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# A geometric approach to passive target localization

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# Drone surveillance can pose a threat

- A popular use of drones is in surveillance
- Some of the surveillance activities may present security concerns in a number of scenarios
- A situational awareness capability of the drone’s presence is desirable

# Drones are getting harder to detect

- Small size, low thermal signature, non-reflective radar materials
- Can be camouflaged and capable of hiding in a non-conspicuous location (e.g., perching on tree branches)
- These drones may present a challenge to EO, IR and radar detection and target localization

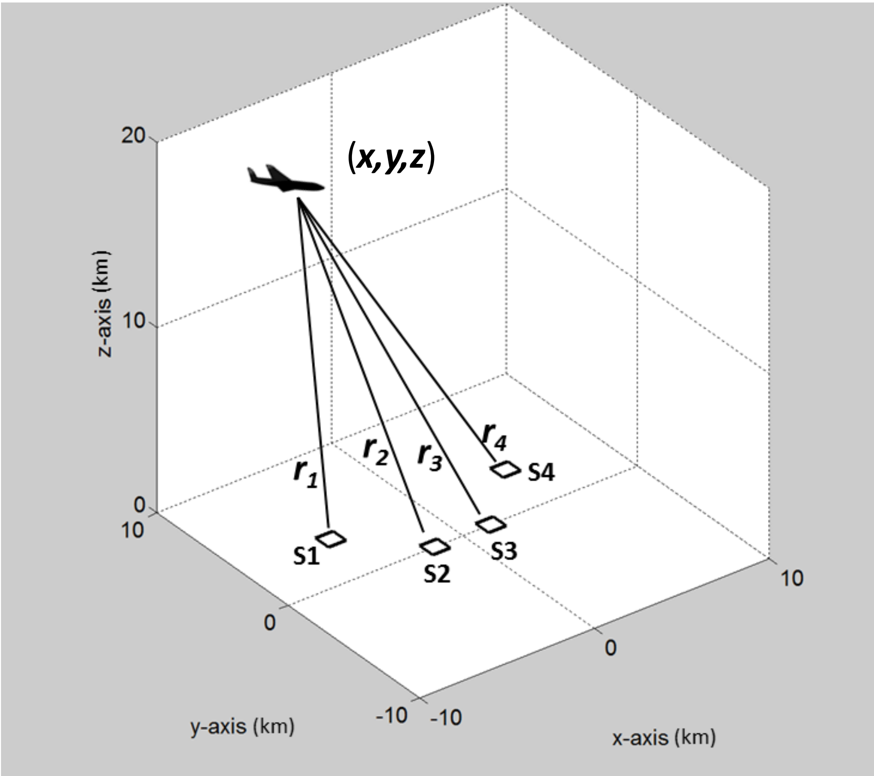
# Radio-Frequency (RF) emission for detection and target localization

- As an alternative, RF signals emitted by the drones can be exploited for detection and localization
- RF: remote piloting (First Person View )

  image transmission (HD/UHD videos and pictures)
- Time-Difference-Of-Arrival (**TDOA**) method can be used to process the detected RF signals and to find the target location
- The TDOA method is also capable of detecting and locating multiple moving targets simultaneously

# Estimating target location by Time-Difference-Of-Arrival (TDOA)

- A geometric approach to solving the TDOA problem will be presented
- It offers a simpler and more intuitive way to solve the problem
    - as an alternative to the conventional iterative numerical methods
- It may offer a means to provide real-time multi-target localization




# The TDOA problem:

$$d_{12} = c\tau_{12} = r_1 - r_2$$

$$d_{34} = c\tau_{34} = r_3 - r_4$$

$$d_{14} = c\tau_{14} = r_1 - r_4$$

$\boldsymbol{d_{ij}}$ is the **TDOA** measurement (range difference)

$\boldsymbol{\tau_{ij} = \tau_i - \tau_j}$ is the **TDOA;** $\boldsymbol{d_{ij} = c\,\tau_{ij}}$

$\boldsymbol{r_i}$ is the distance between the target and receiver $\boldsymbol{i}$

$$r_i(x,y,z) = \sqrt{(x - X_i)^2 + (y - Y_i)^2 + (z - Z_i)^2}$$

- **Equations express the time difference of a signal arriving at a pair of receivers**
- **4 receivers needed to obtain 3 independent TDOA measurements,** $\boldsymbol{d_{ij}}$
- **3 equations to compute the target location (*x,y,z*)**

# TDOA measurements $d_{ij}$

- *The* $\boldsymbol{d_{ij}}$ measurements are made by cross-correlating the signals detected by a pair of receivers
  - $\boldsymbol{d_{12}}$ (receiver-pair S1-S2), $\boldsymbol{d_{34}}$ (S3-S4), $\boldsymbol{d_{14}}$ (S1-S4)
- The cross-correlation is obtained using a matched filter

$$\chi(\tau_{ij}, f_{D,ij}) = \int \mu_i(t - t'(\tau_i, f_{D,i}))\mu_j^*(t - t'(\tau_j, f_{D,j}))\,dt$$

$\tau_{ij} = \tau_i - \tau_j$ (TDOA, time-difference-of-arrival)

$f_{D,ij} = f_{D,i} - f_{D,j}$ (FDOA, frequency-difference-of-arrival)

The peak of the cross-correlation gives the $\boldsymbol{d_{ij}}$ ($=c\tau_{ij}$) value

# Solving the TDOA Equations for the target location (*x,y,z*)

$$d_{12} = r_1 - r_2$$

$$d_{34} = r_3 - r_4$$

$$d_{14} = r_1 - r_4$$

$$r_i = \sqrt{(x - X_i)^2 + (y - Y_i)^2 + (z - Z_i)^2}; \quad i = 1, 2, 3, 4$$

- A set of 3 non-linear equations
- Conventionally solved by iterative numerical methods (e.g., Least Square)
- Complex algorithms and require an initial value; bad guess means slower convergence, hence long computation time

