<td>7</td>
<td>30.8</td>
<td>147.1</td>
</tr>
<tr>
<td>10</td>
<td>89.5</td>
<td>236.0</td>
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</table>
Parallel structure in permutation

<table>
<thead>
<tr>
<th></th>
<th>Target #1</th>
<th>Target #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1-S2</td>
<td>A</td>
<td>D</td>
</tr>
<tr>
<td>S3-S4</td>
<td>B</td>
<td>E</td>
</tr>
<tr>
<td>S1-S4</td>
<td>C</td>
<td>F</td>
</tr>
</tbody>
</table>

permutations:

ABC
ABF
AEC
AEF
DBC
DBF
DEC
DEF
Numerical method and closed-form solutions need to solve 3 TDOA equations simultaneously

\[ d_{12} = c\tau_{12} = r_1 - r_2 \]
\[ d_{34} = c\tau_{34} = r_3 - r_4 \]
\[ d_{14} = c\tau_{14} = r_1 - r_4 \]

Pre-computing needs combinations of 3 \( d_{ij} \) values as one single set. The no. of permutated sets required \( 6417 \times 6417 \times 13334 \approx 5 \times 10^{11} \)
## Coplanar receiver configuration

- 4 receivers are located at the same $z = 0$ level
- $\beta = 1$ MHz, SNR = 16, $\varepsilon = 30$ m

<table>
<thead>
<tr>
<th>Time (arb. unit)</th>
<th>$X_{Tg}$</th>
<th>$Y_{Tg}$</th>
<th>$Z_{Tg}$</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
</tr>
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<tbody>
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**Coplanar receiver configuration**

- 4 receivers are located at the same $z = 0$ level
- $\beta = 20$ MHz, SNR = 16, $\varepsilon = 1.5$ m

**Table 4.5**

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<th>Time (arb.unit)</th>
<th>$X_{Tg}$</th>
<th>$Y_{Tg}$</th>
<th>$Z_{Tg}$</th>
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