# Geometric approach to solving the TDOA equations

TDOA equations:

$$d_{ij} = r_i - r_j$$

$$= \sqrt{(x - X_i)^2 + (y - Y_i)^2 + (z - Z_i)^2} - \sqrt{(x - X_j)^2 + (y - Y_j)^2 + (z - Z_j)^2}$$

- Using geometry, each TDOA equation can be solved individually.
- The solution is given by a hyperboloid,

$$\frac{x'^2}{(d_{ij}^2/4)} - \left( \frac{y'^2}{(d^2/4) - (d_{ij}^2/4)} + \frac{z'^2}{(d^2/4) - (d_{ij}^2/4)} \right) = 1$$

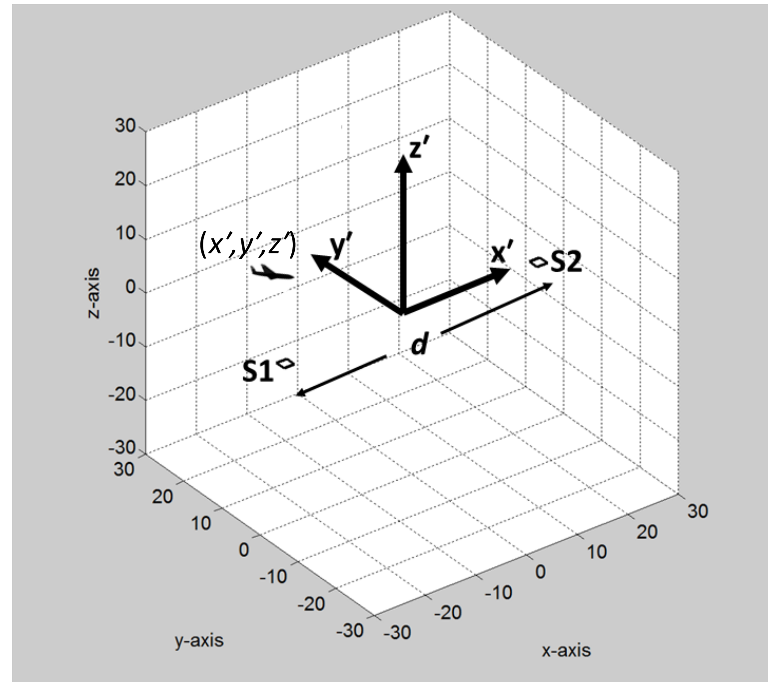
in a local coordinate system $(x', y', z')$ (target)

where $(X_1', Y_1', Z_1') = (-d/2, 0, 0)$, (S1)

$(X_2', Y_2', Z_2') = (+d/2, 0, 0)$, (S2)

$d_{ij}$ = TDOA measurement

$d$ = distance between the two receivers



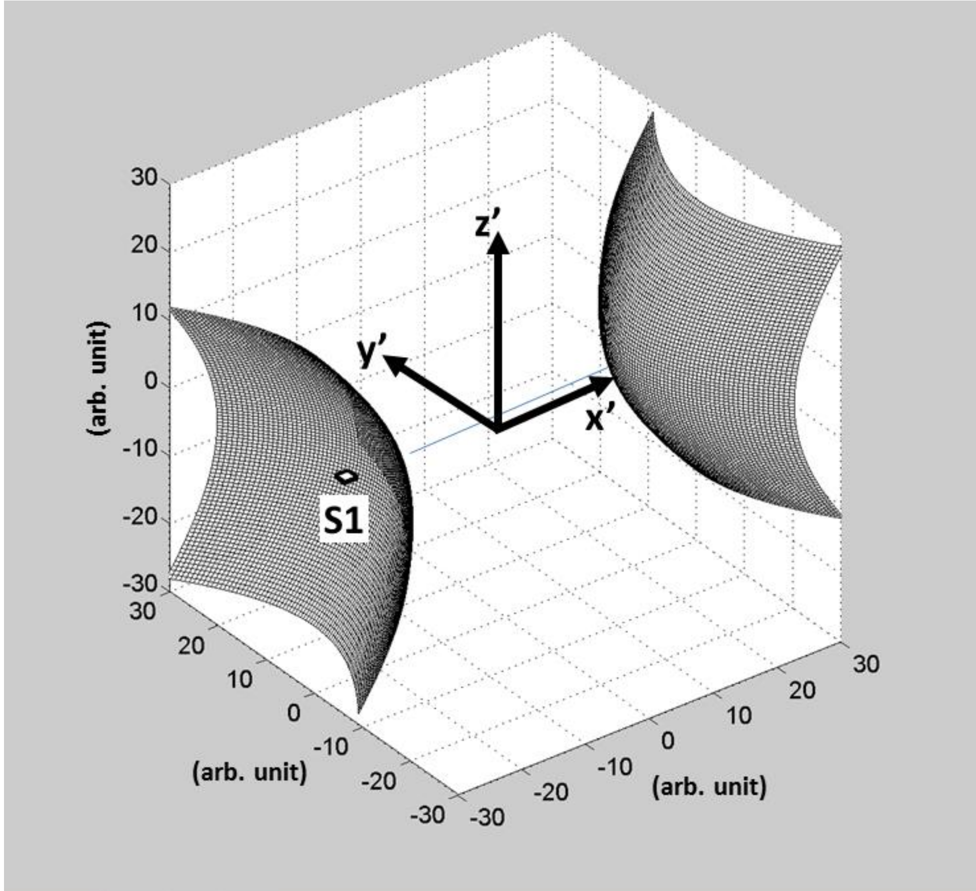
# TDOA solution = hyperboloid

$d_{ij} = r_i - r_j$ TDOA equation

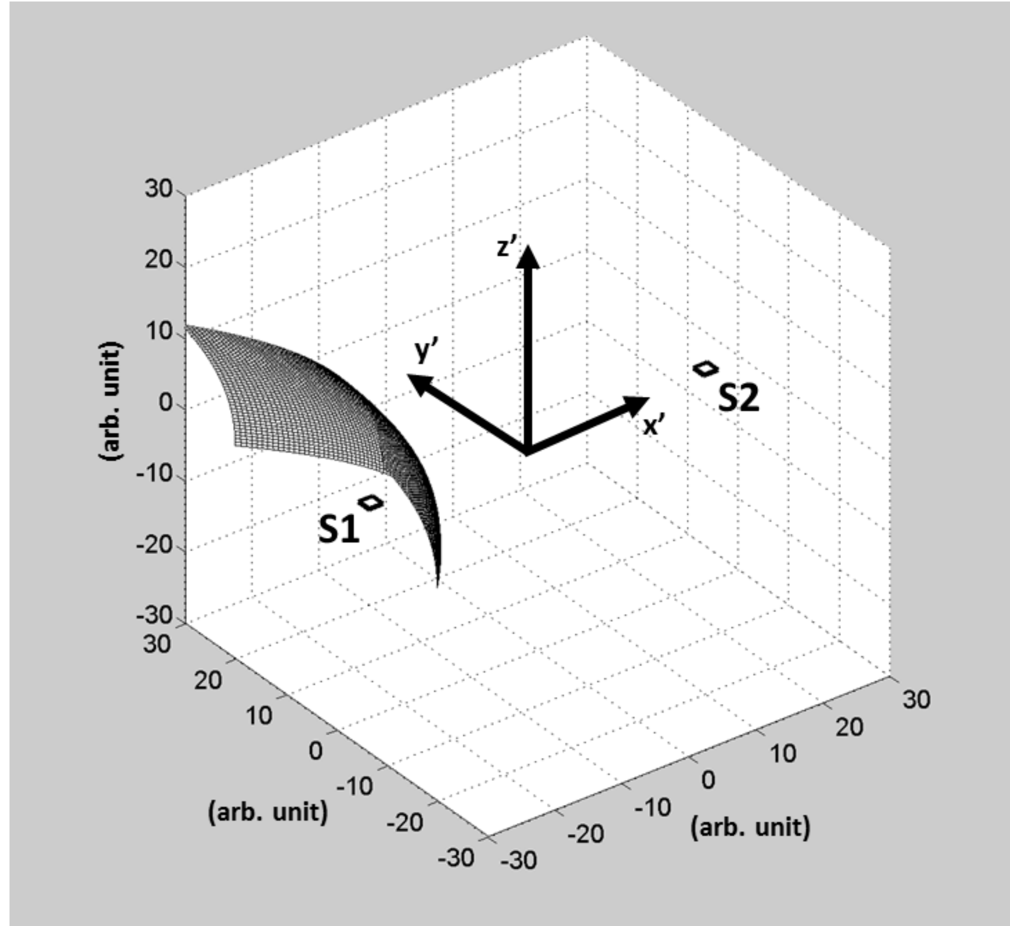
Solution:

$$\frac{x'^2}{(d_{ij}^2/4)} - \left( \frac{y'^2}{(d^2/4)-(d_{ij}^2/4)} + \frac{z'^2}{(d^2/4)-(d_{ij}^2/4)} \right) = 1$$

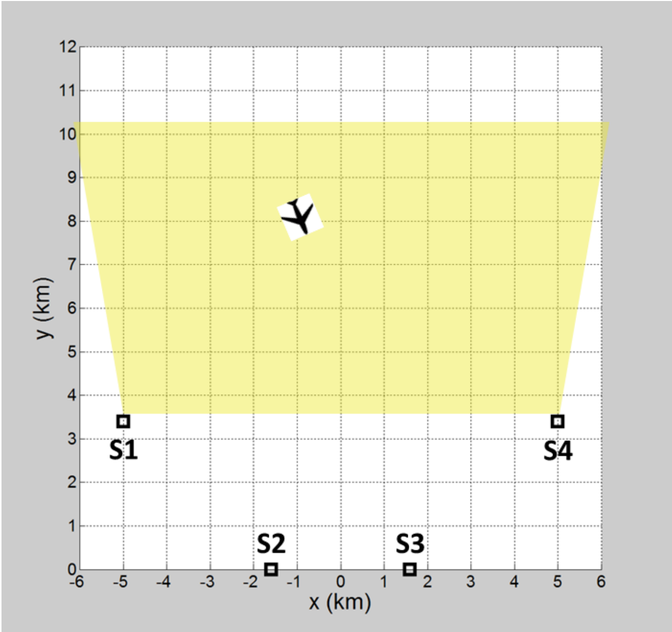
- Positive $d_{ij}$, right hand side surface ($r_i > r_j$); negative $d_{ij,}$ left hand side surface ($r_i < r_j$).
- The target is somewhere on the surface of the hyperboloid



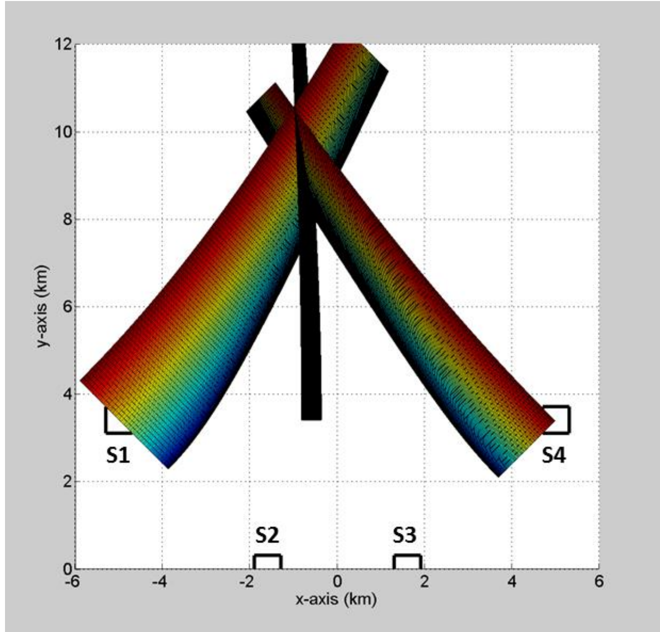
- Since the +/– sign of $d_{ij}$ is known from the cross-correlator, and knowing the target is above ground, we can further narrow down the target's location.
- Do the same for the other 2 equations (i.e., receiver pairs S3-S4 and S1-S4)
- Hence obtain 3 hyperboloids as solutions for the 3 TDOA equations
- The 3 hyperboloids are then used to pinpoint the target's location.



# Target localization from intersection of 3 hyperboloids

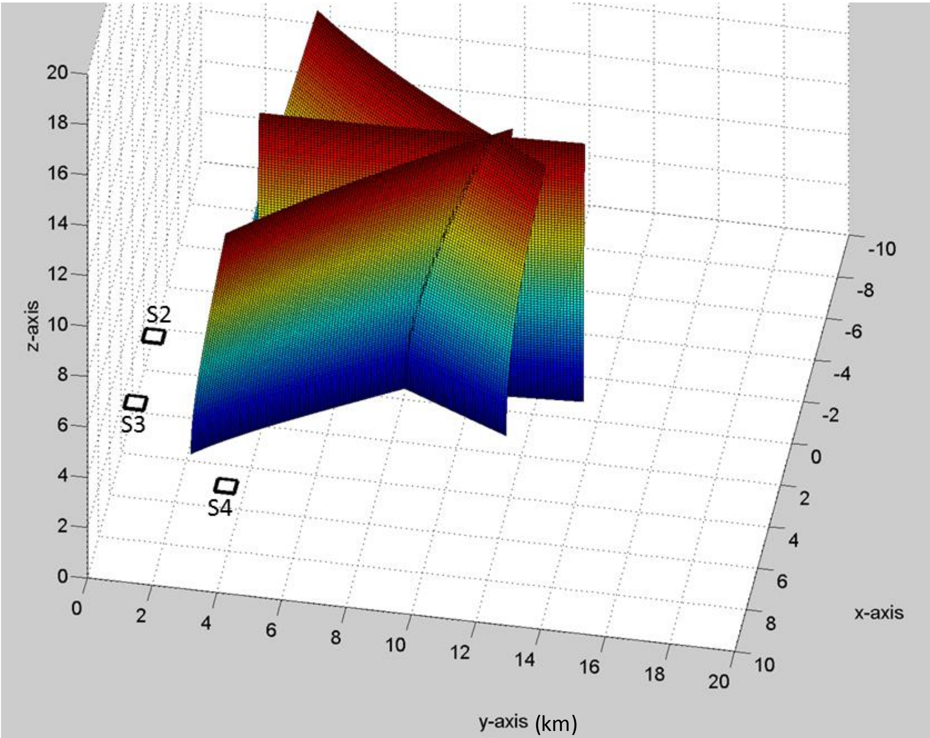


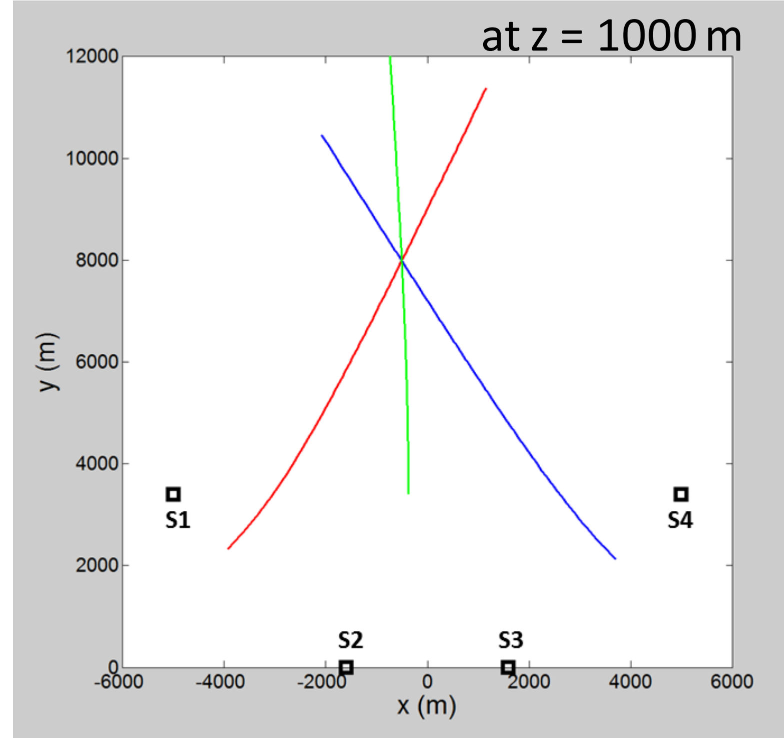

4 receivers in a "forward-looking" system configuration, with 3 receiver-pairs: S1-S2, S3-S4, S1-S4




3 intersecting hyperboloids

- **Place the 3 hyperboloids in the same orientations as the receiver pairs in the system configuration**
- **The 3 hyperboloids will intersect with one another**
- **The target location is where the 3 hyperboloids intersect at one point (*x,y,z*)**




- The intersection point is searched by scanning the intersecting hyperboloids layer by layer along z.

- This intersection point is found at *z* where the 3 intersecting hyperbolic curves form the smallest area (i.e., the sharpest point).

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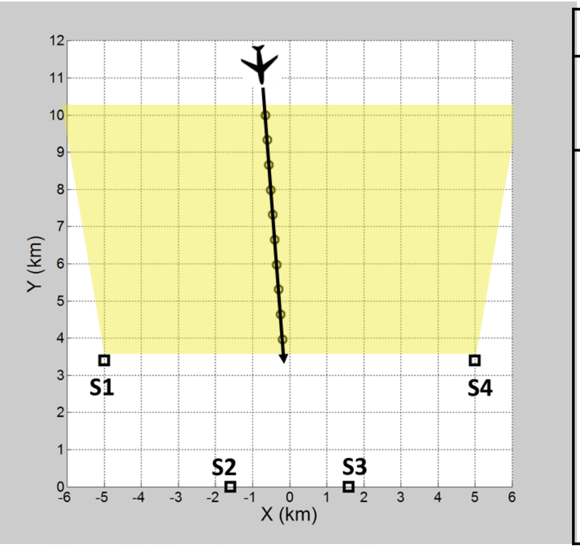
# TDOA measurements ($d_{ij}$) and target localization accuracy

- The positioning precision of the hyperboloid depends on the accuracy of $d_{ij}$

$$\frac{x'^2}{(d_{ij}^2/4)} - \left( \frac{y'^2}{(d^2/4)-(d_{ij}^2/4)} + \frac{z'^2}{(d^2/4)-(d_{ij}^2/4)} \right) = 1$$

- If TDOA measurements ($d_{ij}$) have very small error, then target localization would be very accurate because the hyperboloids can be placed precisely.




Using error-free $d_{ij}$ measurements

| | Target ground truths (m) | | | Computed target locations (m) | | |
|---|---|---|---|---|---|---|
| Time (arb.unit) | $X_{Tg}$ | $Y_{tg}$ | $Z_{Tg}$ | $x$ | $y$ | $z$ |
| 1 | -660.00 | 9998.50 | 1000.00 | -660.00 | 9998.50 | 1000 |
| 2 | -608.44 | 9328.48 | 1000.00 | -608.44 | 9328.48 | 1000 |
| 3 | -556.89 | 8658.46 | 1000.00 | -556.89 | 8658.46 | 1000 |
| 4 | -505.33 | 7988.44 | 1000.00 | -505.33 | 7988.44 | 1000 |
| 5 | -453.78 | 7318.42 | 1000.00 | -453.78 | 7318.42 | 1000 |
| 6 | -402.22 | 6648.40 | 1000.00 | -402.22 | 6648.40 | 1000 |
| 7 | -350.67 | 5978.38 | 1000.00 | -350.67 | 5978.38 | 1000 |
| 8 | -299.11 | 5308.36 | 1000.00 | -299.11 | 5308.36 | 1000 |
| 9 | -247.56 | 4638.34 | 1000.00 | -247.56 | 4638.34 | 1000 |
| 10 | -196.00 | 3968.32 | 1000.00 | -196.00 | 3968.32 | 1000 |

# TDOA measurements deviated from the error-free values

- Real TDOA measurements ($d_{ij}$) have errors
- The errors are characterized by the Cramer-Rao Lower Bound variance $\boldsymbol{\sigma^2}$
- The standard deviation ( ”root mean square error”),

$$\sigma \geq \frac{1}{\beta\sqrt{6.5SNR}}$$

- **σ** is dependent on signal bandwidth β and SNR
- Drone’s emitting signal bandwidths:
  - 1-3 MHz (telemetry data)
  - 15 MHz (first person view)
  - 20 MHz (UHD videos)
- *SNR* = 16 (12 dB) “detection threshold” of signals
- $\boldsymbol{\sigma} \approx 10^{-8} – 10^{-7}$ s
- Error for $\boldsymbol{d_{ij}}$ : **ε = c σ** ≈ 3 – 30 m (**c** = speed of light)

# TDOA measurements ($d_{ij}$) with large deviations from the error-free values

- ε = cσ = 30 m
- **σ** parameters: β = 1 MHz, SNR = 12 dB

| | Target ground truths (m) | | | Computed target locations (m) | | |
|---|---|---|---|---|---|---|
| Time (arb.unit) | $X_{Tg}$ | $Y_{Tg}$ | $Z_{Tg}$ | *x* | *y* | *z* |
| 1 | -660.00 | 9998.50 | 1000.00 | -637.86 | 9837.28 | 400 |
| 2 | -608.44 | 9328.48 | 1000.00 | -624.27 | 9398.05 | 1700 |
| 3 | -556.89 | 8658.46 | 1000.00 | -543.75 | 8603.23 | 600 |
| 4 | -505.33 | 7988.44 | 1000.00 | -490.12 | 7938.34 | 800 |
| 5 | -453.78 | 7318.42 | 1000.00 | -441.23 | 7277.22 | 900 |
| 6 | -402.22 | 6648.40 | 1000.00 | -394.97 | 6562.24 | 900 |
| 7 | -350.67 | 5978.38 | 1000.00 | -344.75 | 5947.75 | 1200 |
| 8 | -299.11 | 5308.36 | 1000.00 | -288.28 | 5238.35 | 400 |
| 9 | -247.56 | 4638.34 | 1000.00 | -254.61 | 4595.83 | 1300 |
| 10 | -196.00 | 3968.32 | 1000.00 | -204.42 | 3923.01 | 1500 |

# TDOA measurements ($d_{ij}$) with a smaller error

- ε = 1.5 m
- **σ** parameters: β = 20 MHz, SNR = 12 dB

| | Target ground truths (m) | | | Computed target locations (m) | | |
|---|---|---|---|---|---|---|
| Time (arb.unit) | $X_{Tg}$ | $Y_{Tg}$ | $Z_{Tg}$ | *x* | *y* | *z* |
| 1 | -660.00 | 9998.50 | 1000.00 | -658.36 | 9985.12 | 1000.00 |
| 2 | -608.44 | 9328.48 | 1000.00 | -607.37 | 9320.40 | 1000.00 |
| 3 | -556.89 | 8658.46 | 1000.00 | -556.09 | 8649.78 | 1000.00 |
| 4 | -505.33 | 7988.44 | 1000.00 | -504.86 | 7981.78 | 1000.00 |
| 5 | -453.78 | 7318.42 | 1000.00 | -452.51 | 7313.34 | 1000.00 |
| 6 | -402.22 | 6648.40 | 1000.00 | -401.72 | 6644.95 | 1000.00 |
| 7 | -350.67 | 5978.38 | 1000.00 | -349.92 | 5975.92 | 1000.00 |
| 8 | -299.11 | 5308.36 | 1000.00 | -298.84 | 5306.21 | 1000.00 |
| 9 | -247.56 | 4638.34 | 1000.00 | -247.41 | 4636.79 | 1000.00 |
| 10 | -196.00 | 3968.32 | 1000.00 | -195.37 | 3966.59 | 1000.00 |

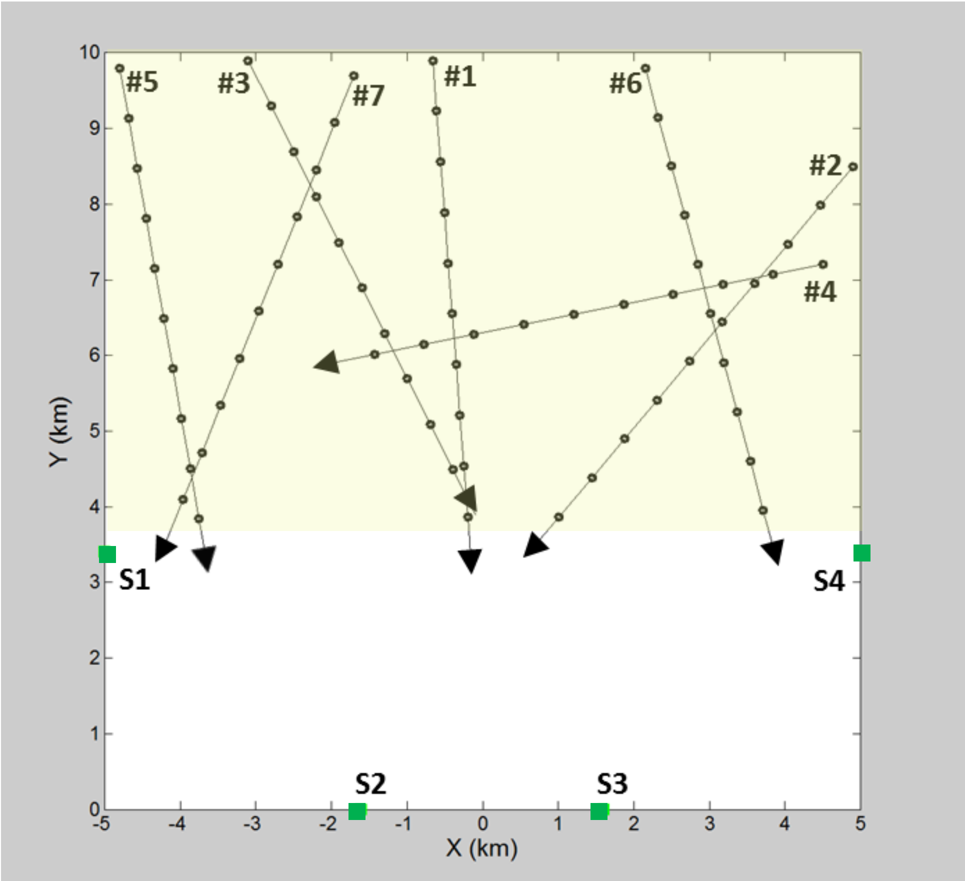
# Summarize briefly:

- The target localization accuracy is fundamentally linked to the signal's bandwidth and the SNR via the Cramer-Rao Lower Bound relation that characterizes the error in the TDOA measurements ($d_{ij}$)

# Multi-targets detection and localization

- Drones are becoming cheaper and more accessible
- Use of multiple drones in surveillance will become more likely and may even be a norm
- An effective drone detection system must be able to detect and localize multiple targets simultaneously and in real-time in order to deal with the threats
- There has not been much work published on multi-target localization
- Applying the geometric method to multi-target localization

# A 7-target scenario




target altitude = 1000 m

Open circles :
TDOA measurements made by the receiver pairs
S1-S2
S3-S4
S1-S4
at 10 time instants

# Multiple Target Localization Scenario

- Each receiver-pair detects 7 targets and generates 7 TDOA $d_{ij}$ ; i.e.,
  - S1-S2: 7 $d_{12}$ *values*
  - S3-S4: 7 $d_{34}$ *values*
  - S1-S4: 7 $d_{14}$ *values*
- 3 sets of 7 TDOA measurements ($d_{ij}$) feeding the TDOA equations
- Need to search a $n^3$ permutation ($7^3$ =343 sets) of TDOA (3-$d_{ij}$) combinations to determine the locations of the 7 targets
- TDOA equations have to be solved 343 times; this requires a bit of computing time

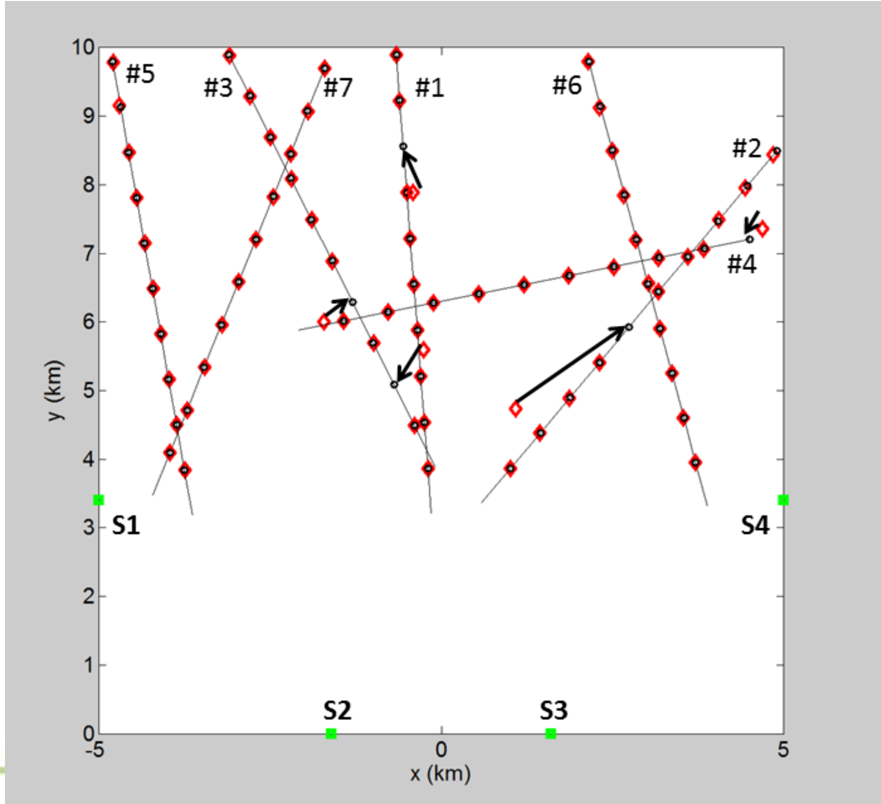
TDOA equations:

$$d_{12} = r_1 - r_2$$

$$d_{34} = r_3 - r_4$$

$$d_{14} = r_1 - r_4$$

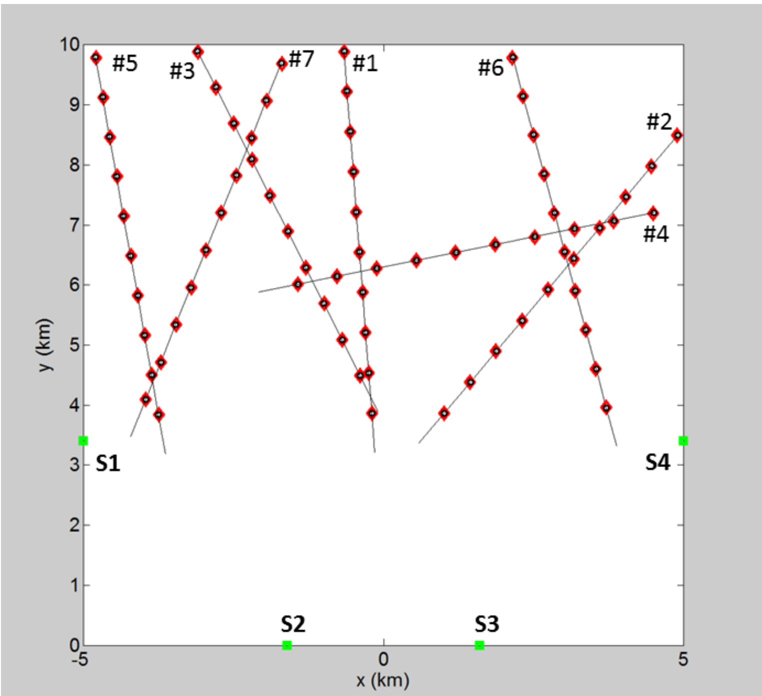
- Target localization results for the case, $\varepsilon = 1.5$ m (TDOA measurement error)
- 5 target mis-locations occur
- They are due to combinations of $d_{ij}$ values in the permutation that are not all from the same target, but have nonetheless generated the sharpest intersection point from the 3 intersecting hyperboloids
- Mis-locations are due to the TDOA measurements ($d_{ij}$) having too large an error ε




○ = target ground truth (x,y)

♦ = computed location (x,y)

- **Reducing TDOA error to ε = 0.15 m (from 1.5)**
- **β = 20 MHz, SNR = 32 dB**




○ = ground truth

♦ = computed target location (x,y)

computed target altitude (z)

| | target altitude (m) | | | | | | |
|---|---|---|---|---|---|---|---|
| | T#5 | T#3 | T#7 | T#1 | T#6 | T#2 | T#4 |
| time | | | | | | | |
| 1 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 |
| 2 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 |
| 3 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 |
| 4 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 |
| 5 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 |
| 6 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 |
| 7 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 |
| 8 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 |
| 9 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 |
| 10 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 1000.00 |

actual target altitude = 1000 m

For multi-target localization, the TDOA error ε should be kept small to minimize mis-locations

# Geometric approach to achieve real-time multi-target localization?

- The geometry-based solution is not real-time
- Most of the computing time is spent on the hyperboloids

| $n$ (no. of targets detected) | $t$ (non-coplanar receiver configuration) |
|---|---|
| 1 | 18.6s |
| 3 | 56.1 |
| 7 | 147.1 |
| 10 | 236.0 |

per time instant of sampling

- Real-time via the geometric method: each of the 3 TDOA Equations is solved individually (i.e., computing a hyperboloid)

$$d_{12} = r_1 - r_2 \rightarrow \text{hyperboloid1}$$

$$d_{34} = r_3 - r_4 \rightarrow \text{hyperboloid2}$$

$$d_{14} = r_1 - r_4 \rightarrow \text{hyperboloid3}$$

- The hyperboloids can be pre-computed for a range of different $d_{ij}$ values for each of the 3 TDOA equations and stored as look-up tables to save considerable computing time

# Approach to real-time multi-target processing

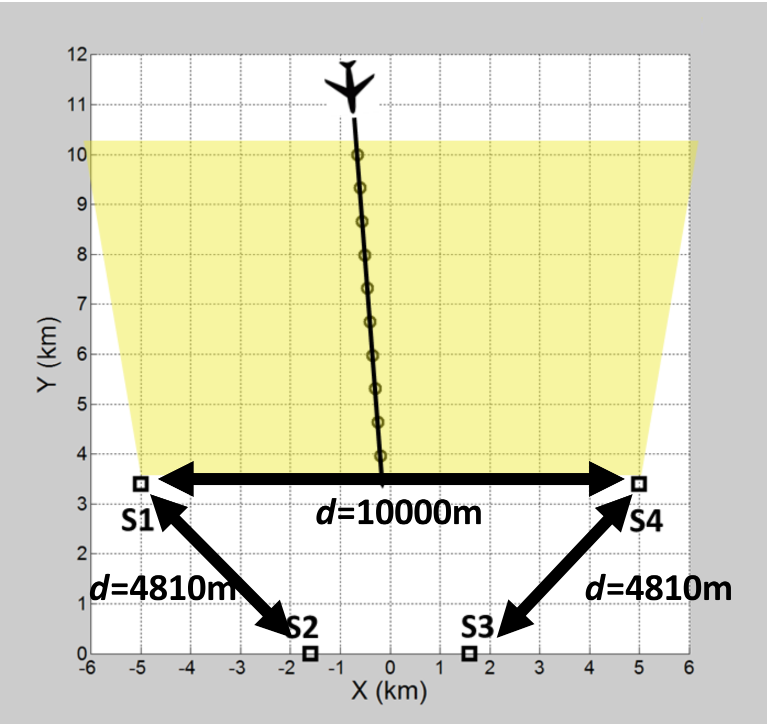
$$d_{ij} = r_i - r_j$$

$$\frac{x'^2}{(d_{ij}^2/4)} - \left( \frac{y'^2}{(d^2/4)-(d_{ij}^2/4)} + \frac{z'^2}{(d^2/4)-(d_{ij}^2/4)} \right) = 1$$

Each single TDOA equation has
a hyperboloid as solution

- *d* is the known separation distance between a pair of receivers
- $-d < d_{ij} < d$
- For a given TDOA error $\varepsilon$ , there are $(2d/\varepsilon + 1)$ possible $d_{ij}$ values
- $(2d/\varepsilon + 1)$ hyperboloids can be pre-computed and stored as look-up tables

# Number of look-up tables for the hyperboloids




**Assume FPV transmitter power = 500 mW**

TDOA error: $\varepsilon$ = 1.5m
(σ parameters: β=20 MHz, SNR=12dB)
+
detection system size with *d* as shown on the left

\# of hyperboloids = (2*d*/$\varepsilon$ +1)
13334 (**S1-S4**)
6417 (**S1-S2**)
6417 (**S3-S4**)
=
26168 (total)
hyperboloids to be pre-computed and stored as look-up tables; each corresponds to a specific $d_{ij}$ value.
This total is applicable to any ***n***-target scenarios, as long as the correlator can resolve 2 targets to within $\varepsilon$.

# How multi-target localization in real-time could be achieved

- Use look-up tables
  - Large data storage capacity and fast data retrieval algorithms make this viable
- Apply parallel computing algorithms
  - The $n^3$ permutation is highly parallel in computing structure

- Using both look-up tables and multi-core parallel computing, real-time (~ 1s) multi-target localization may be realizable

# Thank you

# Computing time: coplanar vs non-coplanar

*Table 5.10: Computation time consumed in target localization processing for different number of targets detected using sequential processing.*

| n (no. of targets detected) | *t* (coplanar configuration) | *t* (non-coplanar configuration) |
|---|---|---|
| 1 | 0.4s | 18.6s |
| 3 | 2.8 | 56.1 |
| 7 | 30.8 | 147.1 |
| 10 | 89.5 | 236.0 |

# Parallel structure in permutation

| | Target #1 | Target #2 |
|---|---|---|
| S1-S2 | A | D |
| S3-S4 | B | E |
| S1-S4 | C | F |

permutations:

**ABC**
ABF
AEC
AEF
DBC
DBF
DEC
**DEF**

Numerical method and closed-form solutions need to solve 3 TDOA equations simultaneously

$$d_{12} = c\tau_{12} = r_1 - r_2$$
$$d_{34} = c\tau_{34} = r_3 - r_4$$
$$d_{14} = c\tau_{14} = r_1 - r_4$$

Pre-computing needs combinations of 3 $d_{ij}$ values as one single set.
The no. of permutated sets required 6417x6417x13334≈$5x10^{11}$

# Coplanar receiver configuration

- 4 receivers are located at the same $z = 0$ level
- $\beta = 1$ MHz, SNR = 16, $\varepsilon = 30$ m

Table 4.4

| | Target ground truth (m) | | | Computed target location (m) | | |
|---|---|---|---|---|---|---|
| Time (arb.unit) | $X_{Tg}$ | $Y_{Tg}$ | $Z_{Tg}$ | $x$ | $y$ | $z$ |
| 1 | -660.00 | 9998.50 | 1000.00 | -649.17 | 9889.12 | 0 |
| 2 | -608.44 | 9328.48 | 1000.00 | -700.88 | 9881.41 | 3660.00 |
| 3 | -556.89 | 8658.46 | 1000.00 | -803.17 | 10144.20 | 6610.00 |
| 4 | -505.33 | 7988.44 | 1000.00 | -490.19 | 7932.89 | 0 |
| 5 | -453.78 | 7318.42 | 1000.00 | -438.48 | 7262.07 | 0 |
| 6 | -402.22 | 6648.40 | 1000.00 | -391.61 | 6549.75 | 0 |
| 7 | -350.67 | 5978.38 | 1000.00 | -485.88 | 6316.99 | 5650.00 |
| 8 | -299.11 | 5308.36 | 1000.00 | -292.53 | 5252.69 | 0 |
| 9 | -247.56 | 4638.34 | 1000.00 | -242.15 | 4595.37 | 0 |
| 10 | -196.00 | 3968.32 | 1000.00 | -187.34 | 3950.14 | 0 |

# Coplanar receiver configuration

- 4 receivers are located at the same $z = 0$ level
- $\beta$ = 20 MHz, SNR = 16, $\varepsilon$ = 1.5 m

Table 4.5

| | Target ground truth (m) | | | Computed target location (m) | | |
|---|---|---|---|---|---|---|
| Time (arb.unit)\ | $X_{Tg}$ | $Y_{Tg}$ | $Z_{Tg}$ | $x$ | $y$ | $z$ |
| 1 | -660.00 | 9998.50 | 1000.00 | -659.84 | 9994.90 | 1080.00 |
| 2 | -608.44 | 9328.48 | 1000.00 | -609.10 | 9332.24 | 1130.00 |
| 3 | -556.89 | 8658.46 | 1000.00 | -561.09 | 8678.75 | 1270.00 |
| 4 | -505.33 | 7988.44 | 1000.00 | -504.69 | 7981.44 | 1020.00 |
| 5 | -453.78 | 7318.42 | 1000.00 | -461.56 | 7355.42 | 1520.00 |
| 6 | -402.22 | 6648.40 | 1000.00 | -403.99 | 6653.53 | 1150.00 |
| 7 | -350.67 | 5978.38 | 1000.00 | -353.01 | 5984.02 | 1250.00 |
| 8 | -299.11 | 5308.36 | 1000.00 | -293.74 | 5301.20 | 0 |
| 9 | -247.56 | 4638.34 | 1000.00 | -258.45 | 4619.69 | 1900.00 |
| 10 | -196.00 | 3968.32 | 1000.00 | -193.72 | 3976.66 | 720.00 |